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Color Gluon Excitation at Fermilab

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In unified gauge theories with color (excitable by lepton probes), color effects become manifest in the asymptotic region through gluon terms. Several new results pertaining to these are derived and quantitatively discussed. These may be testable at energies higher than 50 GeV now available.

Color[1] as a hadron symmetry orthogonal to the usual internal symmetries is an elegant concept. Although invariant under strong interactions, it may be broken by weak and electromagnetic forces.[2] Such is the case in the integer-charge quark model of Han and Nambu[1] incorporated[3] within a unified gauge theory. Thus it may be relatively easy to excite color in deep-inelastic lepton-hadron processes. This question, currently of great theoretical and experimental interest,[4] however hinges on the important issue of the location of color threshold. If the masses of the lowest colored hadrons are chosen in the range of 3 to 5 GeV, simple-minded parton calculations[5] with Han-Nambu quarks predict 50 to 200% increases in the deep-inelastic electromagnetic structure functions from Stanford Linear Accelerator Center (SLAC) energies (< 20 GeV) to Fermi National Accelerator Laboratory energies (> 50 GeV), in gross contradiction with the muon data[6] from the latter laboratory. This has led to the widespread belief[4] that color threshold, if any, must be at enormously high energies.

Recently, a counterargument has emerged against the above logic. In spontaneously broken unified color-gauge theories, mixing[7] among the gauge bosons invalidates naive parton calculations[5] above color threshold. Mixing leads to quadratic couplings[7,8] between various physical vector bosons. In deep-inelastic lepton-induced transitions into colored hadrons, the highly virtual photon or W boson now has not only a direct point coupling but also a gluon-mediated vector-dominance type of coupling to any parton. A negative sign between these two terms leads[9] to a factor $-m_g{}^2(Q^2+m_g{}^2)^{-1}$ in the amplitude, where m_g is the gluon mass and Q^2 is the square of the absolute momentum transfer. This observation has been made independently by Pati and Salam[10] and by the present authors[11] (see also Ref. 8). However, as shown below and in Ref. 11 but contrary to the assertion in Ref. 10, this does not imply the vanishing of the color-excited parts of the deep-inelastic scale functions far above color threshold.

In these theories consistency requires both quark partons and gluon partons[12] although the latter respond to lepton probes only above the color threshold. Direct point coupling leads to Bjorken scaling[13] in the case of quark partons. The same—in the case of gluon partons and in particular with a Yang-Mills vertex—leads to[14] a power violation of Bjorken scaling involving $Q^2m_g{}^{-2}$ in F_1 and both $Q^2m_g{}^{-2}$ and $Q^4m_g{}^{-4}$ in F_2 (using standard deep-inelastic terminology). Thus the suppression factor $m_g{}^4(Q^2+m_g{}^2)^{-2}$ does make the quark contributions to the color-excited structure functions vanish[15] for $Q^2 \gg m_g{}^2$,[10,11] i.e., in *deep inelastic lepton-hadron reactions far above gluon masses and color threshold, integrally charged quarks behave as fractionally charged ones.*[16] In more detail,[11] the Han-Nambu quark now has a momentum-dependent "effective charge" $e(Q)=e_s+e_n m_g{}^2(Q^2+m_g{}^2)^{-1}$, with subscripts s and n referring to the color singlet (Gell-Mann–Zweig) and nonsinglet parts, respectively. Below color threshold $e=e_s$. Above color threshold $e=e_s+e_n$ for $Q^2 \ll m_g{}^2$ and again $e=e_s$ for $Q^2 \gg m_g{}^2$. But while the asymptotic gluon contribution to F_1 vanishes, that to F_2 does not and in fact scales.[11,17,8] The gluon-partons then lead to color excitation in the asymptotic region. If color threshold is presumed to be somewhere between SLAC/CERN energies (< 20 GeV) and those within reach at Fermilab (50–300 GeV) where the asymptotic region with color excitation is assumed to be attainable, *the new color-gluon excitation effect is predicted at around the 15% level in muon scattering at Fermilab for a specific gauge model.*

