Observation of an Obliquely Growing Mode in an Ion-Beam–Plasma System

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By use of a small launcher in an ion-beam—plasma system, a spatially growing mode, which propagates obliquely to the ion beam, was observed. With ion beams, the wave propagation vector was found to be not in the radial direction from the launcher. The observed phase velocity and spatial growth rate of a slow ion-beam—plasma mode are explained by three-dimensional wave theory.

In recent years, instabilities due to interactions of ion beams with plasmas have been investigated theoretically[1-4] with great attention in relation to the heating of plasmas and to the amplification of plasma waves. Observation of oscillations in ion-beam–plasma systems have been reported[5-8] and spatially growing waves have been investigated.[9-11] The previous propagation results were restricted to one-dimensional propagation. An ion beam has also been used as a controlled wave launcher.[12] In this Letter, obliquely propagating waves launched by a small probe in an ion-beam–plasma system are investigated in experiment and in theory.

The experiments were performed in a chamber 18 cm in diameter and 150 cm in length, in which forward-diffusion-type plasmas are produced. The ion beams were produced by an independent plasma source. The experimental setup is schematically indicated in Fig. 1(a). Argon gas was used at a pressure $p \approx 4\times 10^{-4}$ Torr. Typical plasma parameter values were $n_0 \approx 10^9$ cm^{-3} for the plasma density, $T_e \approx 3$ eV for the electron temperature, and $n_b \approx 2\times 10^8$ cm^{-3} for the ion-beam density. Plasma density and electron temperature were obtained with a Langmuir probe. The ion temperature, the density, and the temperature of the ion beams were obtained with a Faraday cup. The excitation of waves in the ion-beam–plasma system was performed with a small cylindrical probe 1 mm in diameter and 3 mm in length. The signals were detected with a rectangular mesh 2 mm$\times$ 3 mm using an interferometer technique. The detector was movable radially and azimuthally. The resolution angle due to the finite width of the detector is $\theta \approx 5°$.

When a continuous sinusoidal signal was applied to the launcher, wave patterns were measured for several fixed directions from the launcher. Typical patterns are shown in Fig. 1(b), in which the velocity of ion beams is fixed at $V_b/C_p = 2.0$ [V_b being the ion beam velocity, $C_p = (kT_e/M_i)^{1/2}$ being the ion acoustic velocity]. The chosen velocity of the ion beams in Fig. 1(b) is a stable one for the two-stream ion-ion instability[1,2] in the one-dimensional problem, i.e., in the case where the wave vector and the beam direction are the same. Spatially growing wave patterns are indicated for some directions in the figure. Those growing waves correspond to a slow mode of the ion beams as shown later. Although there should be other stable modes in ion-beam–plasma systems, their amplitudes were too small to compare with the growing slow mode of the ion beams under present experimental conditions. That is, the effects of those stable modes are

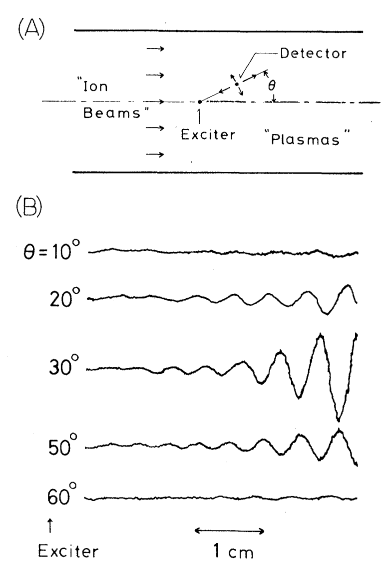

FIG. 1. (a) Schematic experimental setup. The chamber is 18 cm in diameter. (b) Typical wave patterns obtained by an interferometer technique for several directions in the ion-beam–plasma system, when a small probe is used as a launcher. The direction of ion beams is $\theta = 0°$. $f = 450$ kHz, $V_b/C_p = 2.0$.

not included in the wave patterns of Fig. 1(b). It should be noted that the propagating wave shows strong growth for $\theta \approx 30^\circ$, and that the measured wavelength is different for different directions.

