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Josephson Oscillation of a Moving Vortex Lattice

P. Martinoli, O. Daldini, C. Leemann,* and B. Van den Brandt
Laboratorium für Festkörperphysik, Eidgenössische Technische Hochschule Zürich, CH-8049 Zürich, Switzerland
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Experimental evidence is presented for a supercurrent oscillation arising from vortex motion in the flux-flow regime of superconducting films with periodically modulated thickness. An essential condition for detecting the rf electric field associated with the oscillation is matching of the vortex lattice to the periodic pinning structure represented by the thickness modulation.

This Letter reports the observation of radiofrequency (rf) electric fields in the flux-flow regime of type-II superconducting films with periodically modulated thickness when vortex motion is driven by a dc transport current. These oscillating voltages are a manifestation of a Josephson-like supercurrent oscillation arising from coherent vortex motion in the one-dimensional periodic pinning potential represented by the thickness modulation.[1] Evidence for the oscillation is found when the value of the transverse magnetic field $H \simeq B$ corresponds to matching of the vortex lattice to the periodic film structure.

Several years ago, Kulik[2] and Schwartz[3] suggested that there is a close analogy between the flux-flow state in type-II superconductors and the ac Josephson effect in superconducting weak links. In fact, a moving vortex lattice can be thought of as a supercurrent density distribution oscillating both in space and time in a way very similar to that sometimes observed in weak links in connection with the ac Josephson effect.[4] According to this picture, one would expect the existence of electromagnetic radiation from type-II superconductors in the flux-flow regime.[2,3]

As pointed out by Meincke,[5] however, the dramatic mismatch between the flux-flow velocity (or phase velocity of the supercurrent pattern) and the phase velocity of the electromagnetic field almost excludes the possibility of detecting ac Josephson effects in type-II superconductors. Meincke's argument, however, is only valid for uniform motion of an extended vortex lattice. Actually, by considering flux flow in the presence of pinning, Schmid and Hauger[6] have shown that the pinning potential introduces the necessary mechanism for coupling electromagnetic fields to the supercurrent oscillations. This coupling can be significant only when the effect of pinning results in a sufficiently coherent modulation of the vortex velocity. In this case the corresponding modulation of the supercurrent density distribution leads to a net supercurrent oscillation which can therefore interact with the electromagnetic field. This interaction was first demonstrated experimentally for weak random pinning by Fiory[7] and recently for a periodic pinning structure by Martinoli *et al.*,[8] who observed quantum-interference phenomena when the flux-flow state was driven by superimposed dc and rf currents.

At this point, the problem of the direct detection of the supercurrent oscillation associated with the moving lattice arises in a quite natural way. In this connection, we note that Clem[9] predicts the existence of characteristic structures, related to the oscillation, in the noise power spectrum. Such structures, however, were not observed by Fiory[7] for flux flow in a random pinning potential. In this respect our modulated films appear more promising, since a sufficiently coherent motion is expected when the vortex lattice matches the periodic film structure.[8] A simple model accounts for the essential features of vortex motion in thickness-modulated films.[8] Assuming a harmonic film profile $d(x,y) = d + \Delta d \times \sin(qx)$ ($q = 2\pi/\lambda$, where λ is the period of the thickness modulation) and neglecting background pinning effects, the motion of the vortex lattice will be the same as that of a single flux line when the matching condition, $\vec{q} = \vec{g}$, is satisfied ($\vec{g}$ is a reciprocal-lattice vector). The equation of motion can then be written (for $\Delta d/d \ll 1$) as

$$\eta v(t) = \eta \dot{x}(t) = f_L + (\Delta d/d) q \Phi(q) \sin[qx(t)], \qquad (1)$$

where η is the viscosity coefficient describing flux-flow damping, $f_L = \varphi_0 j_{dc}$ is the dc Lorentz driving force, and the last term represents the harmonic pinning force. $\Phi(q)$ is the Hankel transform of the free-energy density distribution within a vortex line.[6, 8]

