trons is included correctly if we allow the plasma wave to damp beyond the shelf region. After all, the momentum carried by the hot electrons is just that of the plasma wave prior to its damping during which it releases its momentum and energy to a small number of electrons. In addition, we have previously shown[2] that fluid equations and particle simulations give the same conversion efficiency to the plasma wave independent of its subsequent damping and generation of energetic electrons. Thus the lowest-order effects of the hot electrons are included in this physical model. As further evidence that the hot-electron effects are present we show in Figs. 3(c) and 3(d) the late-time density profiles obtained from a hybrid-code simulation and a PIC electron-ion simulation of the interaction of a high-frequency capacitor field with an expanding plasma. This PIC simulation shows that 5% of the electrons at critical density are accelerated toward the low-density region to 5 times the initial electron thermal speed. Note that the hot-electron pressure is comparable to the background pressure, typical of high-power laser-plasma simulations.[2] Even in the presence of these energetic electrons near the critical region, the density profile of the PIC simulations shows insignificant deviation from that obtained with the hybrid-code simulation. The detailed profile modification observed in simulations has also been seen in laboratory experiments.[8] In the hybrid model above, the increase in the background temperature resulting from the absorption of the laser energy has been neglected.

The earlier pioneering work by Kidder[9] differs from this calculation because it was not self-consistent and did not include the effects of enhanced resonant absorption.[10]

*Work performed under the auspices of the U. S. Atomic Energy Commission.

[1]K. G. Estabrook, E. J. Valeo, and W. L. Kruer, Phys. Fluids 18, 1151 (1975); D. W. Forslund, J. M. Kindel, K. Lee, and E. L. Lindman, LASL Report No. LA-5542-PR, 1973 (unpublished), p. 67; E. J. Valeo and W. L. Kruer, Phys. Rev. Lett. 33, 750 (1974).

[2]D. W. Forslund, J. M. Kindel, K. Lee, E. L. Lindman, and R. L. Morse, Phys. Rev. A 11, 679 (1975); J. De Groot and J. Tull. Phys. Fluids 18, 672 (1975); D. Biskamp and H. Welter, in *Plasma Physics and Controlled Nuclear Fusion, Tokyo, Japan, 1974* (International Atomic Energy Agency, Vienna, 1974), paper IAEA-CN-331 F5-1.

[3]R. J. Mason, Phys. Fluids 14, 1943 (1971).

[4]R. L. Dewar and E. J. Valeo, in Proceedings of the Sixth Conference on Numerical Simulation of Plasmas, Berkeley, California, 1973 (unpublished), Paper c7.

[5]R. D. Richtmyer and K. W. Morton, *Difference Methods for Initial Value Problems* (Interscience, New York, 1967), pp. 189 and 199 ff.

[6]D. W. Forslund, J. M. Kindel, K. Lee, and E. L. Lindman, Phys. Rev. Lett. 34, 193 (1975).

[7]D. W. Forslund and J. P. Freidberg, Phys. Rev. Lett. 27, 1189 (1971).

[8]H. C. Kim, Ph.D. thesis, University of California, Los Angeles, 1974 (unpublished); A. Y. Wong, B. H. Quon, R. Stenzel, and H. C. Kim, private communication.

[9]R. E. Kidder, University of California Radiation Laboratory Report No. UCRL-74040, 1972 (unpublished).

[10]J. M. Kindel, K. Lee, and E. L. Lindman, Phys. Rev. Lett. 34, 134 (1975).

Thermal-Equilibrium Properties of Vacancies in Metals through Current-Noise Measurements

M. Celasco,* F. Fiorillo, and P. Mazzetti
Istituto Elettrotecnico Nazionale Galileo Ferraris, Gruppo Nazionale Struttura della Materia del Consiglio Nazionale delle Richerche, 10125 Torino, Italy

(Received 10 September 1975)

We report a new method of measuring thermal-equilibrium properties of vacancies in metals through current-noise measurements. Aluminum noise spectra taken at 435 and 475°C directly yield vacancy lifetimes $\tau_0 = 4.7\times10^{-3}$ and 2.8×10^{-3} sec, respectively, corresponding to a migration energy $E_m = 0.6$ eV, and permit estimation of a unit vacancy resistivity $\Delta\rho_v = 1.9\times10^{-8}$ Ω m/at.% from the measured product $(\Delta\rho_v)^2\eta_v$, η_v being the vacancy concentration taken from literature data.

