Supersymmetric and Strangeness Analog States in p-Shell Λ Hypernuclei*

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Hypernuclear (${}_{\Lambda}^{A}Z^*$) states exist which possess higher shell-model permutation symmetry than the Pauli principle allows for nuclear (${}^{A}Z$) states and which are the lowest hypernuclear excitations in the appropriate valent configuration. The nature and location of these supersymmetric states and of strangeness-analog states are discussed within the framework of Sakata SU(3) symmetry restricted to p-shell configurations, for the hypernuclei ${}_{\Lambda}^{9}\mathrm{Be}^*$ and ${}_{\Lambda}^{16}\mathrm{O}^*$, for which observations have recently been reported.

Recent observations[1,2] of Λ hypernuclear states (${}_{\Lambda}^{A}Z^*$) strongly excited in nuclear (K^-,π^-) strangeness-exchange reactions for K^- in flight have aroused much interest in the nature of these states and in the indirect information they give concerning the ΛN interaction. As pointed out[3,4] long ago, such reactions with small momentum transfer lead to a coherent sum of Λ-particle, neutron-hole states, which may act as a doorway strangeness-analog state. Kerman and Lipkin[5] have argued the case that the underlying symmetry relating strangeness-analog states to their parent nuclear ground states is, to a good first approximation, the Sakata version of SU(3) in which the single-particle orbitals coincide for protons, neutrons, and Λ particles; they have conjectured[6] that the strangeness-analog state generally emerges within a residual-interaction calculation as the lowest hypernuclear excitation of the general type

$$\sum_{l,j} A(l,j)(l,j)_n^{-1}(l,j)_\Lambda.$$

However, as pointed out by Hüfner, Lee, and Weidenmuller,[7] the $s_\Lambda s_n^{-1}$ and $(p_{3/2})_\Lambda(p_{3/2})_n^{-1}$ excitations in ${}_{\Lambda}^{12}\mathrm{C}$ may be widely separated in energy and therefore unlikely to be coherent. For this reason, in discussing p-shell hypernuclei, we shall confine our attention to Λ-particle, neutron-hole excitations in the p shell,[8] adopting Sakata SU(3) symmetry as a fruitful first approximation.

In this Letter, we point out the existence of nonanalog supersymmetric states in the valent Λ-hypernuclear configuration $(1p)^{A-4}$, which lie considerably *below* the energy observed or expected for analog states in the same configuration. Just as for the other nonanalog states, which lie in the vicinity of the analog state and to which the strength of the strangeness-exchange reaction may become spread, these supersymmetric states cannot be realized in ordinary nuclei because of the Pauli principle. Observation of their formation, would shed light on the strength and detailed nature (especially space-exchange character) of the ΛN residual interactions.

Consider, for example, the $(1p)^5$ configuration. The orbital permutation symmetry for ordinary ${}^{9}\mathrm{Be}$(g.s.) is dominantly of the type [4,1]; this symmetry type will necessarily hold also for the ${}_{\Lambda}^{9}\mathrm{Be}^*$ analog state in this configuration, as well as for some of the nonanalog states in its vicinity. However, when one of the five valent baryons is a Λ particle, states become possible which have the higher symmetry type [5] for permutations in their orbital wave function. These are necessarily nonanalog states and we shall refer to them as "supersymmetric." These states allow more relative s-state bonds for the baryon-baryon interaction than does the analog state embedded in the supermultiplet with orbital permutation symmetry [4,1]. Hence, since the ΛN interaction is strongly attractive in relative s states, a considerable *symmetry energy* will generally separate the supersymmetric state (or group of states) from all other states of the $(1p)^5$ Λ-hypernuclear configuration. This situation is indicated schematically in Fig. 1. The spectroscopic notation ${}^{2I+1,2S+1}L$ used for nuclei has been generalized here to ${}^{n,2S+1}L$, where n denotes the dimension of the Sakata SU(3) multiplet appropriate for the state considered. Insofar as the spin

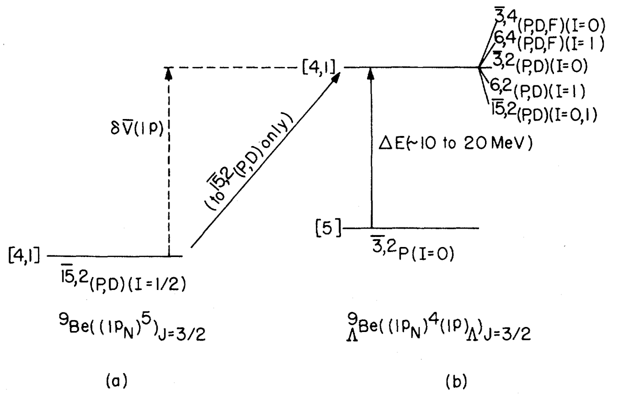

FIG. 1. Schematic representation of low-lying L-S $(1p)^5$ states with $J=\frac{3}{2}$ for (a) ^{9}Be and (b) $^9_\Lambda$Be. The continuous arrowed line marks the dominant strangeness-exchange transition from (a) to the analog state of (b). $\delta\bar{V}(1p)$ denotes the expectation value of the difference in the one-body shell-model potentials experienced by the N and Λ particles in the $1p$ orbit (Ref. 9).