From the wave patterns in several directions from a point source, a typical equivalent phase surface (an envelope of wave fronts) of the growing wave can be obtained for fixed frequency and beam velocity, as shown in Fig. 2. The direction of wave normal to the equiphase surface is found to be not the radial direction from the launcher. In anisotropic media, the ray direction is generally different[13] from the direction of the propagation vector $\vec{k}$. As the ion-beam–plasma system is an anisotropic medium, phenomena such as seen in the experimental results of Fig. 2 can be expected. The wavelengths of the propagating wave indicated in Fig. 1(b) are those in the ray direction (i.e., the radial direction) and are not those in the direction of the propagation vector $\vec{k}$. The direction of the propagation vector is shown to vary with a change of the angle θ. For a fixed direction θ, the propagation vector $\vec{k}$ normal to the equiphase surface is found to lie in the same direction, independent of the distance r from the launcher.

In order to obtain the relation of the phase velocity to the direction θ for such experimental results as Fig. 2, one must take account of the fact that the propagation vector is not in the radial direction. That is, one can obtain from the experimental data for a fixed θ that the propagation vector for the direction $\theta+\alpha$ has a wavelength of $\lambda\cos\alpha$, i.e., that the phase velocity for the direction $\theta+\alpha$ is given by $f\lambda\cos\alpha$, where λ and α indicate the wavelength in the radial direction and the angle between the radial direction and the direction of the propagation vector, respectively. The phase velocities obtained by the above procedure can be compared with the phase velocities for any direction calculated from the dispersion relation.

By the above procedure, one can obtain experimental results for three-dimensional phase velocities as shown in Fig. 3(a), in which the velocity of the ion beam is varied. By a similar procedure, the imaginary part (the growth rate) of the propagation vector was obtained for several beam velocities and is shown in Fig. 3(b), in which the geometrical attenuation due just to the

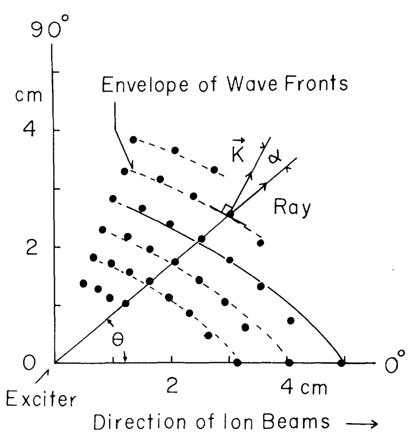

FIG. 2. Typical envelope of wave fronts obtained from experimental wave patterns. For a fixed θ, the distance between the two dots indicates the wavelength, in the θ direction, obtained experimentally by an interferometer technique. $f=400$ kHz, $V_b/C_p=2.0$. Dashed lines are experimental envelopes of wave fronts. The solid line is the theoretical surface of constant phase.