The contribution of vacancies to the resistivity of metals is generally obtained under nonequilibrium conditions.[1-3] So far measurements in thermal equilibrium have only been performed by measuring the resistivity of the metal[4,5] or its temperature coefficient[6,7] up to the melting point and subtracting the estimated resistivity of an ideal lattice without point defects. This last

estimation is the major weakness of such a method.[8]

In the present paper, we propose a new method of measuring the effect of vacancies on the resistivity of a metal in thermal equilibrium, together with other parameters of physical interest. This method is based upon the study of the current noise generated by vacancy number fluctuations in a small specimen. With respect to the techniques previously quoted, this seems to be the most precise and sensitive method to obtain equilibrium values of the resistivity change per unit vacancy concentration, besides other physical quantities, like vacancy lifetime.

Actually, it is a well-known fact that metals far from the melting point do not present detectable current noise.[9] The only noise component in a metal filament is the Johnson noise, which is independent of the current through the filament. However, as shown below, according to the literature data on resistivity and on the heat of formation and the heat of migration of vacancies, the effect of vacancy number fluctuations on the resistivity of most metals in thermal equilibrium at a sufficiently high temperature should be a measurable quantity with the presently available extra-low-noise amplifiers, provided that cross-correlation analysis techniques are used. The small active volumes and the good heat dissipation required to support sufficiently high current densities can be obtained without too much difficulty at least in the case of good conducting materials.

Suppose that each vacancy in the specimen gives rise to a resistance change ΔR during its lifetime τ and that the total resistance change in the specimen is proportional to the number N of vacancies present in the specimen. Further assumptions are that the events of creation or annihilation of a vacancy are statistically independent and that the lifetime τ of each vacancy is a stochastic variable distributed according to an exponential law:

$$P_1(\tau) = \tau_0^{-1} \exp(-\tau/\tau_0), \tag{1}$$

where τ_0 is the average lifetime of the vacancy. Equation (1) is equivalent to assuming that the annihilation probability of each vacancy in each time interval Δt during its life is constant (uniform sink distribution).

The total resistance change of the specimen due to vacancies can be described as a superposition of rectangular pulses of amplitude ΔR and duration τ. Because of the assumed statistical independence of vacancy creation or annihilation events, the power spectrum of voltage noise when a current I_0 is flowing in the specimen containing an average number N of vacancies can thus be easily computed[10]:

$$\varphi(f) = 4(\Delta R)^2 I_0^2 N \frac{\tau_0}{1+4\pi^2 f^2 \tau_0^2}$$

$$= \frac{4\times 10^4}{C_A} \frac{l}{S} J_0^2 (\Delta\rho_v)^2 \frac{\eta_v \tau_0}{1+4\pi^2 f^2 \tau_0^2}. \tag{2}$$

In Eq. (2) J_0 is the current density, l and S are, respectively, the length and the cross-sectional area of the specimen, $\Delta\rho_v$ is the resistivity change per unit vacancy concentration in atomic percent, C_A is the number of atoms per unit volume, and η_v is the vacancy concentration per atom.

Equation (2) gives a shot-noise spectrum whose cutoff frequency is simply related to the vacancy average lifetime. Introducing recent data published in the literature[8] into Eq. (2), we expect a noise spectral density of the order of 5×10^{-20} V^2/Hz, with reasonable values of the other quantities involved. For a temperature of the order of 450°C a cutoff frequency of the order of ~40–60 Hz can be estimated.

Experiments were made on aluminum films 1.5 μm thick, 15μm wide and 3 mm long, having an electrical resistance of about 10 Ω at 400°C, obtained by photoetching techniques on an oxidized silicon wafer. These films, covered by a SiO_2 protecting layer, were enclosed in a transistor-like container filled with N_2 and placed in a Mu-metal-screened dc electrical furnace. Measurements were taken from room temperature up to 475°C, an upper temperature limit for the specimen reliability. Current density was 3.6×10^9 A/m^2, sufficiently low to avoid important electromigration effects. In the analysis frequency range (6 Hz to 5 kHz) used no current noise was detectable at temperatures lower than about 300°C and a good spectral analysis was only possible near the highest temperature.

On account of the expected extremely low current noise, cross-correlation techniques were used, employing a Model No. 3721A Hewlett Packard correlator equipped with a Model No. 3720A Hewlett Packard spectrum display. Two battery-fed extra-low-noise Brookdeal preamplifiers, Model No. 9431, and two General Radio filters, Model No. 1952, were used to select sufficiently narrow bands for best data analysis.

Filtered noise was recorded on an FM eight-channel SE Labs EMI Ltd. Model Eight-Four magnetic tape recorder to make subsequent analysis easier.