Using Anderson's phase-slip mechanism, we have previously shown[8] that Eq. (1) can be easily transformed into an equation predicting how the relative phase between two points of the mixed state evolves in time in the flux-flow regime. In that form, Eq. (1) is very similar to that describing the so-called resistively shunted Josephson junction, a two-fluid model appropriate for various types of weak links.[10-12] In our case, however, we are not dealing with a single weak link but with a two-dimensional array of weak links[13] acting coherently when the matching condition is satisfied. It is straightforward to extend well-known results of the resistive-junction model to our particular situation. Since the total dc current is conserved, Eq. (1) predicts that an oscillating electric field $E(t)$ (corresponding to normal fluid) is generated across the normal cores by dissipative feedback of the supercurrent oscillation [represented by the last term in Eq. (1)]. $E(t)$ is given by the following Fourier expansion[12]:

$$E(t)/E_{dc} = 1 + 2\sum_{n=1}^{\infty}\{[1-(1-\alpha^2)^{1/2}]/\alpha\}^n \times \cos(n\omega t), \qquad (2)$$

where $E_{dc} = \rho_f (j_{dc}^2 - j_c^2)^{1/2}$, $\alpha = j_c/j_{dc}$, and $\omega = qE_{dc}/B = qv_{dc}$ is the fundamental frequency associated with vortex motion in the periodic pinning structure. ρ_f is the flux-flow resistivity and the critical current density, j_c, is given by $j_c = (\Delta d/d) q \times \Phi(q)/\varphi_0$. We note that for $\alpha \ll 1$ the only significant contribution to $E(t)$ arises from the fundamental ($n = 1$) component.

It is the oscillating electric field described by Eq. (2) which has been the object of our investigations. The experiments were performed on granular Al films. The technique used to obtain the thickness modulation has been reported elsewhere.[1] For the present experiments λ is 1.9 μm and ($\Delta d/d$) amounts to ~ 5%. Other typical parameters characterizing our layers can be found in Ref. 1. To detect the oscillating E field, whose fundamental frequency is typically in the high-frequency to very-high-frequency region, we have coupled out part of the energy associated with the oscillation into the combined heterodyne–phase-sensitive-detection system shown schematically in Fig. 1. A resonant LC circuit, tunable to frequencies f_D varying from ~ 25 to ~ 50 MHz, is directly connected across the voltage contacts of the modulated film and serves essentially as a selective matching transformer between the low-impedance Al film and the relatively high input impedance of the detector.[14] The rf-detection system is inductively coupled to the resonant circuit. So far, we have not attempted to optimize the coupling. Therefore the rf-power-transmission coefficient, T_{rf}, between film and detector input is only moderate (somewhat less than 10^{-2}).

As we shall see below, our experiments require a relatively large sensitivity. This is achieved with a standard phase-sensitive technique which takes advantage of the mixing properties of the modulated films. A small, low-frequency (~ 300 Hz) ac current is superimposed on the dc current and modulates the velocity of the vortices. This is equivalent to modulating the intrinsic frequencies of the current-controlled voltage oscillation.

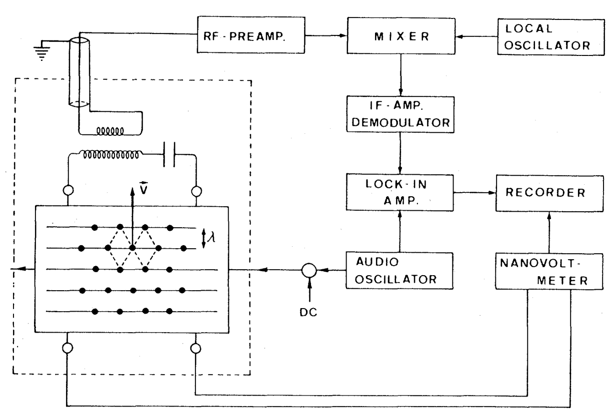

FIG. 1. Block diagram of the detection system. The matching configuration selected for this experiment is also shown. The dashed line delimits the low-temperature part of the apparatus.

