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 1 A. Benvenuti et al., Phys. Rev. Lett. 34, 419 (1975). v^2 V. Barger, T. Weiler, and R. J. N. Phillips, Phys. Rev. Lett. 35, 692 (1975).

 3 A. Pais and S. B. Treiman, Phys. Rev. Lett. 35, 1206 (1975).

 4 J. T. Dakin et al., Phys. Rev. D 10, 1401 (1974).

 5 A. Benvenuti et al., Phys. Rev. Lett. 35, 1199 (1975).

6While the necessity for such a step is clear the precise details are hopelessly complicated. The F is expected to have a Gaussian distribution in transverse momentum $\exp(-BP_T^2)$. Some of this momentum is

transferred to the μ upon decay. A nonrelativistic calculation (valid since $E \gg P_\tau$) gives

$$
(P_T{}^{\mu})_{\text{obs}} = P_T{}^{\mu} + (E_{\mu}/E_F) P_T{}^F, \quad 0 < E_{\mu}/E_F < 1.
$$

But $E_{\mu}/E_{\bm{F}}$ is not observable. I have chosen to consider

$$
\rho((P_T^{\mu})_{obs}) = \int_{-\infty}^{\infty} e^{-AP^2} \rho(P_T^{\mu} + P) \ dP
$$

and obtain reasonable results for A in the range 6 to 27. On the average A is simply related to B :

 $A \simeq B \langle (E_{\mu}/E_F)^2 \rangle^{-1}$.

 ${}^{7}L$. M. Chounet, J. M. Gaillard, and M. K. Gaillard, Phys. Bep. 4C, 199 (1972).

 8 This mode is suggested by the recent observation of μe events at SPEAR. $L^+ \rightarrow \overline{\nu}_e \mu^+ \nu_\mu$ is a possible explanation of these data. See also S. Y. Park and A. Yildiz, "Heavy Lepton Production in e^+e^+ Annihilation" (to be published) .

 $9A.$ Benvenuti et al., Phys. Rev. Lett. 35, 1249 (1975).

The Unresolved $(\pi, \pi N)$ Puzzle

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The importance of charge-exchange interactions of nucleons within the nucleus following $(\pi, \pi N)$ reactions as proposed recently by Sternheim and Silbar may be quantitatively appraised. An experiment datum already exists which contradicts the hypothesis that charge exchange is so influential.

A recent Letter by Sternheim and Silbar' showed that their treatment of final-state interactions in the quasifree-pion-induced one-step neutron knockout reaction on ${}^{12}C$ in the vicinity of the (3, 3) resonance could successfully vitiate an apparent discrepancy between theory and experiment. In particular, for a self-conjugate target nucleus the impulse approximation leads to a neutron knockout cross section ratio $\theta = \sigma^2 / \sigma^+$ which parallels the free- π ⁺ pion-nucleon cross section ratio of about 3 between 100 and 300 MeV. New measurements² on ${}^{12}C$ demonstrated, however, that the ratio @increases from unity to about 1.8 over this region. Sternheim and Silbar proposed that the ratio is altered from the simple theory because of the nonnegligible probability P of charge exchange of the struck nucleon such that

$$
\Re = \sigma^2 / \sigma^2 \approx (9 - 8P)/(3 + 6P). \tag{1}
$$

Estimation of the charge-exchange probability was accomplished with a semiclassical transport model and the effect of the Pauli exclusion principal was accommodated through normalization to the experimental ratio at one point. The contribution of the final-state charge exchange as measured by the factor P accordingly varies from about 50% to 20% over the range of experimental values. A follow-up note by Silbar³ showed that the consideration of a final-state interaction of the scattered pion altered the above conclusion insignificantly.

