Infrared Problem in Non-Abelian Gauge Theory*

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I extend the Bloch-Nordsieck idea to show that in the lowest nontrivial order of radiative correction the fermion-fermion and gauge-meson–fermion scattering rates are finite, provided that they are averaged over the initial and summed over the final internal spin states. Questions of the physical gauge coupling and infrared slavery are discussed.

An outstanding problem in the study of non-Abelian gauge theory[1] has to do with its behavior in the infrared region. Just as in quantum electrodynamics (QED), there are in particular two aspects which are related to the existence of massless particles in the theory. One deals with the long-range nature of the force, which cannot be ascertained definitively by perturbation expansion. The other has to do with the emission and absorption of soft gauge particles. This, in its most fundamental way, addresses the following question: What are the relevant physical matrix elements in the theory?

In this Letter, I will focus attention on the latter problem, which is amenable to perturbation analysis. Let us recall that in QED, the answer is obtained by use of the Bloch-Nordsieck model, wherein each physical electron is accompanied by an indefinite number of soft photons. It turns out that in non-Abelian gauge theory, at least to the lowest nontrivial order that I have looked into so far, this same approach with some modifications also solves the problem.

The necessary modifications are not unexpected. First of all, if to each particle carrying internal quantum numbers there must be attached an indefinite number of soft gauge particles carrying the same kind of quantum numbers, it is perhaps not surprising to learn that the only meaningful physical matrix elements are rates with initial and final states averaged and summed over in the internal symmetry space. This I find to be the case. It should be noted that the "charges" under consideration are the color charges in the current parlance. They are not observable charges.

The next necessary consideration has to do with subtraction of ultraviolet infinities. Again we fall back on QED. There, in the Feynman gauge, if the subtraction is done on shell, then new infrared divergences are introduced to both the electron wave-function renormalization and the vertex renormalization. However, because of the Ward identity, these extraneous infinities cancel out. In non-Abelian gauge theory, we do not have such a simple identity, and we must take note of this fact so as not to introduce spurious infinities.

In the following, I will briefly describe the processes investigated. Details will be published elsewhere.

The Lagrangian consists of a multiplet of fermion fields and a set of gauge fields,

$$\mathcal{L} = -\tfrac{1}{4}(\partial^\mu A_a{}^\nu - \partial^\nu A_a{}^\mu + g\epsilon_{abc}A_b{}^\mu A_c{}^\nu)^2 - \bar{\psi}[\gamma_\mu(-i\,\partial^\mu - \tfrac{1}{2}g\tau_a A_a{}^\mu) + m]\psi, \tag{1}$$

where the τ's act on the fermion internal index and satisfy

$$[\tfrac{1}{2}\tau_a, \tfrac{1}{2}\tau_b] = i\epsilon_{abc}\tau_c/2. \tag{2}$$

The ϵ's are real and totally antisymmetric structure constants of the internal group.

We work in the Feynman–'t Hooft gauge. All integrals are regulated[2] with respect to both ultraviolet and infrared divergences by dimensional continuation ($4 \to n$). Specifically, we look into the lowest nontrivial radiative correction to the rates of fermion-fermion and gauge-meson–fermion scattering. They are described separately.

Fermion-fermion scattering.—There are three kinds of corrections, summarized in Fig. 1.

(a) The net effect of the fermion self-energy is to multiply each external electron line by a factor

$$1 + \tfrac{1}{2}(\tfrac{1}{2}g\tau_a)^2\pi^{n/2}(2\pi)^{-4}\int d\alpha_1\, d\alpha_2\, \delta(1-\alpha_1-\alpha_2)\{\Gamma(3-\tfrac{1}{2}n)\alpha_1\alpha_2 2m[2m+(n-2)\alpha_1 m](\alpha_1{}^2m^2)^{-3+n/2} - \Gamma(2-\tfrac{1}{2}n)(n-2)(1-\alpha_1)(\alpha_1{}^2m^2)^{-2+n/2}\}. \tag{3}$$

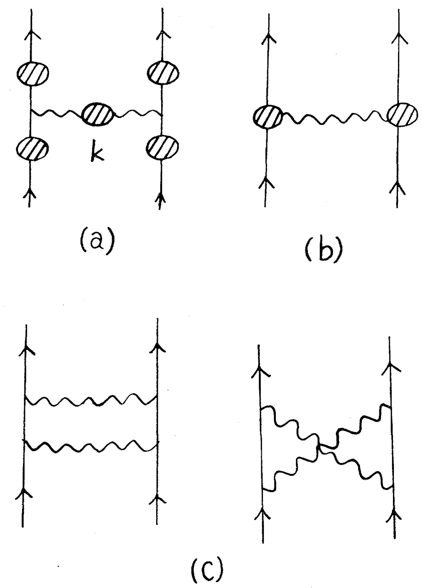

FIG. 1. Three types of radiative corrections to fermion-fermion scattering. The diagonally lined circles denote corrections.