Despite the validity of the general reasoning given above in a large class of gauge theories, for concreteness let us choose the group[3,18] $SU_{\mathrm{color}}(3) \otimes SU_L(2) \otimes U(1)$ incorporating the Weinberg-Salam theory and Han-Nambu quarks interacting with an octet of color gluons. The symmetry breaking is taken to give the gluon octet a single mass (to preserve color symmetry to zeroth order in the semiweak coupling constants); otherwise arbitrary ratios of gluon masses enter (Cheng and Wilczek[8]) the expressions for the scale functions. We then find[11]

$$F_{1,3}{}^{>}(x) = F_{1,3}{}^{<}(x), \tag{1a}$$

$$F_2{}^{>\mathrm{CC}}(x) = F_2{}^{<\mathrm{CC}}(x) + \tfrac{1}{2}xg(x), \tag{1b}$$

$$F_2{}^{>\mathrm{NC}}(x) = F_2{}^{<\mathrm{NC}}(x) + (1+\tfrac{1}{3}\tan^4\theta)\tfrac{1}{4}xg(x), \tag{1c}$$

$$F_2{}^{>\mathrm{EM}}(x) = F_2{}^{<\mathrm{EM}}(x) + \tfrac{1}{3}xg(x). \tag{1d}$$

Here the superscripts < and > refer to scaling regions below and above the color threshold, respectively; and CC, NC, and EM refer to charged weak current,[19] neutral weak current,[19] and electromagnetic current cases, respectively. $F^{<}(x)$ equals $F_{\mathrm{WS}}(x)$, i.e., as calculated[20] in the Weinberg-Salam theory (with θ as the Weinberg-Salam mixing angle) with fractionally charged Gell-Mann–Zweig quarks. The function $g(x)$ is the probability-averaged population density of each type of gluon with a fraction x of the longitudinal momentum of the nucleon in the infinite-momentum frame.

The novel feature of this note is the extraction of quantitative information from Eqs. (1) on color-excitation effects by the use of the momentum-conservation sum rule.[12,13] With an isospin-averaged nucleon and ignoring the square of the sine of the Cabibbo angle, one obtains[21]

$$8\int_0^1 dx\, xg(x) = 1 - 9\int_0^1 dx[F_2{}^{<\mathrm{EM}}(x) - \tfrac{1}{6}F_2{}^{<\mathrm{CC}}(x)]. \tag{2}$$

The Callan-Gross equality is $F_2{}^{<\mathrm{EM}}(x) = 2xF_1{}^{<\mathrm{EM}}(x)$, but far above the color threshold we have $F_2{}^{>\mathrm{EM}}(x) - 2xF_1{}^{>\mathrm{EM}}(x) = \frac{1}{3}xg(x)$. Using the notation $\delta f \equiv f^{>} - f^{<}$ for any object f, we have

$$\int_0^1 dx\, \delta F_2{}^{\mathrm{EM}}(x) = \int_0^1 dx[F_2{}^{>\mathrm{EM}}(x) - 2xF_1{}^{>\mathrm{EM}}(x)] = \tfrac{1}{3}\int_0^1 dx\, xg(x). \tag{3}$$

Evaluating $\int_0^1 dx\, xg(x)$ from SLAC/CERN data[22] through Eq. (2), we predict

$$\int_0^1 dx\, \delta F_2{}^{\mathrm{EM}}(x)\{\int_0^1 dx\, F_2{}^{<\mathrm{EM}}(x)\}^{-1} = \int_0^1 dx[F_2{}^{>\mathrm{EM}}(x) - 2xF_1{}^{>\mathrm{EM}}(x)]\{\int_0^1 dx\, F_2{}^{<\mathrm{EM}}(x)\}^{-1} \simeq 15\%. \tag{4}$$

