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Atomic Structure Effects in Negative Meson Capture*

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Correlations are noted in the variation with atomic number of kaonic, pionic, and muonic x-ray yields, and positron-annihilation lifetimes in annealed metals. These variations are believed to be due to the variation with Z of the electron density in the outer part of the target atoms.

The extensive data of Wiegand and Godfrey[1] on kaonic-x-ray absolute yields exhibit a striking variation with atomic number. In the present note (i) we point out the existence of correlated variations in related data from muonic[2-4] and pionic[5] atoms, [6] and in recent data on positron annihilation in annealed metals[7]; (ii) we comment on a previous explanation of the kaonic-yield variations[8]; and (iii) we put forth a different explanation of these correlated variations, one which, in fact, is strongly supported by recent theoretical work on negative meson capture.[9]

In Fig. 1(a) are plotted the measured absolute yields of several x-ray transitions (from 6→5 up to 11→10) from kaonic atoms[1]; the higher and lower transitions for a given element, those

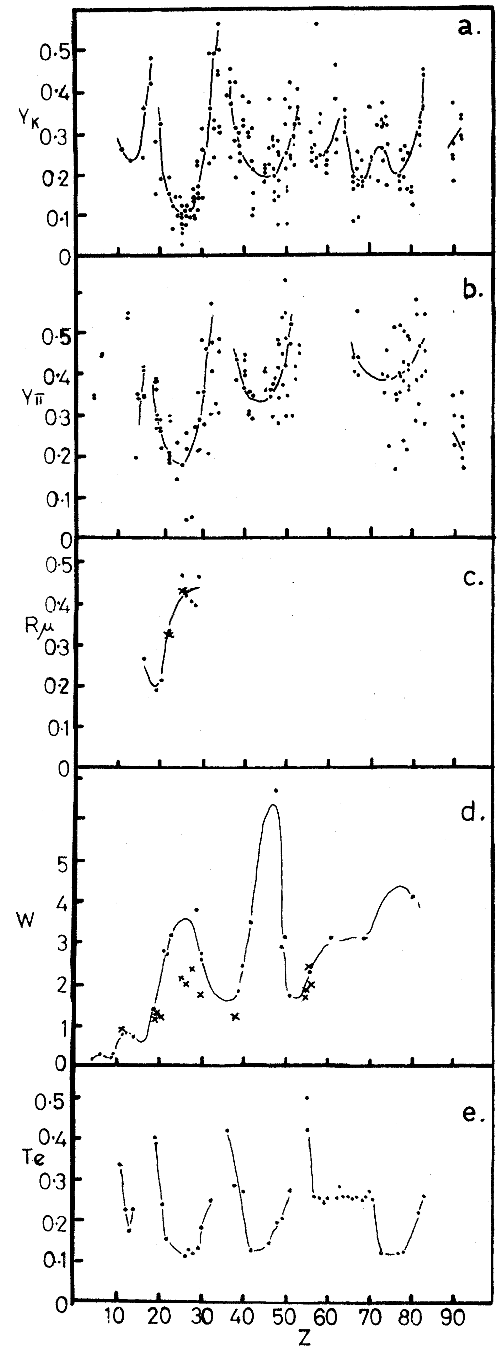

FIG. 1. (a) Kaonic-atom x-ray yields Y_K as a function of atomic number Z (Ref. 1). (b) Pionic-atom x-ray yields Y_π (Ref. 5). (c) Muonic-atom ratios of yields $R_\mu = (K_\beta + K_\nu)/K_\alpha$; the points are from Ref. 2 and the crosses from Ref. 3. (d) Muon-capture probability ratios W in oxides (points, from Ref. 4), and analogous data for pions in chlorides (crosses, from Ref. 5). (e) Positron lifetimes T_e in annealed metals in units of 100 nsec (Ref. 7). The errors are not plotted for each datum point but are typically 15% [except for (e) where they are negligible on this scale]. The lines drawn are suggested shapes to guide the reader's eye.

which are significantly reduced in intensity by cascade effects (higher) or nuclear absorption (lower), are omitted from this plot. The solid line represents the average over these transitions as a function of Z. In Fig. 1(b) are shown analogous data from pionic atoms (transitions in the range $3\to 2$ to $8\to 7$).[5]

These yield variations are clearly correlated with position in the periodic table (and hence with atomic radius)[8] and must reflect the atomic physics of the capture and de-excitation process. Presumably the yields are determined by the shape of the population distribution in orbital angular momentum l at some (principal quantum number) n value [conventionally taken as $n = (M/m)^{1/2}$, where M is the meson mass and m the electron mass]. Heavier weighting at low l implies lower yields because of increased capture by the nucleus from high-n, low-l states.

Such a variation in l distribution for muons should be reflected in the intensity ratios, $(K_\beta + K_\nu)/K_\alpha$, $\nu > 2$, etc., with heavier weighting at low l implying increased ratios. The limited amount of muonic data available[2,3] appear to conform to this trend and are plotted in Fig. 1(c). Somewhat different information comes from measurements of the muon capture ratios of elements in compounds. Data from some oxides[4] are plotted in Fig. 1(d). Here, too, one sees a correlation with the absolute yield data. (More precise, systematic, and extensive muon data would make these comparisons much more interesting.) Analogous data for pions in chlorides[5] are also shown.[10]

In Fig. 1(e) we show an entirely different kind of data: positron-annihilation lifetimes in annealed metals.[7] The correlation with the yield data is very striking.

Condo[8] has recently emphasized the correlation between the kaonic yields[1] and atomic radii, and has suggested that smaller atomic radius causes the l distribution to be truncated at smaller l, producing lower x-ray yields. However, in initial capture into an atom, a critical role is played by the centrifugal barrier seen by the meson[11]; as l increases, this barrier moves in to a smaller radius, which removes any *direct* connection between atomic radius and a cutoff l. This view is supported by the recent analysis of Godfrey and Wiegand,[12] who fitted cutoff l's to the kaonic-yield data.

In a recent paper, Leon and Miller[9] have proposed a model for negative meson capture (the "fuzzy Fermi-Teller model"). By varying in this model the strength of the *electron-meson scatter-*

ing in the outer part of the atom, one can vary the initial capture distribution and its evolution during the early part of the de-excitation, and hence the l distribution at $n=(M/m)^{1/2}$; these changes are about the right size to give the observed absolute-yield variations (increased scattering producing lower yield). This same change for a given element in a compound will increase capture on that element, which accounts for the inverse correlation between the yield data [Figs. 1(a)–1(c)] and the capture-ratio data [Fig. 1(d)]. Now positron annihilation is mostly on loosely bound electrons, and the valleys in Fig. 1(e) presumably reflect an increased density of electrons in the outer part of the atom.[13] Since such an increase necessarily implies increased electron-meson scattering in the outer parts of the atom, we think that this is the basic physical property underlying the variations in yield and capture ratios.

Future theoretical and (hopefully) experimental work will attempt to test this idea quantitatively.

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Comment on de Haas–van Alphen Measurements in Gadolinium*

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Fermi-surface cross sections for field along principal symmetry directions for Gd are presented which are in serious disagreement with those of previous investigators. This disagreement stems from the use of an incorrect value for the saturation magnetization. The new frequency values are compared with the results of a relativistic augmented-plane-wave band calculation.

The only direct measurements of the Fermi surface of a rare-earth metal with an incomplete $4f$ shell were reported by Young, Jordan, and Jones, who observed several de Haas–van Alphen (dHvA) frequencies[1] for Gd and reported effective masses associated with some of these frequencies.[2] Assuming a rigid-band splitting of 1.14 eV for a relativistic augmented-plane-wave