dependence of the ΛN and NN forces can be neglected, in first approximation, it is useful to go further and to group the $^9_\Lambda$Be* excitations into irreducible SU(6) supermultiplets, their substates being labeled by the appropriate combinations for $\mathrm{SU(3)}\otimes\mathrm{SU(2)}_\sigma$. The $^9_\Lambda$Be* analog state and its ^{9}Be(g.s.) parent are then common members of a fifteen-dimensional SU(3) multiplet characterized by the SU(3) labels $(\lambda,\mu)=(1,2)$. Besides the nuclear doublet (^{9}Be, ^{9}B), this multiplet contains two hypernuclear analog states, an isosinglet $^9_\Lambda$Be* and an isotriplet ($^9_\Lambda$Li*, $^9_\Lambda$Be*, $^9_\Lambda$B*). With the target ^{9}Be, the strangeness-exchange reaction will lead to both $^9_\Lambda$Be* states; with the target ^{9}B, it could only reach $^9_\Lambda$B*. The other SU(3) multiplets with [4, 1] orbital symmetry are a six-dimensional (2, 0) and a three-dimensional (0, 1) and each includes one Λ-hypernuclear multiplet. None of these SU(3) multiplets includes any ordinary nucleus and these are therefore "nonanalog states" of the kind referred to by Lipkin.[3] The supersymmetric state necessarily belongs to a (spin doublet) three-dimensional (0, 1) multiplet, since it has orbital symmetry [5]; it has only one Λ-hypernuclear state, namely $^9_\Lambda$Be*.

Next we estimate the symmetry energy ΔE [see Fig. 1(b)] separating the $^9_\Lambda$Be* supersymmetric state from the analog state. For simplification, we shall consider only the dominant component ($^2P_{3/2}$) for the ^{9}Be wave function. Adopting, for convenience, the baryon-baryon potential

$$V_{BB}=\sum_{i<j}W_{ij}[1-\epsilon+\epsilon P_x(ij)], \tag{1}$$

we then have,[10] in the Sakata SU(3),

$$\Delta E^{(0)}(^9_\Lambda\mathrm{Be}^*)=-[\tfrac{6}{5}(1-\epsilon)F^{(2)}+\epsilon(5F^{(0)}+\tfrac{1}{5}F^{(2)})] \tag{2}$$

for the energy difference between the ^2P states with orbital permutation symmetries [4, 1] and [5]. Soper's parameters for the nuclear p shell[11] correspond to $\epsilon=\frac{1}{2}$ and to the Slater integrals $F^{(0)}=-5.6$ MeV and $F^{(2)}=-10.1$ MeV for the potential W. Expression (2) then gives $\Delta E^{(0)}(^9_\Lambda\mathrm{Be}^*)=21.1$ MeV.[12] As we shall see, this is an overestimate, since the ΛN forces are in fact appreciably weaker than NN forces.

We shall take the ΛN interaction to have the form

$$V_{\Lambda N}=sW(1-\epsilon'+\epsilon' P_x). \tag{3}$$

Its dominant effect is to produce a mean nuclear potential (known to have depth $\bar{V}_\Lambda\simeq 30$ MeV) for the Λ particle appreciably weaker than the mean nuclear potential ($\bar{V}_N\simeq 50$ MeV) for a corresponding nucleon, a difference which simply shifts all the $^9_\Lambda$Be* levels upward by the same amount, the quantity δV_p indicated in Fig. 1. We shall not attempt to calculate δV_p here, since the nuclear shell model is not an appropriate basis for the calculation of nuclear well depths, but we do calculate energy separations between levels of $^9_\Lambda$Be*, which is a well-established use for the shell model.

