M. S. Chen and Professor G. L. Kane are appreciated.

*Work supported in part by the U. S. Energy Research and Development Administration.

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[3]It is possible that by choosing a particular symmetry group, the sum and average procedure is too restrictive. However, this is not the emphasis here. I do want to point out that rates for a reaction channel with specific quantum numbers are in general not finite. Take SU(2) as an example. Call the fermions $p$ and $n$ and the gauge mesons $\rho^+$, $\rho^-$, and $\rho^0$. It is a simple exercise to show that to the order considered here the rates for $p+p\rightarrow p+p$, $p+p\rightarrow p+n+\rho^+$, and $p+p\rightarrow p+p+\rho^0$ do not sum up to a finite result. In this case the channel quantum numbers are $I=1$, $I_z=1$. This peculiar feature bears on what we mean by a proton, for example, when it can emit and absorb charged soft gauge mesons at any time.

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# Energy Dependence of $^{51}$V($d$, $^3$He)$^{50}$Ti; Implications for Distorted-Wave-Born-Approximation Analysis and Rearrangement*

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(Received 22 September 1975)

The differential cross section as a function of energy has been carefully measured and analyzed for the ground and first three excited states in the reaction $^{51}$V($d$, $^3$He)$^{50}$Ti. The results show no evidence for rearrangement effects. Finite-range, distorted-wave Born-approximation calculations, which are necessary to fit the data, yield spectroscopic factors about 0.6 of those expected on the basis of an $(f_{7/2})^3$ description of $^{51}$V and a conventional ($d$, $^3$He) normalization.

We have performed a careful measurement and a detailed distorted-wave Born-approximation (DWBA) analysis of the single-particle transfer reaction $^{51}$V($d$, $^3$He)$^{50}$Ti, as a function of the incident deuteron energy. Particular emphasis was placed on the proton pickup configuration reached by $l_p=3$ transfers which populate the first four states of $^{50}$Ti of $J^\pi=0^+$, $2^+$, $4^+$, and $6^+$. The measurements were made at 30 and 80 MeV and the data of Hinterberger *et al.*[1] at 52 MeV were reanalyzed in a manner consistent with the analysis of our own data.

This careful study was motivated by the desire to test the applicability of the DWBA analysis over a wide range of incident energies. In particular, we were interested in the possibility that rearrangement effects[2,3] (hole-state lifetime effects) might be observed as energy-dependent spectroscopic factors deduced from the DWBA analysis. Should the reaction tend away from the sudden limit, interactions which result in "rearrangement" could redistribute the spectroscopic strengths among those states which are excited in the reaction. In the present analysis we have centered attention on the energy dependence of the relative spectroscopic factors (ratio of the excited-state spectroscopic factor to the ground-state spectroscopic factor) since these quantities can be measured more accurately than absolute spectroscopic factors and the rearrangement which occurs may be very small. Furthermore ratios are much less sensitive to the details of the manner in which the DWBA is used.