The exact number in the right-hand side of Eq. (4) is 0.15 ± 0.06; but if one ignores the "sea" of quark-antiquark pairs, Eq. (2) can be replaced by $8\int_0^1 dx\, xg(x) = 1 - \int_0^1 dx\, F_2{}^{<\mathrm{CC}}(x)$ in which case, from neutrino data alone,[22] $\int_0^1 dx\, xg(x) = 0.061 \pm 0.006$ and the right-hand side of Eq. (4) becomes 0.15 ± 0.02. *We hope that this particular color-excitation test will receive the urgent attention of the muon experimentalists at Fermilab.*[23] Next, we convert the integrated weak scale functions into total cross sections to obtain[23]

$$\frac{4\pi}{G^2M}\delta\left(\frac{\sigma_{\nu N}{}^{\mathrm{CC}}}{E}\right) = \frac{4\pi}{G^2M}\delta\left(\frac{\sigma_{\bar{\nu}N}{}^{\mathrm{CC}}}{E}\right) = \int_0^1 dx\, xg(x) = 0.061 \pm 0.006, \tag{5a}$$

$$\frac{8\pi}{G^2M}\delta\left(\frac{\sigma_{\nu N}{}^{\mathrm{NC}}}{E}\right) = \frac{8\pi}{G^2M}\delta\left(\frac{\sigma_{\bar{\nu}N}{}^{\mathrm{NC}}}{E}\right) = (1+\tfrac{1}{3}\tan^4\theta)\int_0^1 dx\, xg(x) = (1+\tfrac{1}{3}\tan^4\theta)(0.061 \pm 0.006). \tag{5b}$$

These effects are in fact quite small. For instance, using the known data[22] on $(\nu+\bar{\nu})N$ total cross sections we find

$$\delta(\sigma_{(\nu+\bar{\nu})N}{}^{\mathrm{CC}}/E)(\sigma_{(\nu+\bar{\nu})N}{}^{<\mathrm{CC}}/E^{<})^{-1} \sim 5\%. \tag{6}$$

This is somewhat comforting since the charged-current total cross sections at Fermilab are known to be consistent with smooth linear (energywise) extrapolations at CERN.

We next observe that if we neglect the sea of quark-antiquark pairs in the nucleon, we obtain the following additional results which, although less reliable, are nevertheless interesting. (A) Firstly,

$$\sigma_{\bar{\nu}N}{}^{>\mathrm{CC}} - \tfrac{1}{3}\,\sigma_{\nu N}{}^{>\mathrm{CC}} = (G^2ME^{>}/\pi)\int_0^1 dx\, x g(x). \tag{7}$$

Putting in the numbers and the relevant experimental information,[22] one finds that

$$(1/E^{>})(\sigma_{\bar{\nu}N}{}^{>\mathrm{CC}} - \tfrac{1}{3}\,\sigma_{\nu N}{}^{>\mathrm{CC}})[(1/E^{<})\sigma_{\bar{\nu}N}{}^{<\mathrm{CC}}]^{-1} = 0.028 \pm 0.004. \tag{8}$$

(B) Secondly,[25]

$$\int_0^1 dx[F_2{}^{>\mathrm{EM}}(x) - \tfrac{5}{18}F_2{}^{>\mathrm{CC}}(x)] = \tfrac{7}{36}\int_0^1 dx\, x g(x). \tag{9}$$

Numerically, Eq. (9) leads to

$$\int_0^1 dx[F_2{}^{>\mathrm{EM}}(x) - \tfrac{5}{18}F_2{}^{>\mathrm{CC}}(x)]\left[\int_0^1 dx F_2{}^{<\mathrm{EM}}(x)\right]^{-1} = 0.085 \pm 0.007. \tag{10}$$

(C) Thirdly,

$$\frac{\pi G^{-2}}{M}\,\delta\left[\frac{d^2\sigma_{\bar{\nu}N}{}^{\mathrm{CC}}}{E\,dx\,dy}\right] = \frac{\pi G^{-2}}{M}\,\delta\left[\frac{d^2\sigma_{\nu N}{}^{\mathrm{CC}}}{E\,dx\,dy}\right] = (1-y)\frac{x}{2}g(x),$$

where y is the inelasticity variable.[13] The linear gluon term in y, if neither too large nor too small, will produce a relatively more significant effect in the $\bar{\nu}$ case where the colorless y distribution is proportional to $(1-y)^2$ than in the ν case where the same is y independent. This may have something to do with the y anomaly[26] in $\bar{\nu}$ scattering.