The introduction of $V_{\Lambda N}$ is equivalent to adding to V_{BB} an SU(3)-breaking term $V^{(1)}$, of the form

$$V^{(1)}=\sum_{i<j}(S_i+S_j)[w_{ij}+v_{ij}P_x(ij)], \tag{4}$$

where S_i denote the strangeness of the ith baryon, with values -1 for Λ and 0 for the nucleon. Comparison of $V_{\Lambda N}$ given by the sum $V_{BB}+V^{(1)}$ with $V_{\Lambda N}$ from Eq. (3), leads to the relationships

$$w=W[1-\epsilon-s(1-\epsilon')],\quad v=W(\epsilon-s\epsilon'). \tag{5}$$

The Slater integrals for w and v will be denoted by $f^{(l)}$ and $g^{(l)}$, respectively. For quantitative discussion, the estimate $s\simeq 0.6$ may be adopted,[13] whereas acceptable values for ϵ' lie in the range 0 to 0.5.[14]

As we shall see below, the calculation of the levels which then result for $^9_\Lambda$Be* must include the nonanalog states which stem from the same SU(6) supermultiplet as ^{9}Be, since these states are degenerate with the analog state, in zeroth approximation. However, for $I=1$, it happens that the central interaction (3) does not mix the states $^{6,2}P$ and $^{6,4}P$ with the state $^{\overline{15},2}P$, so that the latter is uniquely specified. We shall therefore give the energies of ${}^9_\Lambda\mathrm{Be}^*$ levels relative to that for the $I=1$ ${}^{\overline{15},2}P$ level.

For the supersymmetric state, the use of the physical $V_{\Lambda N}$ through the addition of the SU(3)-breaking term (4) to the potential V_{BB} increases the symmetry energy difference ΔE between it and the $I=1$ ${}^9_\Lambda\mathrm{Be}^*$ analog state from the value $\Delta E^{(0)}$, by the addition

$$\Delta E^{(1)}({}^9_\Lambda\mathrm{Be}^*) = \tfrac{29}{125} f^{(2)} + g^{(0)} + \tfrac{4}{125} g^{(2)} \simeq -2.73 \text{ to } -1.65 \text{ MeV}, \qquad (6)$$

as ϵ' ranges from 0 to 0.5. One-body spin-orbit interactions of the form $\zeta\vec{s}\cdot\vec{l}$ can also be introduced readily for N and Λ, and the mixings produced between the various substates of the hypernuclear [4, 1] and [5] supermultiplets can be calculated. However, all these effects are estimated to be negligible for the separation $\Delta E({}^9_\Lambda\mathrm{Be}^*)$. There are also contributions generated by intermediate-coupling effects, such as the admixture of ${}^2D[4,1]$; these can be estimated from existing nuclear calculations[11] and increase $\Delta E^{(0)}$ by about $\Delta E_{\mathrm{IC}} \simeq -3$ MeV. Thus, our estimate is that the ${}^9_\Lambda\mathrm{Be}^*$ supersymmetric state will lie about

$$\Delta E = \Delta E^{(0)} + \Delta E^{(1)} + \Delta E_{\mathrm{IC}} \simeq 21 - 2 - 3 = 16 \text{ MeV} \qquad (7)$$

below the $I=1$ analog state. The situation is depicted in Fig. 1(b).

At this point, it is clearly necessary to discuss the identification of the ${}^9_\Lambda\mathrm{Be}^*$ analog states in the data.[2] The $I=0$ levels of ${}^9_\Lambda\mathrm{Be}^*$ turn out to be more complicated than those for $I=1$. In the SU(3) approximation, the $I=0$ states ${}^{\overline{15},2}P$, ${}^{\overline{3},2}P$, and ${}^{\overline{3},4}P$ are degenerate, but the SU(3)-breaking central forces (4) have a large off-diagonal matrix element between the first two of these states, with value $U = 5.4$ to 3.3 MeV as ϵ' varies from 0 to 0.5, which dominates the situation. The result is that the two $I=0$ states ${}^{\overline{15},2}P$ and ${}^{\overline{3},2}P$ become strongly mixed, almost one-to-one and with a substantial energy separation (approximately $2U$), so that there are predicted to be two $I=0$ states ${}^9_\Lambda\mathrm{Be}^*$ which can be excited from ${}^9\mathrm{Be}$ in small-momentum-transfer strangeness-exchange reactions. In more detail, with the parameter values given above and as ϵ' varies from 0 to 0.5, the upper level (originally ${}^{\overline{3},2}P$) is shifted upward to an energy value essentially coincident with the $I=1$ analog state, and the lower level (originally ${}^{\overline{15},2}P$) is shifted down to a position between 11.2 and 6.7 MeV below the $I=1$ analog state, the mixing angle being given by $\tan\theta \simeq 0.75$. In strangeness-exchange reactions, these upper and lower $I=0$ states will be excited in proportion $\sin^2\theta:\cos^2\theta$, and the $I=1$ analog state in proportion 0.5 relative to their sum.