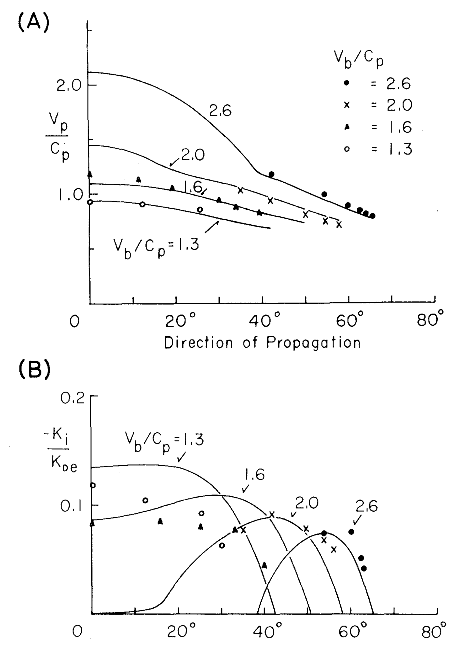

FIG. 3. (a) Experimental and theoretical (solid curves) phase velocities for propagation at an arbitrary angle to the ion beams. The velocity of the ion beams is chosen as a parameter. $N_{b0}/N_0=0.2$, $T_e/T_i=10.0$, $\omega/\omega_{p0}=0.5$, $T_{b0}/T_i=1.5$, $T_{b\perp}/T_i=1.5$, $\omega_{p0}=[4\pi(N_{b0}+N_{i0})e^2/m_i]^{1/2}$. N_{i0} and $T_{b\perp}$ are the density of plasma ions and the beam temperature perpendicular to the ion beam flow. T_{b0} is the beam temperature in the beam direction and is an extrapolated value at zero beam velocity, $V_b=0$. (b) Growth rate corresponding to the case of (a). The values plotted as experimental results were 1.3 times the value obtained from experimental results.

diverging three-dimensional propagation has been removed. The direction $\theta = 0°$ in Figs. 3(a) and 3(b) indicates that of ion-beam flow. The theoretical curves indicated by the solid lines in Fig. 3 will be discussed later. As indicated in the figure, the fastest growing mode was in the beam direction for low beam velocities and was oblique to the beam for high velocities. As will be shown later, the latter case corresponds mainly to a stable condition in the one-dimensional problem of the two-stream ion-ion system.

In order to explain the experimental results, we may formulate the potential $\varphi(\vec{r})$ due to an oscillating point charge in the ion-beam–plasma system as follows[14]:

$$\varphi(\vec{r}) = \frac{\rho e^{-i\omega t}}{2\pi^2}\int_{-\infty}^{\infty} d^3k\, \frac{\exp(i\vec{k}\cdot\vec{r})}{k^2 D(\vec{k})}, \tag{1}$$

$$D(\vec{k}) \equiv 1 - \frac{k_{De}^2}{2k^2} Z'\left(\frac{\omega}{ka_e}\right) - \frac{k_{Di}^2}{2k^2} Z'\left(\frac{\omega}{ka_i}\right) - \frac{k_{Db}^2}{2k^2} Z'\left(\frac{\omega - \vec{k}\cdot\vec{V}_b}{ka_b}\right), \tag{2}$$

where ρ, k_{Dj}, a_j, k, $\vec{k}$, ω, $\vec{V}_b$, and Z' denote the charge density of an oscillating point charge located at the origin, the Debye wave number, the thermal velocity of the jth particle (electrons, plasma ions, and beam ions), the wave number, the wave vector, the angular frequency, the velocity of the ion beam, and the derivative of the plasma Z function,[15] respectively. The dielectric constant of the ion-beam–plasma system is indicated as $D(\vec{k})$, and $D = 0$ is the dispersion relation.[1,2] For the case of $T_e \gg T_i$, $k_{De}^2 \gg k^2$ and without the beams, one can easily obtain the solution of Eq. (1) as $\varphi(R) \propto \exp[i(k_p R - \omega t)]/R$, $k_p = \omega/C_p$, where R is the distance from the origin and C_p is the ion acoustic velocity. The solution shows a spherical wave propagating with the same velocity as the plane-wave propagation, its amplitude decreasing geometrically in proportion to R^{-1}. The general asymptotic solution of Eq. (1) had been obtained by Lighthill.[16] From the Lighthill solution, one can obtain the surface of constant phase. For the experimental conditions, the surface of constant phase can be obtained by using a graphical technique and is indicated partly in Fig. 2 by a solid line. The experimental envelope of the wave fronts is shown to be in rough accord with the theoretical surface of constant phase.