The experiments were carried out using con-

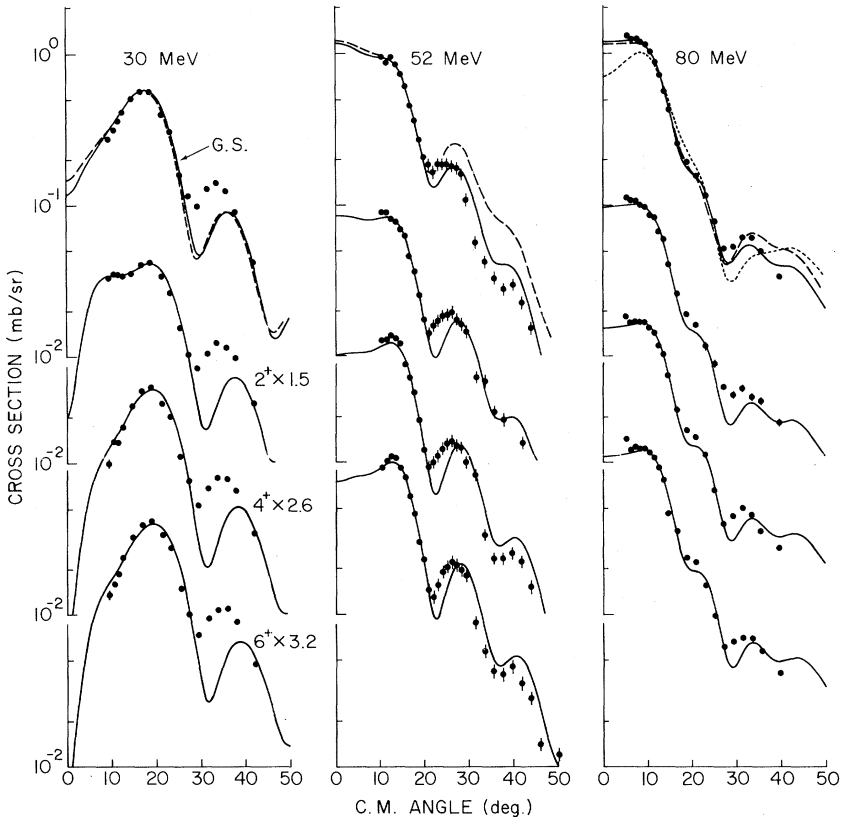


FIG. 1. The differential cross sections for the reaction $^{51}$V($d$, $^{3}$He)$^{50}$Ti to the ground state and first three excited states of $^{50}$Ti. The data at 30 and 80 MeV are from this work and the data at 52 MeV are from Ref. 1. The errors in the cross-section values are discussed in the text and the statistical errors shown are for those data points for which the error exceeds the size of the plotted points. The solid curves shown are for the best-fit DWBA calculations (in Table I). The dashed curves illustrate the zero-range, local calculations at 52 and 80 MeV. The dotted curve at 80 MeV shows the effect of using the so-called unique shallow potential for the elastic scattering. Further tests and details are given in Ref. 4.

ventional solid-state techniques for particle identification and energy measurement. As the minimum spacing of the levels of interest in $^{50}$Ti is 0.53 MeV no particular effort had to be made to achieve the overall energy resolution necessary to separate cleanly the desired levels such that the relative cross sections could be measured to an accuracy of approximately 3%. Care was taken to assure that no systematic errors were introduced by the particle identification scheme. The largest sources of error in the absolute-cross-section determinations were target thicknesses, detector solid angles, and statistical uncertainties. The overall absolute error is evaluated to be ± 6% for both the 30- and 80-MeV data. Figure 1 shows the observed differential cross sections and best-fit analysis of the data.[4] Space limitations prevent giving the optical-model parameters used in the DWBA calculations (they are however from previous works[5-9]). Note in Fig. 1 that the so-called deep potential gives a much better fit to the data.[9]

The data have been analyzed, mainly using DWUCK4.[10] At lower energies it has been shown by several groups[1,11,12] that finite-range (FR) effects [in the local energy approximation (LEA)] do not change the shape of the calculated curves but do affect the normalization and therefore the deduced spectroscopic factors. Previous zero-range (ZR) analyses of this reaction at 28, 34, and 52 MeV yielded spectroscopic factors consistent with that expected on the basis of the $^{51}$V ground state as being largely $(f_{7/2})^3$. Recent shell-model calculations[13] indicated a ground-state configuration at least 96% $(f_{7/2})^3$ with the remaining amplitudes being in the main $f_{7/2}(p_{3/2})^2$ and $(f_{7/2})^2 p_{3/2}$.