In e^+e^- annihilation into hadrons in the asymptotic region with color excitation, gluon contributions are small in the present gauge model; in fact they change[11] R (the ratio of the annihilation cross sections into hadrons and muons) in the asymptotic region by only $\frac{1}{8}$. This is independent of whatever may be the interpretation of the recently observed narrow resonances in e^+e^- annihilation.

We have presented the numerical results for a particular gauge model. The 15% effect predicted in muon scattering is of course encouraging in the light of the strength of the apparent scaling violations observed at Fermilab.[6] However, the basic point that we wish to make holds independent of the gauge group chosen and many of our results are easily generalizable to bigger gauge groups. Actually, for the electromagnetic part, it is straightforward to write the results in a more general form. A gluon pair of charges $\pm Q_g$ contributes $\frac{1}{6}Q_g{}^2xg(x)$ to $F_2{}^{\mathrm{EM}}$ and $\frac{1}{16}Q_g{}^2$ to R in the asymptotic region above color threshold (provided all the gluons, neutral as well as charged, have equal mass).

In this note we have pursued the consequences of color-symmetry breaking by weak and electromagnetic forces treated within a unified gauge model. There exists the alternative point of view that color is an exact symmetry.[27] The observation of gluon excitations (or the lack of it) in very deep-inelastic lepton-hadron scattering in the way suggested here will thus distinguish between the two points of view.

In conclusion, Han-Nambu and Gell-Mann–Zweig quarks behave alike in deep-inelastic lepton-hadron scattering as well as in e^+e^- annihilation far above color threshold where color effects emerge only through gluon scaling terms. If the said threshold is presumed to have set in below energies within reach at Fermilab, color effects are not of the order of 50 to 200% as naively expected[5] but are at a smaller level, for instance around 15% in muon scattering. Color is excited not too predominantly in the asymptotic region but it should nonetheless be visible through gluons.

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[15]For $Q^2 \sim m_g{}^2$, the parton description is invalid; the structure functions may not be separable into quark and gluon parts; any numerical estimate (Ref. 10) of the impact of the suppression factor for the quark part is dubious. Note that m_g is the effective mass of the gluon-parton which may be quite small compared to the masses of colored hadrons just as the effective quark-parton mass is believed to be smaller than usual hadronic masses.

[16]Color-singlet parts cannot distinquish between these; H. Lipkin, Phys. Rev. Lett. 28, 63 (1972).

[17]No gluon contribution to F_3 exists in gauge theories with only vector currents exciting color gluons.

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[19]Gluon contributions to the ν and $\bar{\nu}$ scale functions are identical.

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[21]Equation (2), valid without Glashow-Iliopoulos-Maiani charm, becomes an inequality (with left-hand side $\geq$ right-hand side enlarging the gluon contribution) if charmed quarks and antiquarks are subsumed in the nucleon, and the parton framework is maintained. However, the parton picture may not be valid (*vis à vis* the heavier charmed quark) at SLAC/CERN energies < 20 GeV (where, of course, charm is supposedly unexcited in the scattering experiments), in which case Eq. (2) is true if the charm contribution to the sea of quark-antiquark pairs is summarily ignored.

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[23]The predicted violation of the Callan-Gross relation given in Eq. (4) distinguishes color excitation from pure charm excitation in the asymptotic region.

[24]If charm is excited at Fermilab, there should be additional positive contributions to the numbers given in Eqs. (5a) and (6).

[25]Charge symmetry holds so that $F_{2\nu N}=F_{2\bar{\nu}N}$.

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