Verification of the Principle of Equivalence for Massive Bodies*

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Analysis of 1389 measurements, accumulated between 1970 and 1974, of echo delays of laser signals transmitted from Earth and reflected from cube corners on the Moon shows gravitational binding energy to contribute equally to Earth's inertial and passive gravitational masses to within the estimated uncertainty of 1.5%. The corresponding restriction on the Eddington-Robertson parameters is $4\beta-\gamma-3=-0.001\pm0.015$. Combination with other results, as if independent, yields $\beta=1.003\pm0.005$ and $\gamma=1.008\pm0.008$, in accord with general relativity.

Verification of the principle of equivalence has been of concern to physicists at least since the time of Ioannes Grammaticus in the 5th Century.[1] Laboratory experiments performed over the past 300 years have allowed increasingly stringent limits, from 1 part in 10^3 to 2 parts in 10^{12}, to be placed on the independence on composition and size of the ratio of the inertial to the passive gravitational masses of diverse objects.[2] However, despite their impressive accuracy, these experiments fail utterly to test whether gravitational binding energy contributes equally to inertial and gravitational mass. For a meter-sized laboratory object, the gravitational binding energy represents only about 1 part in 10^{23} of the total energy, about eleven orders of magnitude too small to detect with present laboratory techniques. To test this aspect of the principle of equivalence, the cornerstone of general relativity, it is necessary to utilize planetary-sized bodies since the ratio, Δ, of the magnitude of the gravitational binding energy to the total energy scales as the square of a typical length. For a homogeneous sphere, $\Delta=0.8\pi G\rho R^2/c^2$, with G the constant of gravitation, c the speed of light, ρ the density, and R the radius.

To verify the principle of equivalence or to detect a violation for such massive bodies, one must monitor their orbital behavior. However, without independent measurements of mass, three or more bodies are required to detect a violation. Nordtvedt[3] pointed out that for this purpose the Earth-Moon-Sun system would be useful since laser measurements[4] of the Earth-Moon separation, made possible by the optical corner reflectors on the moon, would allow a significant test to be made.

To describe the orbital effects of a violation, we consider the simplified Newtonian equations of motion for the geocentric orbit of the Moon, neglecting terms of order Δ^2 and perturbations of all bodies except the Sun:

$$\ddot{\vec{r}}_{me}\cong -G(M_e+M_m)[\vec{r}_{me}/r_{me}^3]+GM_s[\vec{r}_{es}/r_{es}^3-\vec{r}_{ms}/r_{ms}^3]+\eta G(M_e+M_m)(\Delta_e+\Delta_m)[\vec{r}_{me}/r_{me}^3]$$
$$-\eta GM_s(\Delta_s+\Delta_m)[\vec{r}_{es}/r_{es}^3-\vec{r}_{ms}/r_{ms}^3]-\eta GM_s(\Delta_e-\Delta_m)[\vec{r}_{es}/r_{es}^3], \qquad (1)$$

where M_i denotes the inertial mass of body i (m, e, and s for Moon, Earth, and Sun, respectively), and $\vec{r}_{ij}$ the vector extending from body j to body i. The ratio of gravitational to inertial mass is represented by $(1-\eta\Delta)$, where for Δ, introduced above, we find $\Delta_m\simeq 0.2\times10^{-10}$, $\Delta_e\simeq 4.6\times10^{-10}$, and $\Delta_s=O(10^{-5})$.[5] The parameter η ($\equiv 0$ in general relativity) was first calculated in terms of the parametrized post-Newtonian formalism by Nordtvedt[3]; for "fully conservative" theories of gravitation $\eta=4\beta-\gamma-3$,[6] where β and γ are the so-called Eddington-Robertson parameters[7].

