the Kondo effect, and detailed derivation of the results quoted above, will be published elsewhere.

The three-dimensional Hubbard model[5] incorporates interactions $U[n_\uparrow(R_i)-\frac{1}{2}][n_\downarrow(R_i)-\frac{1}{2}]$ at every site R_i of a three-dimensional lattice. Because the Tomonaga operators defined on the ith site do not commute with those on a jth site, the transformation to the ± operators is no longer exact. Ignoring the correction terms, we avail ourselves of (10a) and, after summing over sites, obtain

$$:\mathcal{H}_{\mathrm{Hubbard}}: = \sum_{k,\tau}\{\epsilon_k + \tau U[S(\epsilon_k) - S(\epsilon_F)]\} c_{k\tau}^{\dagger} c_{k\tau}. \tag{20}$$

Note that the parameter $\frac{1}{2}U$ of Eqs. (12) *et seq.* is now replaced by U. As U is increased, the susceptibility rises until it diverges at U_0, given by

$$(\partial/\partial\epsilon_k)[\epsilon_k - U_0 S(\epsilon_k)]_{\epsilon_F} = 0, \text{ i.e., at } U_0\mathcal{N}(\epsilon_F) = 1, \tag{21}$$

which is the usual ball-park figure for a phase transition. The bandwidth of the (+) quasiparticles has approximately doubled at U_0.

Unlike the reduction of the Wolff model, the result (20) is not exact. Nevertheless, it contains all the qualitative features one expects, except for the finite lifetime of quasiparticles away from the Fermi energy. Many such aspects of the problem appear to be of interest, and further details will be published in due course.

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[4] At high temperature or with strong interactions, the number of degrees of freedom associated with bosons greatly surpasses that for fermions so that correct thermodynamic properties near U_c are difficult to extract from a pure Tomonaga model. Criteria for the validity of the Tomonaga operators have been given, e.g., by M. Aizenman and H. Gutfreund, J. Math. Phys. (N. Y.) 15, 643 (1974).

[5] See the review by C. Herring, in *Magnetism,* edited by G. Rado and H. Suhl (Academic, New York, 1966), where the model and several alternative solutions are compared.

[6] K. Wilson, in *Nobel Symposia, Proceedings of the 1973 Symposium,* edited by B. Lundqvist *et al.* (Academic, New York, 1974), p. 68; K. Wilson, Rev. Mod. Phys. 47, 773 (1975).

Ion-Temperature Measurements of Fission-Fragment Tracks in CsBr Films*

R. D. Macfarlane and D. F. Torgerson
Department of Chemistry and Cyclotron Institute, Texas A & M University, College Station, Texas 77843
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The velocity spectrum of Cs^+ and Br^- ions emitted from thin CsBr films irradiated with fission fragments show that the ions are not ejected by direct collisional interactions with the fission fragments but appear to be thermionically emitted with an average temperature of 6.6×10^4 K. This suggests the existence of a transient hot thermal core within the fission fragment track. Transient fission tracks in condensed phases may be convenient media for studying fast high-temperature atomic and molecular interactions.

For heavy ions penetrating condensed media at fission-fragment energies, Rutherford scattering from electrons and charge exchange are the important energy transfer mechanisms. The free secondary electrons produced deposit energy in the vicinity of the heavy-ion trajectory and form a cylindrical track which contains free ions and which can have a transient excess temperature relative to the surrounding medium. Mozumder has carried out a detailed theoretical study of the problem and has shown that tracks produced by α particles and protons can be expected to have a diameter of ~ 30 Å and an excess temperature of $\sim 100^\circ$.[1] Mozumder finds that very heavy ions, such as fission fragments, should produce a different track containing a central core with a much higher transient excess temperature, possibly greater than 10^4 K, as a result of the overlap of a high density of secondary-electron tracks. The mean life of the hot core is predicted to be $\sim 10^{-11}$ sec, a time sufficiently long that fast reactive atomic and molecular interactions could take place at these high temperatures and densities. This would offer possibilities for studying processes such as charge transfer, charge exchange, ion-pair formation, and vibrational fragmentation in a high-temperature environment. The critical question is whether a high effective thermodynamic temperature is actually attained in the fission track. This has been a subject with considerable uncertainty but it has never been directly investigated experimentally.

We report here the results of experiments where we have obtained the average temperature of thermal pulses in thin films of CsBr by measuring the velocity distribution of free Cs^+ and Br^- ions emitted as a result of the formation of a fission track in the film.

Thin films of CsBr (4 $\mu g/cm^2$) were deposited uniformly on 4×10^{-3}-mm-thick aluminized Mylar by the electrospray technique. Samples were mounted in the ion-source region of a newly developed time-of-flight mass spectrometer and positioned in front of a ^{252}Cf fission source.[2] Fission fragments penetrating the foil produced free positive and negative secondary ions which were accelerated through a potential of ±10 kV. The complementary fission fragment, emitted in the opposite direction, was used to generate a "time zero" pulse. The time of flight of the secondary ions was measured over a 1.7-m flight path using a time digitizer (EGG TDC 100) to obtain precise flight times. The instrumental time resolution was 1.5 nsec. We measured spectra at various acceleration voltages and determined that at 10 kV, at least 95% of the Cs^+ and Br^- ions emitted from the foil were detected.