The use of a shielded room was also necessary during data detection. With use of an average time of the order of 2×30 min (to correct for slight phase shifts, cross correlation was taken twice with inverted channels and was averaged by using a special feature of the Hewlett Packard spectrum display), sensitivity turned out to be better than 5×10^{-21} V^2/Hz down to 5 Hz.

Current noise was measured as a difference between noise spectra taken, respectively, with and without current in the specimens. In the second case, results were coincident with the expected Johnson noise spectra. Preliminary results are reported in Fig. 1, where the voltage spectra refer to a single specimen (two identical specimens were actually used as arms of a bridge to avoid a dc component at the preamplifier inputs).

In order to confirm that the structure shown by these spectra has a real physical meaning and is not due, for instance, to a systematic error of the measuring device, measurements were also performed by simply substituting the aluminum specimens with two carbon resistors of the same electrical resistance ($\sim 10\ \Omega$) and supplying sufficiently low current to get a current noise of the same order of magnitude as the one measured on the films. As expected, the power spectrum was in this case a perfect straight line having a $1/f$ slope. Checks were also made to exclude noise contributions due to ion diffusion through the SiO_2 insulating layer.

As shown in Fig. 1 the experimental spectra can be considered as a superposition of a shot-noise component, of the type given by Eq. (2), and a low-frequency component which decreases very rapidly with frequency, with a slope of absolute magnitude greater than 2.

This last component could be due to the current noise generated by spurious effects, such as grain boundary movements in the specimen. If such movements occur by steps, as highly probable, there are sudden, small changes of conductivity which give a current noise having just this type of spectrum. Another possibility is that vacancy creation or annihilation processes are not statistically independent, as assumed in

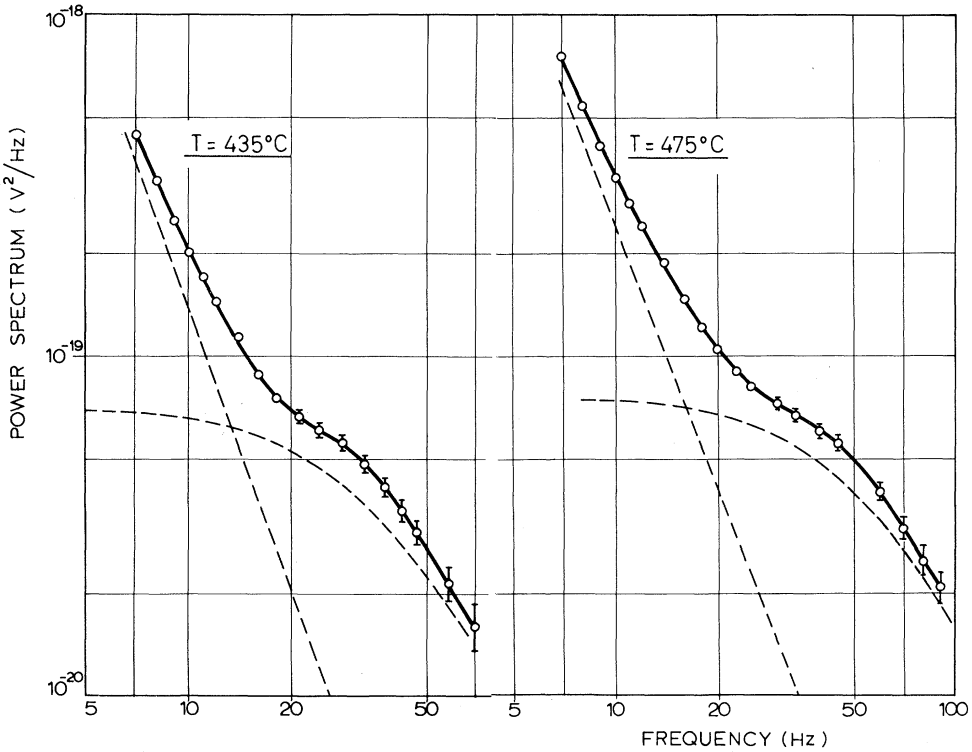

FIG. 1. Power spectra of current noise in Al thin films taken at two different temperatures with a current density $J_0=3.6\times10^9$ A/m^2. Dashed curves represent a best-fit decomposition of experimental data. Data fluctuations for a single spectrum point are represented by vertical bars. Points are averages over ten samples.

TABLE I. Single-vacancy lifetime τ_0, product of the square of unit vacancy resistivity and the vacancy concentration $(\Delta\rho_v)^2\eta_v$, average number of vacancy jumps n_s, vacancy migration energy E_m, and unit vacancy resistivity $\Delta\rho_v$ in aluminum from noise-spectra measurements.