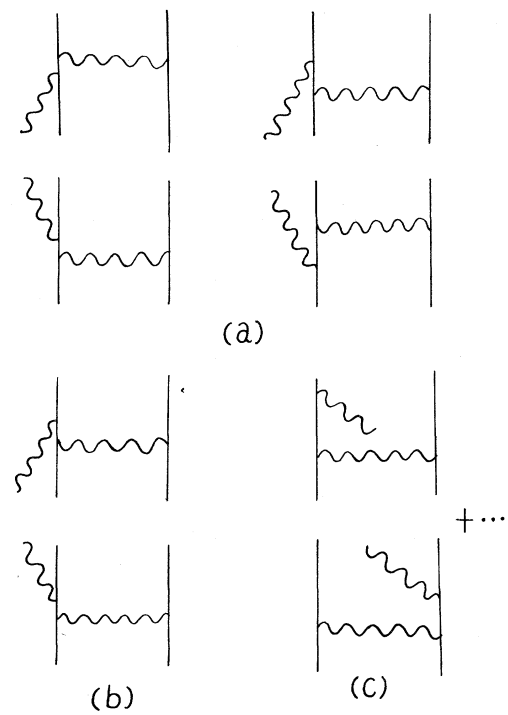

FIG. 2. Soft gauge-meson (gluon) emission in fermion-fermion collision. Each diagram here denotes a term in the square of the production amplitude with the corresponding topology.

The second term in the angular brackets has an ultraviolet divergence, whereas the first term has an infrared singularity.

It is easy to show that the latter singularity is canceled by the soft-emission diagrams of Fig. 2(a) plus the left-to-right reflected diagrams, provided that we average and sum over the initial and final spin variables, respectively.[3]

The gauge-meson vacuum polarization due to fermion loops is infrared finite. Gauge mesons and Faddeev-Popov ghosts give

$$\Pi_{ab}{}^{\mu\nu}(k) = (g^{\mu\nu}k^2 - k^\mu k^\nu)\tfrac{1}{2}(-ig\epsilon_{acd})(-ig\epsilon_{bdc})\pi^{n/2}(2\pi)^{-4}\Gamma(2-\tfrac{1}{2}n)$$
$$\times\int d\alpha_1 d\alpha_2\,\delta(1-\alpha_1-\alpha_2)(2+8\alpha_1{}^2+n-4n\alpha_1{}^2)/(\alpha_1\alpha_2k^2)^{-2+n/2}. \qquad (4)$$

The divergence when n approaches 4 is ultraviolet in origin. Note also the bad behavior as the square of the momentum transfer k^2 approaches 0. This has to do with both the range of binding, which is not being considered, and mass singularities, which will be discussed below.

(b) The vertex correction due to fermion-fermion-gluon coupling is

$$\Lambda_a{}^\mu(k)_{FFV} = (\tfrac{1}{2}g)^3\pi^{n/2}(2\pi)^{-4}\int d\alpha_1\, d\alpha_2\, d\alpha_3\,\delta(1-\alpha_1-\alpha_2-\alpha_3)$$
$$\times\{N_a{}^\mu[m^2(\alpha_1+\alpha_3)^2+k^2\alpha_1\alpha_3]^{-3+n/2}$$
$$+\tfrac{1}{2}(n-2)^2\Gamma(2-\tfrac{1}{2}n)[m^2(\alpha_1+\alpha_3)^2+k^2\alpha_1\alpha_3]^{-2+n/2}$$
$$\times\bar{u}(p+k)\tau_b\tau_a\tau_b\gamma^\mu u(p)\}, \qquad (5)$$

with

$$N_a{}^\mu = \bar{u}(p+k)\big(\{-4m^2[1-(\alpha_1+\alpha_3)-\tfrac{1}{2}(\alpha_1+\alpha_3)^2]-2k^2(1-\alpha_1)(1-\alpha_3)\}\gamma^\mu+2m(1-\alpha_1-\alpha_3)\alpha_1[\gamma^\mu,\gamma\cdot k]$$
$$+(n-4)\{[(\alpha_1+\alpha_3)^2m^2-\alpha_1\alpha_3k^2]\gamma^\mu-(\alpha_1+\alpha_3)\alpha_1[\gamma^\mu,\gamma\cdot k]\}\big)\tau_b\tau_a\tau_b u(p). \qquad (6)$$

The first term in Eq. (5) has an infrared singularity, which is tamed by that of Fig. 2(b), together with their left-to-right and up-to-down mirror terms.