The inability to determine η from two-body motion follows from Eq. (1): The third term on the right can be absorbed into the first with merely a redefinition of the Earth-plus-Moon mass. Similarly, the fourth can be absorbed into the second with a redefinition of the Sun's mass. The last term, $-\eta GM_s(\Delta_e-\Delta_m)\vec{r}_{es}/r_{es}^3$, is of nearly constant magnitude and is always directed towards the Sun; its orbital consequences are precisely the same (except for scale!) as those due to sunlight pressure[8] and were described for the present context first by Nordtvedt.[3] Of primary interest here is the added variation in the Earth-Moon distance, approximated for purposes of discussion by[9]

$$\delta r_{em} \simeq \eta(\Delta_e - \Delta_m) r_{es}[\Omega(3-\Omega)/(2-\Omega)(1-\Omega)] \cos[(\omega_{me}-\omega_{es})t] \simeq 8\eta \cos[(\omega_{me}-\omega_{es})t] \text{ m}, \quad (2)$$

where ω_{ij} is the mean orbital angular velocity of body i about body j and $\Omega \equiv \omega_{es}/\omega_{me} \simeq 0.075$. The period of this variation is a synodic month.

What other perturbations of the Earth-Moon distance could mask this effect of a violation of the principle of equivalence? Radiation pressure, as mentioned, has precisely the same signature, but its effect here is negligible. The standard Newtonian perturbations, due to the Sun, when decomposed into sinusoidal components, also show a large contribution with a synodic monthly period and an amplitude of approximately 110 km.[10] The largest term[11] in the usual expansion of this amplitude in powers of Ω is proportional to $a_m[M_s/(M_e+M_m)]^{1/2}[a_m/a_e]^{5/2}[1-2(M_m/M_e)]$ which contains all the significant factors found in the smaller terms as well. Let us examine these factors. The semimajor axis, a_e, of Earth's orbit is known from analysis of other data to more than sufficient accuracy for present purposes.[12] The ratio, M_m/M_e, is known from prior analyses[13] with an uncertainty of about 5×10^{-8} which introduces an uncertainty in the 110-km amplitude of about 1 cm. The semimajor axis, a_m, of the Moon's orbit and the ratio $(M_e+M_m)/M_s$, on the other hand, are both estimated best from the laser data, primarily through their other orbital effects, and any masking is automatically taken into account in the simultaneous estimation of these parameters and η.

We adapted our "planetary ephemeris program"[14] to obtain these estimates. The basic coordinate system used was inertial, centered at the solar-system barycenter (Newtonian definition). We integrated the post-Newtonian equations of motion for the Moon with respect to Earth, along with the variational equations for the six initial conditions of the Moon's orbit. All relevant perturbations were included. The orbits of the Earth-Moon barycenter and of the other planets which perturb the Moon's orbit were obtained from similar integrations, with the initial conditions and masses having been determined mainly from a comparison with optical and radar observations of the planets.[12,15] The equations of motion for the Moon also included the effects of the second and third zonal harmonics of Earth's gravitational field, the second zonal and sectorial harmonics of the Moon's field, and a model of the tidal effects.[16] Except for the η term in the equations of motion, all were in accord with general relativity. The implied inconsistency in the parametrization of the post-Newtonian equations is more apparent than real for two reasons: First, we seek to isolate the effects of a possible violation of the principle of equivalence. Second, use in the other terms of the equations of motion of values of β and γ different from unity, but within the bounds set by other experiments, causes a totally insignificant change in the estimate of η.

We next computed, for each observation, the round-trip light time from the laser site on Earth (the McDonald Observatory in Texas) to the appropriate retroreflector site on the Moon and return. For these computations, we used (i) the orbit of the Moon, calculated as described above; (ii) the orientation of the Moon determined in the conventional manner (but with the physical libration as given on a magnetic tape provided by Williams[17]); and (iii) the orientation of Earth obtained from the standard expressions, with minor corrections, for precession, nutation, polar motion, and variations in the rate of rotation. The effects of the propagation medium and of post-Newtonian relativity[18] on the delays were also included. Our total data set consisted of 1389 delay measurements made between January 1970 and November 1974.[19] With a weighted-least-squares filter, we then estimated η along with the geocentric coordinates of the McDonald Observatory, the selenocentric coordinates of the retroreflectors, all of the initial conditions of the Moon's orbit and of the Moon's motion about its center of mass, six parameters characterizing some of the low-order harmonics of the Moon's gravitational field,[20] the mass of Earth plus Moon, three parameters characterizing a possible angular velocity of the dynamical system with respect to a truly inertial frame, and two bias parameters (one to account for a possible bias in the 1972 delays[21] and a second to account for a possible bias in the other delay measurements). The estimate of η was not highly correlated with the estimate of any other parameter. In particular, the largest correlations, with a_m, M_e+M_m, and the eccentricity of the Moon's orbit, were only 0.8 in magnitude; all others were less than 0.5.