Table I summarizes the results of our calculations. When the normalization of the DWBA calculation is adjusted to fit the data by a least-

TABLE I. The absolute and relative spectroscopic factors deduced from various DWBA calculations at 30, 52, and 80 MeV. The number in parentheses below each entry is the $\chi^2/N$ obtained by the fitting procedure over the entire angular range. The errors shown for the spectroscopic factors reflect experimental errors and errors in the fitting procedure and in no way reflect the large uncertainties as to the best recipes to use theoretically. The best fit at each energy is indicated by B.F. ZRL = zero-range local; FRL = finite-range local; FRNL = finite-range nonlocal.

| | $0^+$ | $2^+$ | $4^+$ | $6^+$ | $2^+/0^+$ | $4^+/0^+$ | $6^+/0^+$ |
|---|---|---|---|---|---|---|---|
| $(f_{7/2})^3$ | 0.75 | 0.42 | 0.75 | 1.08 | 0.56 | 1.00 | 1.44 |
| | | | 80 MeV | | | | |
| ZRL | 0.62±.01 (18.6) | 0.33±.01 (14.4) | 0.65±.01 (18.0) | 1.01±.02 (24.9) | 0.53±.01 (3.5) | 1.06±.01 (2.5) | 1.65±.01 (8.6) |
| FRL | 0.54±.01 (9.5) | 0.29±.01 (7.2) | 0.57±.01 (11.4) | 0.87±.01 (26.6) | 0.525±.010 (3.11) | 1.04±.01 (3.65) | 1.62±.01 (8.6) |
| B.F. FRNL (optical potentials only) | 0.49±.01 (7.4) | 0.26±.01 (10.8) | 0.51±.01 (9.4) | 0.79±.01 (14.0) | 0.525±.01 (3.6) | 1.04±.01 (2.6) | 1.62±.01 (7.6) |
| FRNL (all potentials) | 0.35±.01 (7.5) | 0.18±.01 (11.6) | 0.35±.01 (10.8) | 0.54±.01 (15.5) | 0.515±.01 (3.7) | 1.01±.01 (2.5) | 1.56±.01 (7.7) |
| FRNL (no s-o deuteron pot) | 0.36±.01 (11.4) | 0.18±.01 (11.3) | 0.36±.01 (13.4) | 0.56±.01 (23.2) | 0.52±.01 (4.2) | 1.02±.01 (2.8) | 1.68±.01 (7.9) |
| Exact FRNL | 0.40±.01 (11.6) | 0.21±.01 (13.0) | 0.42±.01 (13.4) | 0.66±.01 (21.9) | 0.53±.01 (4.3) | 1.05±.01 (2.8) | 1.65±.01 (7.8) |
| | | | 52 MeV | | | | |
| ZRL | 0.74±.02 (34.2) | 0.41±.02 (31.0) | 0.79±.01 (51.0) | 1.35±.01 (45.0 | 0.56±.01 (1.50) | 1.00±.05 (4.1) | 1.75±.10 (3.9) |
| B.F. FRNL (all potentials) | 0.41±.02 (5.8) | 0.23±.01 (5.5) | 0.42±.04 (8.9) | 0.73±.04 (11.6) | 0.56±.01 (1.6) | 1.00±.01 (2.9) | 1.80±.10 (3.5) |
| | | | 30 MeV | | | | |
| ZRL | 0.84±.02 (181) | 0.44±.02(ℓ=3) 0.014 .01(ℓ=1) (92) | 0.87±.02 (129) | 1.32±.01 (188) | 0.53±.01(ℓ=3) 0.016±.005(ℓ=1) (.20) | 1.05±.02 (0.24) | 1.58±.02 (0.4) |
| B.F. FRNL (all potentials) | 0.45±.02 (185) | 0.23±.01(ℓ=3) 0.008±.002(ℓ=1) (90) | 0.45±.01 (119.6) | 0.68±.01 (175) | 0.52±.01(ℓ=3) 0.015±.005(ℓ=1) (0.20) | 1.02±.02 (0.3) | 1.50±.10 (0.5) |