Although the correction from three-vector coupling is finite in the infrared limit, I prefer to write it down so as to point out a certain feature,

$$\Lambda_a{}^{\mu}(k)_{VVV} = (-ig^3/4)\pi^{n/2}(2\pi)^{-4}\int d\alpha_1 d\alpha_2 d\alpha_3\,\delta(1-\alpha_1-\alpha_2-\alpha_3)$$
$$\times[\overline{N}_a{}^{\mu}(\alpha_2{}^2m^2+\alpha_1\alpha_3k^2)^{-3+n/2}\Gamma(3-n/2)$$
$$+(2n-2)\Gamma(2-n/2)(\alpha_2{}^2m^2+\alpha_1\alpha_3k^2)^{-2+n/2}$$
$$\times\epsilon_{abc}\overline{u}(p+k)\tau_b\tau_c\gamma^{\mu}u(p)], \qquad (7)$$

with

$$\overline{N}_a{}^{\mu} = \overline{u}(p+k)\big(\{[-2(n-4)\alpha_2{}^2-6\alpha_2{}^2]m^2+2(\alpha_1+\alpha_3-\alpha_1\alpha_3)k^2\}\gamma^{\mu}$$
$$-\alpha_2[\alpha_1+\alpha_3+(2-n/2)\alpha_2][\gamma^{\mu},\gamma\cdot k]m\big)\epsilon_{abc}\tau_b\tau_c u(p). \qquad (8)$$

Note that when $k^2=0$, the induced magnetic moment is singular. This is a general feature of even the static quantities when mass singularity proliferates.

(c) The infrared singularities that originate from multigluon exchange can be shown to match those in the soft-emission graphs of Fig. 2(c), upon summing and averaging over final and initial internal spin states, respectively.

Gluon-fermion scattering.—An added complication must be dealt with immediately. Since we now have the possibility of some external massless particles going through intermediate states which consist of only massless particles which propagate in the same direction, we run into the mass-singularity problem.[4] I prefer to discuss this at a later time.[5]

To simplify algebra, we will consider rates in which the spatial spin of the final gluon is summed over. The types of radiative correction in this process are similar to those of Fig. 1, if we replace one of the two external fermions by a gluon. Now, the gluon self-energy has no infrared difficulty [see Eq. (4)]. The singularities induced by multigluon exchange [Fig. 1(c)] are compensated by those in Fig. 2(c).

Three-vector vertex corrections via a fermion loop are well behaved in the infrared limit. The correction due to gauge fields and ghosts has the following form:

$$\Gamma_a{}^{\nu}(k) = \epsilon_{a_2}{}^{\mu_2\dagger}(p+k)\epsilon_{a_1}{}^{\mu_1}(p)(-ig\epsilon_{c_1ac_3})(-ig\epsilon_{a_1c_1c_2})(-ig\epsilon_{a_2c_2c_3})\pi^{n/2}(2\pi)^{-4}$$
$$\times\int d\alpha_1 d\alpha_2 d\alpha_3\,\delta(1-\alpha_1-\alpha_2-\alpha_3)(\alpha_1\alpha_3k^2)^{-3+n/2}\Gamma(2-n/2)$$
$$\times\{\tfrac{1}{2}[2(g_{\mu_1}{}^{\nu}k_{\mu_2}-g_{\mu_2}{}^{\nu}k_{\mu_1})\pm g_{\mu_1\mu_2}(k+2p)^{\nu}]k^2A_{\pm}+k_{\mu_1}k_{\mu_2}(k+2p)^{\nu}B\}, \qquad (9)$$

where $A_{\pm}$ and B are functions of $\alpha_{1,3}$ and n, which I do not display here to save space. As A_- and B are coefficients of induced moments, they are finite in the ultraviolet limit. A_-, like the fermion magnetic moment, cannot be defined for $k^2=0$. In the infrared limit,

$$A_+ \simeq -\tfrac{5}{2}(2-n/2), \quad A_- \simeq \tfrac{1}{2}(2-n/2). \qquad (10)$$

$\Gamma_a{}^{\nu}(k)$ is therefore divergent, but this divergence is removed by terms with similar behavior in Fig. 2(b).