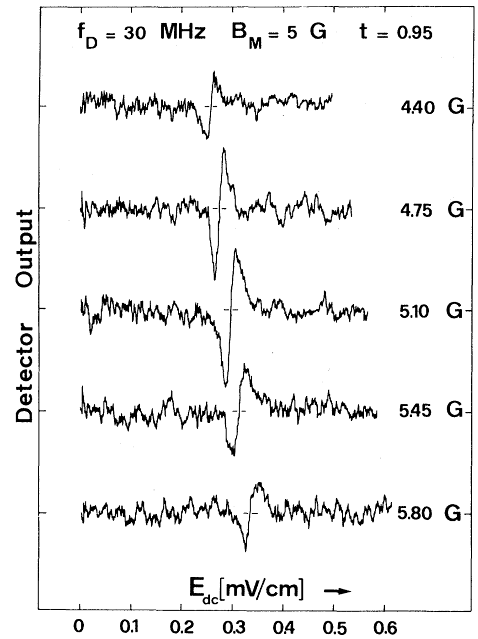

FIG. 2. Recorder traces of the detector output as a function of the dc electric field for a modulated ($\lambda = 1.9$ μm), granular Al film. Parameter is the transverse magnetic field, B. B_M corresponds to the matching configuration shown in Fig. 1. The signal positions (indicated by two dashes) satisfy $E_{dc} = \lambda f_D B$. $t = T/T_c$.

After demodulation the signal can be detected with a conventional lock-in amplifier. With this technique signals of 10^{-16} W can be easily resolved. With a detector frequency f_D, signals revealing the presence of the E oscillation are expected whenever the condition

$$E_{dc} = \lambda f_D B/n \qquad (3)$$

is satisfied. As already discussed in Ref. 8, Eq. (3) is expected to be valid also when B deviates from the matching value, B_M, even though this reduces the signal intensity. Recorder traces of the rf-detector output (with $f_D = 30$ MHz) for several values of B are shown in Fig. 2 as a function of E_{dc}. The matching configuration $\vec{q} = \vec{g}_1$, where $\vec{g}_1$ is a nearest-neighbor reciprocal-lattice vector, is that illustrated in Fig. 1 and corresponds to $B_M = (\sqrt{3}/2)(\varphi_0/\lambda^2) = 5$ G. The antisymmetric shape of the signals is a result of the experimental method and is unimportant in the present context. One can immediately verify that the positions of the signals in Fig. 2 agree with those deduced from Eq. (3) when $n = 1$. This indicates that we have detected the fundamental component of the oscillation. Simultaneously, we have also verified the linear dependence on B predicted by Eq. (3). Moreover, Fig. 2 shows that, as B deviates from B_M, the amplitude of the signals decreases and gradually vanishes, as one expects from the reduced coherence of the moving vortices when the lattice distortions induced by mismatch of the vortex lattice with the periodic pinning structure become too important. Similar signals obeying Eq. (3) for $n = 1$ have been observed when f_D was gradually increased up to ~50 MHz. For the results shown in Fig. 2, α is ~0.3, so that appreciable contributions from higher harmonics are not expected. Harmonic generation should be present if f_D could be significantly reduced below 30 MHz, a frequency range unfortunately not accessible to our detector.

In our experiments the rf power available at the detector input amounts to ~10^{-15} W. This is much less then the expected rf power, $T_{rf}(R_f I_c^2)/8$, calculated from Eq. (2)[15] and the transmission coefficient T_{rf}. This turns out to be ~10^{-9}–10^{-10} W for typical values of T_{rf} ($\lesssim 10^{-2}$), R_f (~10^{-2} Ω), and I_c (10^{-2} A). Although this discrepancy is not understood in detail, it is clear that partly uncorrelated vortex motion due to the random background pinning is responsible. In this connection, additional information concerning vortex motion in superimposed periodic and random pinning potentials may be deduced from a detailed study of the shape of the emitted rf signals.