Figure 1 shows the time-of-flight spectra (a) for Cs^+ ions and (b) for Br^- ions. When we transform the velocity coordinates for the accelerated ions to that prior to acceleration, we find that the distribution is symmetric about zero velocity and can be fitted by a normal distribution function. This means that there is little contribution from a nonisotropic component to the secondary Cs^+ or Br^- spectra from nuclear Rutherford scattering (or sputtering) or from other direct processes that could produce high-energy ions by localized energy deposition. The spectra resemble what one would expect for a Maxwell-Boltzmann distribution of velocities for a very high-temperature gas. This suggests a delocalization of the energy deposited and equipartition to the translational degrees of freedom of the ions. The crystal lattice energy of CsBr is 6.35 eV. This amount of energy must be deposited into vibrational degrees of freedom of the crystal in order to produce a separated Cs^+-Br^- pair, requiring a vibrational temperature (kT) of the order of 10^4 K. We can obtain an estimate of the average temperature of the ions from an analysis of the velocity spectra of the ions.

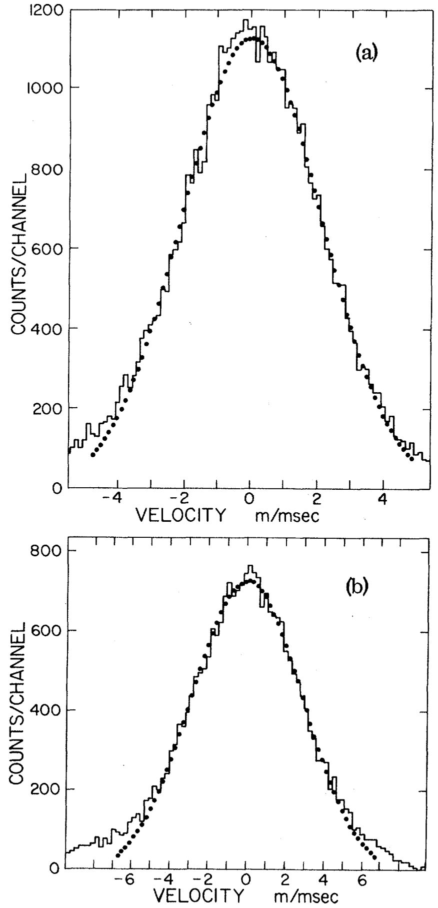

FIG. 1. (a) Velocity distribution of $^{133}Cs^+$ ions produced by a fission fragment track in CsBr. (b) Velocity distribution of $^{79}Br^-$ ions. The dotted lines are Gaussian distribution functions fitted to the experimental data.

The velocity spectrum contains the thermal history of the ions, since they are emitted immedi-

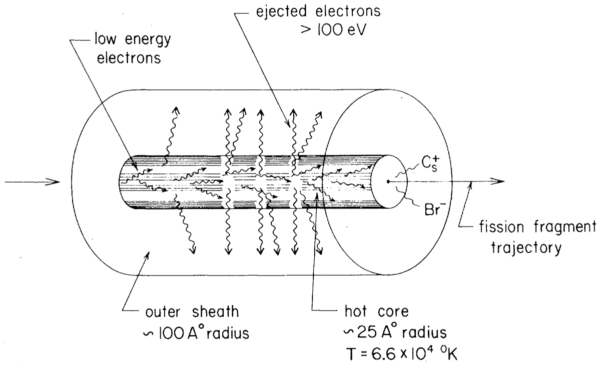

FIG. 2. Characteristics of a fission fragment track in CsBr. The track consists of a hot core, which has an average temperature of 6.6×10^4 K for CsBr, and a cooler outer sheath created by secondary electrons of energy >100 eV.

ately at the onset of the track development when it is at its maximum temperature and emission continues until the track cools to a temperature where ion pair recombination begins to dominate. The spectrum that we observe can be treated as a superposition of an ensemble of velocity distributions with temperatures ranging from the maximum value to the temperature where ion emission ceases. Thus, the width of the distribution represents a weighted average temperature for the ensemble. The Cs^+ spectrum corresponds to an average temperature of $(6.6\pm0.1)\times10^4$ K and the Br^- spectrum, $(6.5\pm0.1)\times10^4$ K. That the two ions give essentially the same value is further evidence that a meaningful thermodynamic temperature is developed in the hot core. A schematic representation of the fission track derived from these results and the Mozumder model is shown in Fig. 2.

We can calculate the initial temperature of the hot core using the Mozumder model.[1] The initial temperature is given in the theory as

$$T_0 = S/\pi\rho C_v r_0^2,$$

where S is the average energy loss per unit length per fission fragment (taken to be 900 eV/Å), ρ and C_v are the density and heat capacity of CsBr (crystal), and r_0 is the radius of the hot core. Mozumder assumes that the main contribution to the core development is from Rutherford-scattered electrons in the energy range of ~100 eV. The radius of the core is established from the attenuation length of 100-eV electrons in CsBr. We have estimated this to be ~25 Å by extrapolating from attenuation-length data for NaCl reported by Battye *et al.*[3] This gives 2×10^5 K for the calculated initial temperature. Although we are uncertain about the effective radius of the hot core, and the calculation contains many untested assumptions, the theory does quite well in predicting the order of magnitude of the excess temperature. The half-life of the thermal pulse can also be calculated from the Mozumder model and this is on the order of 10^{-11} sec.

We conclude that a transient high-temperature thermal pulse is generated when fission fragments traverse condensed media. The Mozumder track model gives a satisfactory description of details of the thermodynamic properties of the thermal pulse. The question remains whether chemical reactions can occur during the thermal pulse. Earlier we reported evidence for free quasimolecular ion production possibly by charge transfer in thin organic films bombarded with fission fragments.[4] Now that the existence of a high-temperature pulse associated with the fission track has been established, it seems reasonable to interpret these results as evidence that fast chemical reactions may take place in the high-temperature environment created by fission tracks in condensed media.

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