This picture shows some concordance with the data[2] on ${}^9_\Lambda\mathrm{Be}^*$ excitation, where two prominent excitations are seen, with comparable intensity; the spacing between the peaks is observed to be 11 MeV, quite comparable with the value calculated above. This interpretation of the data is novel, in that the upper peak consists of *two* states, with $I=0$ and 1, and the lower peak has $I=0$ only. Some such multiplicity for the strangeness analog states should be considered the normal situation for most hypernuclei, according to the above discussion, since an SU(6) symmetry implies the existence of a number of nonanalog states intimately related with the analog state and approximately degenerate with it. In general, the strength of the strangeness-exchange excitation peak will be distributed over a number of hypernuclear excitations, and the broad peaks observed will generally have detailed structure.

If we accept the above picture of the ${}^9_\Lambda\mathrm{Be}^*$ data, and locate the $I=1$ analog state with the upper peak, so that it lies about 27 MeV above the ground state ${}^9_\Lambda\mathrm{Be}$, then the supersymmetric state would be expected to be, according to the calculation (7) above, at excitation energy about 11 MeV above ${}^9_\Lambda\mathrm{Be}$.

Qualitatively similar situations are expected for heavier p-shell hypernuclei, where the L-S description becomes less and less appropriate. We turn now to ${}^{16}_\Lambda\mathrm{O}$ at the opposite end of the p shell, where j-j coupling provides a good approximation. Here two $J^\pi = 0^+$ states occur, whose basis states may be arbitrarily chosen as $(p_{3/2})_n^{-1} \times (p_{3/2})_\Lambda$ and $(p_{1/2})_n^{-1}(p_{1/2})_\Lambda$. For an SU(3)-invariant one-body spin-orbit potential, which means $\zeta_\Lambda = \zeta_N$, these two basis states are degenerate in first approximation. The ΛN residual interaction removes the degeneracy in such a way that the upper state ψ_a is precisely the analog state ${}^1S_0$, and the lower state ψ_s, necessarily ${}^3P_0$, is the 0^+ supersymmetric state. The direct formation of the latter in the (K^-,π^-) reaction is forbidden in the forward direction because the transition ${}^{16}\mathrm{O}({}^1S_0) \to {}^{16}_\Lambda\mathrm{O}({}^3P_0)$ requires spin flip.

If $\delta\zeta \equiv \zeta_\Lambda - \zeta_N \neq 0$, then *both* 0^+ ${}^{16}_\Lambda\mathrm{O}$ eigenstates will have a component of ψ_a and their observation as two separate peaks is expected, unless their splitting is rather small or their decay widths particularly large. Diagonalization of the $(n^{-1}\Lambda)$ energy matrix [in the basis (ψ_a, ψ_s)]

$$\begin{pmatrix} -(1-\epsilon')\frac{2}{5}F_{\Lambda N}{}^{(2)}+\epsilon'(-F_{\Lambda N}{}^{(0)}+\frac{4}{5}F_{\Lambda N}{}^{(2)}) & -\delta\zeta/(2)^{1/2} \\ -\delta\zeta/(2)^{1/2} & (1-\epsilon')\frac{1}{5}F_{\Lambda N}{}^{(2)}+\epsilon'(2F_{\Lambda N}{}^{(0)}+\frac{4}{5}F_{\Lambda N}{}^{(2)})-\delta\zeta/2 \end{pmatrix}$$

then gives the relative energies of the two physical states, and their mixing angle, as defined by the form $\cos\theta\psi_a+\sin\theta\psi_s$ for the upper state. The conclusions depend strongly on $\delta\zeta$ but, with $\epsilon'=\frac{1}{4}$ and $-1.3\leq\delta\zeta\leq 2$ MeV, the relative intensity for production of the supersymmetric state (given by $\tan^2\theta$) is less than 5% (while the energy separation varies from 4.5 to 6.9 MeV). This is consistent with the empirical observation of only *one* strong and narrow ${}^{16}_{\Lambda}$O excitation in the appropriate energy interval.[2]

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[6]Cf. p. 742 of Kerman and Lipkin, Ref. 5.

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[8]In particular, we use the term "analog state" here and elsewhere in this Letter to refer to the state obtained by substituting a p-shell Λ for a p-shell neutron in the same space-spin state and by summing the result over all p-shell neutrons in the nuclear parent state.

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[12]This may be compared with the corresponding symmetry energy difference for the lowest states ^{8}Be*($I=1$) and ^{8}Be($I=0$), which have orbital permutation symmetries [3,1] and [4], respectively. With the same parameters a value $\Delta E({}^8\text{Be})=18.1$ MeV is obtained, rather close to the observed value of 17.6 MeV.

[13]Here and elsewhere in this Letter, we take the view that the term proportional to $f^{(0)}$ is already contained within the SU(3)-breaking single-particle potential, the expectation value of which is given by $\delta\overline{V}(1p)$.

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