In order to explain the experimental results of Fig. 3, we make a simple consideration, namely, we assume that the diverging propagating wave is "locally" a plane wave at a given point of observation. The measured damping factor has been corrected by taking account of the geometrical attenuation which is proportional to R^{-1}, and is compared with the theoretical results of the plane-wave propagation. That is, in what follows we use the dispersion relation of plane-wave propagation, i.e., $D(\vec{k}) = 0$. The numerical results of the unstable mode of ion beams in $D(\vec{k}) = 0$ are shown in Fig. 3 as solid lines for $k = k_r + ik_i$ and $\omega = \omega_r$, where the ion-beam velocity is chosen near the two-stream ion-ion instability according to the experimental conditions. Convectively unstable modes result from the coupling of the plasma ions with the slow mode of the ion beam. As the experimental half-width of the distribution function of the ion beam in energy units is nearly independent of the ion-beam velocity, the temperature of the ion beam in the beam direction depends on the ion beam velocity.[1,10] In the above numerical calculation, the resulting change of ion-beam temperature has been considered.

Propagating waves for the direction $\theta + \alpha$ can couple with ion beams with a velocity V_b, where $V_b \cos(\theta + \alpha)$ is roughly the ion acoustic velocity. That is, all phase velocities of the observed growing waves for arbitrary directions are near the ion acoustic velocity. As shown in Fig. 3, the theoretical phase velocities for any direction are found to be roughly in accord with the experimental results. The observed growing wave can be considered to result from the instability due to the coupling of the plasma ions and the slow mode of the ion beam. The theoretical spatial growth rate[17] as shown in Fig. 3(b) is also in rough accord with the experimental results in its dependence on beam velocity. As shown in the figure, the growing mode occurs near the beam directions for low beam velocities (roughly $V_b < 1.5C_p$ under present experimental conditions) and occurs mainly in the oblique direction for high beam velocities ($V_b \gtrsim 1.5C_p$). The ion-beam velocity of $V_b \approx 1.5C_p$ corresponds to the upper limit for the unstable region of the two-stream ion-ion instability in the one-dimensional case under present experimental conditions. That is, for high-velocity ion beams, the growing mode is preferentially launched in a direction different from that of the ion beam. The accord between experimental and theoretical results confirms that the essential characteristics of the experimental data for oblique propagation may be explained by the dispersion relation $D = 0$.

In conclusion, we observed obliquely growing waves launched by a small probe in an ion-beam–plasma system. In the system, the direction of the wave propagation vector was found to be not in the radial direction from the launcher. The experimental phase velocity and the spatial growth rate of the slow mode of ion beam were found to be roughly in accord with three-dimensional wave theory. That is, the fastest growing mode was observed in the beam direction for low-velocity ion beams, and for higher velocity, the fastest growing mode was observed in a direction oblique to the ion-beam flow. Moreover, the direction of the fastest growing wave inclines toward the transverse direction with increasing beam velocity.

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Simulation of Dissipative Trapped-Electron Instability in Linear Geometry*

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A simulation model has been developed to study the low-frequency trapped-particle instability in a linear geometry. Low-frequency drift instabilities identified as the dissipative trapped-electron modes grow, accompanied by the turbulent transport of plasma. Measured electron heat conductivity is a few times larger than the particle diffusion constant.

Low-frequency microinstabilities such as drift-wave and trapped-particle instabilities have been of wide interest in relation to research on controlled thermonuclear fusion in low-density magnetic-confinement systems. While there is certain experimental evidence that these low-frequency instabilities may be responsible for the anomalous plasma transport in both linear geometry[1] and toroidal systems,[2] positive identification of the instability and the scaling of the associated plasma transport are difficult in many cases where the experimental situation is far more complex than the theoretical treatments.

As for the trapped-electron instability, several experiments have been performed in linear devices for the identification of the instability.[1] It is, however, very difficult to measure cross-field transport in linear devices since the motion along the field lines is much faster than the cross-field diffusion. In a toroidal system it is possible to measure the cross-field transport; extensive investigations on plasma transport have been carried out for the FM-1 spherator.[2]

We have developed a simulation model shown