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*Present address: Department of Physics and Astronomy, Clemson University, Clemson, S.C. 29631.

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No Giant Two-Ion Anisotropy in the Heavy-Rare-Earth Metals*

Per-Anker Lindgård
Ames Laboratory–ERDA and Department of Physics, Iowa State University, Ames, Iowa 50010, and Department of Physics, Danish Atomic Energy Commission Research Establishment Risø, 4000-Roskilde, Denmark†
(Received 8 December 1975)

A new Bose-operator expansion of tensor operators is applied to the heavy-rare-earth metals. The Er data for the cone phase have been analyzed successfully with *single-ion anisotropy* and *isotropic exchange interaction.* The Tb data can be understood on the same basis. The previously found large two-ion anisotropy was due to an inadequate treatment of the large single-ion anisotropy leading to an incorrect expression for the spin-wave energy.

Recently very accurate and detailed experiments of spin-wave spectra in the heavy-rare-earth metals[1-3] revealed that the magnetic interactions might be exceedingly complex. It is in particular difficult to understand the origin of the reported large two-ion anisotropy. Nicklow *et al.*[2] pointed out that this might be due to an inadequate model Hamiltonian. I shall show for the first time that the complexity is nonphysical and arises from an inadequate traditional treatment of the model Hamiltonian.

The previous analysis of the spin-wave data has been done on the basis of the theory of Niira[4] for the ferromagnet and by Cooper *et al.*[5] for the more general case including the spiral and cone phases. They included an anisotropic exchange interaction and the crystal field in the model Hamiltonian. Several subsequent theories added finer details and extra interactions, such as the magnetoelastic effect.[6] However, all theories are based either on the Holstein-Primakoff[7] (HP) transformation or on random-phase-approximation decoupled-spin Green's functions.[8] The anisotropy is treated incorrectly to lowest order by these theories.

A new transformation of spin (tensor) operators in terms of Bose operators has been developed. This makes it feasible to treat a strongly anisotropic magnetic system consistently to a given order of perturbation in the crystal field versus the exchange interaction. In terms of Bose operators, the Hamiltonian can generally (when neglecting higher-order terms) be written as[9,10]

$$\mathcal{H} = N^{-1}\sum_q [A_q \tfrac{1}{2}(a_q^\dagger a_q + a_q a_q^\dagger) + B_q \tfrac{1}{2}(a_{-q} a_q + a_q^\dagger a_{-q}^\dagger)], \tag{1}$$

where A_q and B_q are wave-vector-dependent functions. The spin-wave energy is then[5,10]

$$E_q = \tfrac{1}{2}(A_q - A_{-q}) + [\tfrac{1}{4}(A_q + A_{-q})^2 - B_q^2]^{1/2}. \tag{2}$$

It is therefore not possible[10] from a measurement of E_q alone to determine A_q and B_q uniquely. The neutron scattering intensity contains sufficient extra information in principle[10]; however, in practice it is difficult to measure with sufficient accuracy. Experimental information about A_q and B_q separately has been obtained for two types of structure. For the cone structure $E_q \neq E_{-q}$ so that two functions are available to determine A_q and B_q from (2). This was done for Er by Nicklow *et al.*[2] For a ferromagnet the effect of an external magnetic field H is (predominantly) to change A_q to $A_q + g\mu_B H$. It is therefore possible, although less direct to find A_q and B_q separately by studying the field dependence of (2). This was done for Tb by Houmann and co-workers.[3] Both experiments revealed a strongly q-dependent B_q. On the basis of the existing theories[4-6,8] this could only be understood as a re-