In summary, what has been shown is that with some extension of the Bloch-Nordsieck idea, infrared-finite matrix elements can be constructed for non-Abelian gauge theory.

In the course of this investigation, I have been particularly troubled by one issue, which stems from the subtraction procedure. It is evident that the rates are finite only with respect to the bare coupling constant g or with respect to some renormalization procedure which amputates only the ultraviolet infinities. This does not provide an operational definition of the physical gauge charge g_{phys}. In other words, what is the equivalence of the Thomson limit in non-Abelian gauge theory? An answer to this may well be a true dynamical revelation.

Let us touch on the subject of infrared slavery[6] in view of the present analysis. In my opinion, if quarks and gluons are ultimately shown to be enslaved, it must be for reasons of binding. I see no inconsistency in the idea that suitably constructed coherent states can exist for them.

I have learned that an analysis along the same general direction has been independently carried out by T. Appelquist, J. Carrazone, H. Kluberg-Stern, and M. Roth. I would like to thank Professor Appelquist for sharing some thoughts on this subject. Encouragement and interest from Dr.

M. S. Chen and Professor G. L. Kane are appreciated.

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[1]C. N. Yang and R. L. Mills, Phys. Rev. 96, 191 (1954).

[2]G. 't Hooft and M. Veltman, Nucl. Phys. B44, 189 (1972); for examples of infrared isolation, see W. J. Marciano, to be published, and references therein.

[3]It is possible that by choosing a particular symmetry group, the sum and average procedure is too restrictive. However, this is not the emphasis here. I do want to point out that rates for a reaction channel with specific quantum numbers are in general not finite. Take SU(2) as an example. Call the fermions p and n and the gauge mesons ρ^+, ρ^-, and ρ^0. It is a simple exercise to show that to the order considered here the rates for $p+p\rightarrow p+p$, $p+p\rightarrow p+n+\rho^+$, and $p+p\rightarrow p+p+\rho^0$ do not sum up to a finite result. In this case the channel quantum numbers are $I=1$, $I_z=1$. This peculiar feature bears on what we mean by a proton, for example, when it can emit and absorb charged soft gauge mesons at any time.

[4]T. Kinoshita, J. Math. Phys. (N. Y.) 3, 650 (1962); T. D. Lee and M. Nauenberg, Phys. Rev. 133, B1549 (1964).

[5]I am presently evaluating the hard bremstrahlung integrals to demonstrate explicitly the cancelation of mass singularities.

[6]E.g., S. Weinberg, J. Phys. (Paris), Colloq, 34, C1-45 (1973).

Energy Dependence of ^{51}V(d, ^{3}He)^{50}Ti; Implications for Distorted-Wave-Born-Approximation Analysis and Rearrangement*

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The differential cross section as a function of energy has been carefully measured and analyzed for the ground and first three excited states in the reaction ^{51}V(d, ^{3}He)^{50}Ti. The results show no evidence for rearrangement effects. Finite-range, distorted-wave Born-approximation calculations, which are necessary to fit the data, yield spectroscopic factors about 0.6 of those expected on the basis of an $(f_{7/2})^3$ description of ^{51}V and a conventional (d, ^{3}He) normalization.

We have performed a careful measurement and a detailed distorted-wave Born-approximation (DWBA) analysis of the single-particle transfer reaction ^{51}V(d, ^{3}He)^{50}Ti, as a function of the incident deuteron energy. Particular emphasis was placed on the proton pickup configuration reached by $l_p=3$ transfers which populate the first four states of ^{50}Ti of $J^\pi=0^+$, 2^+, 4^+, and 6^+. The measurements were made at 30 and 80 MeV and the data of Hinterberger *et al.*[1] at 52 MeV were reanalyzed in a manner consistent with the analysis of our own data.

This careful study was motivated by the desire to test the applicability of the DWBA analysis over a wide range of incident energies. In particular, we were interested in the possibility that rearrangement effects[2,3] (hole-state lifetime effects) might be observed as energy-dependent spectroscopic factors deduced from the DWBA analysis. Should the reaction tend away from the sudden limit, interactions which result in "rearrangement" could redistribute the spectroscopic strengths among those states which are excited in the reaction. In the present analysis we have centered attention on the energy dependence of the relative spectroscopic factors (ratio of the excited-state spectroscopic factor to the ground-state spectroscopic factor) since these quantities can be measured more accurately than absolute spectroscopic factors and the rearrangement which occurs may be very small. Furthermore ratios are much less sensitive to the details of the manner in which the DWBA is used.

The experiments were carried out using con-