squares procedure, the best fit (lowest $\chi^2/N$) results from a FRNL calculation. In Fig. 1 we also illustrate several of the possibilities used in attempting to judge the best fit to the 80-MeV data. It should be appreciated that the statistical errors are less than the size of the points shown. As can be seen in Fig. 1, the smallest-angle datum point for all the levels is abruptly high. While we believe that this is related to an experimental, small-angle problem, we could not establish this conclusively and so we have shown the data. The other point to be kept in mind is that the various other calculations shown in Fig. 1 lead to different spectroscopic factors, and that the use of a ZRL calculation in the analysis would cause a roughly 25% variation in the energy dependence of the deduced spectroscopic factor for all states. Such an energy dependence, affecting all four states similarly, is not an effect of the type which we are calling a rearrangement effect. It is presumably a result of an inadequate calculation of the DWBA reaction. The introduction of the nonlocality correction also improves the fit to the data.

We have also compared the LEA with an exact FR calculation[14] and it was found that the exact FR calculation produced a 15% or less increase, relative to the LEA, in the spectroscopic factors. The exact FR calculation was carried out using a Gaussian form for the range function, in the

notation of Ref. 14:

$$V(s)Y_{00}(\hat{s}) = D_0(\pi R^2)^{-3/2}\exp(-s^2/R^2),$$

where $D_0 = -172.8$ MeV fm$^{3/2}$ and $R = 1.54$ fm.[16] This is equivalent to a ZR calculation in which $V(s)Y_{00}(\hat{s}) = D_0\delta(\vec{s})$.

We thus conclude that requiring the best fit to the 80-MeV data necessitates a FRNL calculation and similar conclusions are reached for the 52-MeV data. At 30 MeV there is not such a marked preference for FRNL calculations; however, FRNL calculations yield spectroscopic factors consistent with those found at 52 and 80 MeV. There is a slight trend which can be noted in Table I for the spectroscopic factors from similar FRNL calculations in the LEA to decrease with increasing energy. We believe that this effect is a result of the incorrect energy dependence of the LEA, and that a correct treatment of the finite-range integral will eliminate this apparent energy dependence.[14] That is, the results in Table I are calculated within the LEA approximation except for the test calculations using the exact finite-range formulation. These calculations increase the spectroscopic factor less than 20% at 80 MeV. At 30 MeV the differences between the LEA and the exact FR calculations are negligible. (The exact FR calculations were carried out with no spin-orbit term in the optical potentials and the measure of the FR effects upon which we base our belief was deduced by comparison to LEA calculations which also included no spin-orbit potential.)

A number of tests of the sensitivity of the deduced spectroscopic factors to the parameters of the calculation were carried out at each energy and it was found that the results were insensitive to such parameter variations. The only parameters which showed a significant sensitivity were the radius of the bound-state potential and the separation energy. We believe that we have used as small a radius for the bound-state well as is conventionally assumed ($r_0 = 1.20$ fm compared to $r_0 = 1.25$ fm). Use of the larger radius would decrease the spectroscopic factors even further, while not changing the shape of the angular distribution. We also found that to maintain a good fit to the data it is necessary to use the exact separation energy for each state.[15]

We conclude that these results indicate a discrepancy of the order of 40% in the extraction of spectroscopic factors by DWBA analyses in the LEA approximation as described in Ref. 10 with the normalization and range parameters of Bassel.[16] It is clear that a significant fraction of this discrepancy can arise from an incorrect normalization $D_0$.[14] It is not surprising that FR effects become more important as the energy increases. The constancy of the discrepancy of the spectroscopic factors with energy suggests either a failure, in the sense above, of DWBA for this reaction[17] or effects arising from other than the simple $(f_{7/2})^3$ configuration that we assumed for the ground state of $^{51}$V. It is necessary that such effects do not change the shape of the form factor for the pickup reaction in the tail region but only its magnitude. The incorrect spectroscopic factors may also arise because of the neglect of strongly coupled channels as might be incorporated in a coupled-channels Born-approximation calculation, although the constancy with energy of the spectroscopic factors and the ratios of cross sections, as seen by examination of Table I, would seem to argue against such an interpretation.