We performed various sensitivity tests in which we varied both the data and the parameter sets, the latter mainly by adding parameters relating to M_e/M_m, the tidal model, and the small, but erratic, variations in Earth's rotation. Up to 100 parameters were added to describe these variations alone. We also repeated several of the solutions with the η term dropped from the equations of motion and with the right-hand side of Eq. (2) added instead to the Earth-Moon distance.[22]

The rms of the postfit residuals from these sensitivity tests was typically about 2.8 nsec ($\simeq 42$ cm equivalent distance), slightly over 3 times the average uncertainty in the delay measurements. Taking into account possible residual systematic errors responsible for this large rms and possibly caused by (small) inadequacies in the models used to represent the rotational motions and tidal distortions of the crusts of Earth and Moon, we conclude that $\eta \simeq -0.001 \pm 0.015$, corresponding to a ratio of gravitational to inertial mass that deviates from unity by no more than about 7 parts in 10^{12}. The quoted uncertainty is about 4 times the typical formal (statistical) standard error, and is based on the extremes in the estimate of η obtained in the sensitivity studies, primarily those involving extra parameters to represent variations in Earth's rotation. Our result is in substantial agreement with the most recent value obtained by the LURE team[23] from an analysis of virtually the same data set with an independent computer program; only the lunar libration model, and corresponding partial derivatives, were in common in the two programs.[24]

Combining our estimate for η from the lunar-data analysis with those from recent experiments on the solar deflection[25] and retardation[26] of radio waves and on the perihelia advances[27] of the inner planets, as if all were independent, leads to $\beta = 1.003 \pm 0.005$ and $\gamma = 1.008 \pm 0.008$, with the correlation being 0.6. The validity of this result, of course, is contingent upon the negligible contribution of the other parameters[6] present in the more general expression for η. Without inclusion of the result for η, we obtain $\beta = 1.03 \pm 0.04$, $\gamma = 1.02 \pm 0.02$, and a correlation of 0.9.

What other possibilities are there for independent verification of the principle of equivalence for massive bodies? The Mars-Sun-Jupiter system, a close analog of the Moon-Earth-Sun system, is the most promising. The effect of a violation on Mars's orbit is about three orders of magnitude larger than for the lunar case, nearly counterbalancing the present three orders of magnitude larger uncertainties in the measurements of the distance to Mars. All told, we expect a result of comparable accuracy to be obtainable.

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[2]*Sir Isaac Newton's Mathematical Principles*, edited by F. Cajori (University of California, Berkeley, 1962), p. 411; F. W. Bessel, Pogg. Ann. 25, 401 (1832); R. v. Eötvös, D. Pekar, and E. Fekete, Ann. Phys. (Leipzig) 68, 11 (1922); P. G. Roll, R. Krotkov, and R. H. Dicke, Ann. Phys. (N. Y.) 26, 442 (1964); V. B. Braginsky and V. I. Panov, Zh. Eksp. Teor. Fiz. 61, 873 (1971) [Sov. Phys. JETP 34, 463 (1972)].

[3]K. Nordtvedt, Jr., Phys. Rev. 169, 1014, 1017 (1968), and 170, 1186 (1968).

[4]These measurements were carried out by members of the Lunar Ranging Experiment (LURE) Team; see, for example, P. L. Bender *et al.*, Science 182, 229 (1973).

[5]For the Moon the excellent approximation of constant density was made. For Earth a two-component model —core and mantle—was employed; the uncertainty in the result—under 5%—is of no consequence in this context because of the nearly null result for η (see below).

[6]See, for example, C. M. Will, in *Experimental Gravitation*, edited by B. Bertotti (Academic, New York, 1973), p. 1. If the "fully conservative" assumption is dropped, other post-Newtonian parameters appear in the expression for η; these are treated by Will and will not be discussed in this paper.

[7]H. P. Robertson, in *Space Age Astronomy*, edited by A. J. Deutsch and W. B. Klemperer (Academic, New York, 1962), p. 228

[8]See, for example, I. I. Shapiro, in *Dynamics of Satellites*, edited by M. Roy (Springer, Berlin, 1963), p. 257.

[9]A more accurate approach [see, e.g., K. Nordtvedt, Jr., Phys. Rev. D 7, 2347 (1973)] yields ~9.2η m for the amplitude of δr_{em}.