Temperature (°C)	τ_0 (sec)	$(\Delta\rho_v)^2\eta_v$ [(Ω m/at.%)^2]	n_s	E_m (eV)	$\Delta\rho_v$ (Ω m/at.%)
435	4.68×10^{-3}	1.3×10^{-20}	8×10^{7}	0.6	1.87×10^{-8}
475	2.80×10^{-3}	2.3×10^{-20}			1.85×10^{-8}

deducing Eq. (2), but that sources and sinks are present in the specimen, which create or annihilate vacancies in groups. Even in this case the power spectrum is the sum of a spectral component, which is the same as if events where uncorrelated [and is thus given by Eq. (2)], and a correlation term which, in case of clustering, gives a low-frequency component with a slope of absolute magnitude ≥ 2.[11]

In both cases, through Eq. (2), the shot-noise component can be used to obtain the single-vacancy lifetime τ_0 (from the cutoff frequency of the spectrum) and the quantity $(\Delta\rho_v)^2\eta_v$. Introducing the values of τ_0 at two different temperatures in the well-known formula for vacancy jump rate,[12] one obtains (with a frequency factor 3.66×10^{12} sec^{-1} as suggested by Seeger and Mehrer[13]) the average number of jumps n_s of a vacancy before annihilation and the migration energy E_m. The contribution to resistivity per unit vacancy concentration $\Delta\rho_v$ can be derived at each temperature by computing η_v from the expression

$$\eta_v = \exp(S_f/k)\exp(-E_f/kT). \quad (3)$$

The formation energy E_f and the formation entropy S_f are assumed to be, according to recent literature data for aluminum,[8] $E_f = 0.66$ eV and $S_f = 0.6k$.

The values of all measured and derived quantities are listed in Table I. Good agreement with recently published data is found for E_m.[13] $\Delta\rho_v$ values found in recent literature range from 1.07×10^{-8}[14] to 2.9×10^{-8} Ω m/at.%.[7] Also the values of n_s given in the literature are rather uncertain and only rough estimates are available[2] ($10^7 \leq n_s \leq 10^8$).

Finally it may be pointed out that this method allows a good precision in the measurement of $\Delta\rho_v$ because the spectrum amplitude depends on the square of this quantity. The error involved as a result of uncertainties in spectra detection and subsequent graphical decomposition is estimated to be less than 10%. As to the divacancy contribution to resistivity which, because of the different lifetime, should be observed as a further step in the spectrum, it results below the sensitivity limit of the present experimental setup.

These results must, however, be considered as preliminary. Better specimens are now being prepared to extend the spectral analysis to temperatures higher than 475°C.

The authors wish to thank SGS ATES Electronic Components SpA, which provided the specimens, and Professor G. Montalenti for helpful and stimulating discussions.

*Permanent address: Istituto di Fisica dell'Università di Torino, 10125 Torino, Italy.

[1]J. W. Kauffmann and J. S. Koehler, Phys. Rev. 88, 149 (1952).

[2]T. Federighi, in *Lattice Defects in Quenched Metals*, edited by R. M. Cotterill, U. Doyama, J. J. Jackson, and M. Meshii (Academic, New York, 1965), p. 217.

[3]*Vacancies and Interstitials in Metals*, edited by A. Seeger, D. Schumacher, W. Schilling, and J. Diehl (North-Holland, Amsterdam, 1970).

[4]C. J. Meechan and R. R. Eggleston, Acta Metall. 2, 680 (1954).

[5]R. O. Simmons and R. W. Balluffi, Phys. Rev. 117, 62 (1960).

[6]Ya. A. Kraftmakher and P. G. Strelkov, in *Vacancies and Interstitials in Metals*, edited by A. Seeger, D. Schumacher, W. Schilling, and J. Diehl (North-Holland, Amsterdam, 1970), p. 59.

[7]Ya. A. Kraftmakher and G. G. Sushakova, Phys. Status Solidi (b) 53, K73 (1972).

[8]A. Seeger, Cryst. Lattice Defects 4, 221 (1973).

[9]M. Celasco and F. Fiorillo, Appl. Phys. Lett. 26, 211 (1975).

[10]See, for instance, A. Van der Ziel, Physica (Utrecht) 19, 742 (1953).

[11]P. Mazzetti, Nuovo Cimento 31, 88 (1964).

[12]A. C. Damask and G. J. Dienes, *Point Defects in Metals* (Gordon and Breach, New York, 1963), p. 31.