Finally, this experiment shows no evidence for rearrangement effects of the type suggested by Brueckner, Meldner, and Perez.[2] The basis for this statement is that the ratios of the cross sections are essentially independent of energy and the specific prescription for the DWBA calculation, as is shown in Table I. It may be that rearrangement effects are subtly incorporated into the DWBA parametrization. This point has been emphasized by Friedman[3] in his remark "that reaction models must 'undo' the rearrangement if correct spectroscopic factors are to be found."

We would like to acknowledge many critical and useful discussions with N. S. Chant. We would like to thank Professor P. D. Kunz for making his code DWUCK available to us and explaining some of its intricacies. A nontrivial amount of computer time was supplied by the Computer Science Center of the University of Maryland.

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*Work supported by the U. S. Energy Research and Development Administration.

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‡Guest scientist.

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# Microscopic Approach to the Monopole and Quadrupole Giant Resonances

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(Received 2 December 1975)

We present a microscopic approach to the monopole and quadrupole isoscalar giant resonances based on appropriate sum rules. The energies of both modes contain the same model-independent contribution ($\sqrt{2}\hbar\omega$) and a contribution involving antisymmetrization effects and the nuclear potential. Several potentials are considered and the results of many other approaches are derived in a simple and compact way.

Recent systematic measurements on giant multipole resonances have stimulated a massive effort towards the theoretical exploration of the monopole and quadrupole modes.[1-4] The simple results of the various collective hydrodynamical models (Copenhagen approach) have been compared with calculations made using Skyrme interactions and density-dependent forces.[5-9]

The aim of this Letter is to illustrate a simple and as far as possible exact microscopic treatment of the monopole and quadrupole isoscalar energies based on the fundamental nucleon-nucleon interaction currently used in nuclear spectroscopy. A comparison is made of our results with the previous ones.

Our treatment is based on the well-known first-energy-weighted sum rules[10,11] and on the third-energy-weighted sum rules, not yet explored. Let the isoscalar quadrupole and monopole operators be

$$Q = \tfrac{1}{2}\sum_i y_i z_i \quad \text{and} \quad M = \tfrac{1}{2}\sum_i r_i^2,$$

and let the nuclear Hamiltonian be

$$H = \sum_i p_i^2/2m + \sum_{i<j} V_{ij},$$

where $V_{ij}$ is a realistic two-body *local* potential ($V_{ij} = V_{\text{central}} + V_{\text{tensor}} + V_{\text{Coulomb}}$). We study the quantities

$$S_3^X = \langle 0|[[[X, H], H], [H, X]]|0\rangle, \tag{1}$$

where $X$ is either $Q$ or $M$; and combining $S_3^X$ with

$$S_1^X = \langle 0|[X, [H, X]]|0\rangle, \tag{2}$$

we study the quantity

$$E_X = (S_3^X/S_1^X)^{1/2}. \tag{3}$$

We will compare $E_X$ with the results of other different approaches.

Let us just remember in passing that when $X = Q$, then

$$S_3^Q = [2(\hbar c)^3/\pi^2 e^2]\int \sigma^{SE2}(\omega)\, d\omega = \sigma_0^{SE2};$$

i.e., $S_3^Q$ is proportional to the integrated *isoscalar* quadrupole photo cross section. Similarly

$$S_1^Q = [2(\hbar c)^3/\pi^2 e^2]\int [\sigma^{SE2}(\omega)/\omega^2]\, d\omega = \sigma_{-2}^{SE2}.$$

Other sum rules could be used, in principle, to obtain information on the energy of these modes; one could for example study $E_X' = S_2^X/S_1^X$ or $E_X'' = S_1^X/S_0^X$. It happens, however, that in the calculations of $S_2^X$ and $S_0^X$ we are faced with very complicated spatial correlations ($S_0^X$) or with complicated correlations between coordinates and momenta ($S_2^X$). Finally one could analyze