[10]*Improved Lunar Ephemeris 1952–1959* (U. S. GPO, Washington, D. C., 1954), p. 317.

[11]See, for example, D. Brouwer and G. M. Clemence, *Methods of Celestial Mechanics* (Academic, New York, 1961), p. 329.

[12]M. E. Ash, I. I. Shapiro, and W. B. Smith, Astron. J. 72, 338 (1967), and Science 174, 551 (1971).

[13]$M_e/M_m = 81.3007 \pm 0.0003$ [S. K. Wong and S. J. Reinbold, Nature 241, 111 (1973)]; the inverse of this ratio has an uncertainty of about 5×10^{-8}.

[14]M. E. Ash, Lincoln Laboratory Technical Note No. 1972-5, 1972 (unpublished).

[15]See also J. D. Anderson and L. Efron, Bull. Am. Astron. Soc. 1, 231 (1969); G. W. Null, Bull. Am. Astron. Soc. 1, 356 (1969).

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[20]For discussion of the effects of these low-order harmonics on the orientation of the Moon, see J. G. Williams *et al.*, Moon 8, 469 (1973); R. W. King, Ph.D. thesis, Massachusetts Institute of Technology, 1975 (unpublished). Note also that we used the partial derivatives with respect to the parameters affecting the Moon's orientation as given on William's tape (Ref. 17).

[21]E. C. Silverberg, private communication, concluded from an analysis of the electronics at the McDonald Observatory that the 1972 ranging observations might have been biased negatively by as much as 1.5 nsec. Our estimate of this bias was -0.9 ± 0.2 nsec.

[22]We also did a numerical experiment with the cosine on the right-hand side of Eq. (2) replaced by the sine. The resultant estimate of η was smaller than its uncertainty.

[23]See J. G. Williams *et al.*, preceding Letter [Phys. Rev. Lett. 36, 551 (1976)].

[24]We are, however, now developing an independent formulation of the lunar libration.

[25]C. C. Counselman *et al.*, Phys. Rev. Lett. 33, 1621 (1974); E. B. Fomalont and R. A. Sramek, Astrophys. J. 199, 749 (1975).

[26]I. I. Shapiro *et al.*, Phys. Rev. Lett. 28, 1594 (1972); J. D. Anderson *et al.*, Astrophys. J. 200, 221 (1975).

[27]I. I. Shapiro *et al.*, to be published, obtain $(2+2\gamma-\beta)/3 = 1.003\pm 0.005$ with the use of a solar gravitational quadrupole moment derived for a uniformly rotating Sun. See, also, I. I. Shapiro *et al.*, Phys. Rev. Lett. 26, 1132 (1971).

[28]This approach, suggested by one of us (I. I. S.), was analyzed by G. N. Sherman, Ph.D. thesis, Massachusetts Institute of Technology, 1973 (unpublished).

Anomalous Production of High-Energy Muons in e^+e^- Collisions at 4.8 GeV*

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In view of the possible production of heavy leptons or charmed states in e^+e^- collisions, we searched for anomalous muons with momenta $p_\mu \gtrsim 1$ GeV/c. The inclusive cross section for $n_{\text{ch}} \geq 3$ has an upper limit of 96 pb (assuming isotropy). For $n_{\text{ch}} = 2$ and noncoplanarity $> 20°$, an excess of muonic events is observed, corresponding to $(d\sigma/d\Omega)|_{90°} = 23_{-9}^{+12}$ pb/sr; the probability that known processes produce the observed events is 2×10^{-4}.

Single- or double-lepton production has been observed in hadron-hadron,[1] lepton-hadron,[2] and e^+e^- collisions[3] with rates significantly higher than expected from known physical processes. We have examined our data[4] on e^+e^- collisions for the occurrence of anomalous high-energy muons. This search addresses in particular the questions of production of heavy leptons[5] and charmed states[6] since both, if produced, would give rise to decay muons.

In this Letter, we report on muons with $p_\mu \gtrsim 1.05$ GeV/c from e^+e^- collisions at $\sqrt{s} = 4.8$ GeV. Following a description of the apparatus, centering on its muon-detection characteristics, we discuss the $\mu\mu$ events and compare them with quantum-electrodynamic (QED) predictions. We