[13]A. Seeger and H. Mehrer, in *Vacancies and Interstitials in Metals*, edited by A. Seeger, D. Schumacher, W. Schilling, and J. Diehl (North-Holland, Amsterdam, 1970), p. 1.

[14]K. Furukawa, J. Takamura, N. Kuwana, R. Tahara, and M. Abe, Acta Crystallogr. A28, S159 (1972).

1/f Noise from Systems in Thermal Equilibrium

Richard F. Voss and John Clarke
Department of Physics, University of California, Berkeley, California 94720, and
Inorganic Materials Research Division, Lawrence Berkeley Laboratory, Berkeley, California 94720
(Received 7 July 1975)

The power spectra of fluctuations in the mean square of the Johnson-noise voltage across small semiconductor and metal films in thermal equilibrium were measured down to 10^{-4} Hz. The spectra have a $1/f$-like behavior that matches the resistance-fluctuation spectra obtained by passing a current through the samples. These measurements contribute strong evidence that $1/f$ noise is due to equilibrium resistance fluctuations.

In this Letter we report the observation of a $1/f$-like power spectrum for low-frequency fluctuations of the mean square of the Johnson-noise voltage across a very small sample of semiconductor or discontinuous metal film in thermal equilibrium. The $1/f$ spectrum is shown to be due to resistance fluctuations in the sample, and closely matches the resistance-fluctuation spectrum obtained by passing a current through the sample. Our measurements are the first observation of $1/f$ noise from a system in thermal equilibrium. The fact that $1/f$ noise had until now been observed only under nonequilibrium steady-state conditions led some authors[1,2] to propose nonequilibrium theories for its origin. Our present results, however, together with earlier work showing that a theory based on equilibrium temperature fluctuations quantitatively predicts the magnitude of $1/f$ noise in continuous metal films,[3] superconducting films[4] biased at T_c, and Josephson junctions,[5] constitute strong evidence that $1/f$ noise is an equilibrium effect.

Consider a resistance, R, of total heat capacity C_v shunted by a capacitance, C, and in thermal contact with a reservoir at temperature T_0. The voltage across the capacitor, $V(t)$, represents a single degree of freedom that can exchange energy with the resistor via the charge carriers in the resistor. This exchange takes place on time scales of order $\tau = RC$. In thermal equilibrium the average energy of the capacitor, $\langle E_c \rangle = \frac{1}{2}C\langle V^2 \rangle = \frac{1}{2}k_B T_0$. These voltage fluctuations (Johnson noise) are limited to a bandwidth of $1/4\tau$, and consequently have a spectrum of the form $S_V(f) = 4k_B T_0 R/[1+4\pi^2 f^2 \tau^2]$. If the resistor is assumed to exchange energy with the reservoir on a time scale of order τ_R that is much greater than τ, the capacitor is able to reach equilibrium with the internal degrees of freedom of the resistor before the internal energy of the resistor can change. The temperature of the capacitor is then the same as the temperature of the resistor. $V^2(t)$, like $V(t)$, is a rapidly fluctuating quantity in time due to this exchange of energy between the resistor and capacitor. However, the average of $V^2(t)$ over a time, θ, such that $\tau \ll \theta \ll \tau_R$, $\langle V^2(t) \rangle_\theta = k_B T/C$ (T is now the instantaneous temperature of the resistor), is sensitive to slow energy or temperature fluctuations in the resistor on time scales τ_R or longer.

Experimentally, the Johnson-noise voltage, $V(t)$, is passed through a filter with a band pass from f_0 to f_1, squared, and averaged over a time $\theta > 1/f_0$ to give $P(t)$, a slowly varying signal proportional to the Johnson-noise power in the band width f_0 to f_1. Thus,

$$P(t) \approx 4k_B TR \int_{f_0}^{f_1} df/(1+4\pi^2 f^2 \tau^2) + P_0(t), \qquad (1)$$

where, if R and T are fixed, the first term on the right-hand side represents the average of $P(t)$, and $P_0(t)$ represents fluctuations in $P(t)$ about the average due to the rapid exchange of energy between capacitor and resistor. Because this exchange is so rapid, $P_0(t)$ has a spectrum, $S_{P_0}(f)$, that is independent of f for the low frequencies in which we are interested. S_{P_0} may be reduced by increasing the bandwidth or by moving the bandwidth to higher frequencies, but in practice $P_0(t)$ severly limits the accuracy of measurements of $P(t)$.

If the bandwidth in Eq. (1) is either totally above