Extension of the Sternheim-Silbar model with no additional assumptions offers the means for a very stringent test of the charge-exchange hypothesis. In particular the following nucleon knockout reactions succeeded by the implied charge exchanges are speeifieally predicted:

$$
{}^{12}\mathrm{C}(\pi^+,\pi^0p)^{11}\mathrm{C} \xrightarrow{(\rho,\,n)} {}^{11}\mathrm{N},\tag{2}
$$

$$
^{12}\mathrm{C}(\pi^-, \pi^0 n)^{11}\mathrm{B} \xrightarrow{(n, p)} {^{11}\mathrm{Be}}.\tag{3}
$$

Ignoring double-charge -exchange contributions, ⁴ and two-step paths with their small cross sections, one has according to the Sternheim-Silbar model

$$
\frac{\sigma[^{12}\text{C}(\pi^+,\pi^0 n)^{11}\text{N}]}{\sigma[^{12}\text{C}(\pi^+,\pi^+ n+\pi^0 p)^{12}\text{C}]} \approx \frac{2P}{3+6P},\tag{4}
$$

and

$$
\frac{\sigma[^{12}C(\pi^-, \pi^0 p)^{11}Be]}{\sigma[^{12}C(\pi^-, \pi^- n)^{11}C]} \approx \frac{2P}{9-8P}.
$$
\n(5)

Ratios (4) and (5) predict that the cross sections for Reactions (2) and (3) should attain the rather sizable values of \sim 5 mb at and just above the (3, 3) resonance even in the absence of contributions from other reaction paths.

¹¹Be is a hard- β (11.5 MeV maximum) emitting isotope with a 13.8-sec half-life and is naturally suited to the type of straightforward radioactivation determination employed in the 11 C production experiments. Furthermore, since the production of ¹¹Be relative to ¹¹C from π ⁻ activation of carbon is sufficient to verify the above implication, absolute beam determinations and their inherent uncertainties are completely unnecessary. Production of ¹¹Be from ¹²C by π ⁻ mesons across the (3, 3) resonance would consequently be an extremely worthy venture.

As it turns out, there does already exist an indication that the success of the Sternheim-Silbar model may be illusory. The production cross section of ${}^{10}C$ from ${}^{11}B$ at the (3, 3) resonance has been reported to be 0.85 mb.⁵ Formation of ^{10}C from ^{11}B should follow a process similar to Reaction (2); that is

$$
^{11}\mathrm{B}(\pi^{+},\pi^{0}p)^{10}\mathrm{B} \xrightarrow{(\hat{p},n)} {^{10}\mathrm{C}}.
$$
 (6)

However, a σ less than 1 mb is dramatically inconsistent with the strength of the hypothetical charge-exchange effect. Although the $^{11}B(\pi^+$, $\pi^+ n + \pi^0 p$ ¹⁰B formation cross section has not been completely defined, neutron knockout systematics suggest it is most likely large. As seen with high-energy proton-induced neutron knockout re- \arctan^6 all high-energy pion-induced neutron knockout reactions' with the understood exception⁸ of $14N$ have very roughly the same cross sections, $30-70$ mb near the $(3, 3)$ resonance. Consequently, the Sternheim-Silbar model would predict a cross section for Reaction (6) roughly a factor of ⁵ larger than observed, perhaps attesting to the unimportance of the charge-exchange contribution.

The possible failure of the simple semiclassical picture may not be so surprising in view of

the inability of intranuclear cascade calculations, which inherently include the Sternheim-Silbar effect in addition to further aspects omitted by the linear transport model, to predict correctly all pion-induced neutron-knockout cross sections.⁹ The $(\pi, \pi N)$ puzzle may just reflect unsuitability of the impulse approximation for these reactions. One of the necessary conditions for its applicability is that the incident particle never interact strongly with two constituents of the nucleus at the same time.¹⁰ Similarly, for lower energy incident or scattered pions and low-energy scattered nucleons, the de Broglie wavelengths become so large and/or mean free paths become so short as to weaken the justification for using two-particle collision kinematics at the (3, 3) resonance in the models employed. This point was elegantly discussed and demonstrated recent-
ly by Dover and Lemmer.¹¹ ly by Dover and Lemmer.¹¹

¹M. M. Sternheim and R. R. Silbar, Phys. Rev. Lett. 34, 824 (1975).

 \overline{P} B.J. Dropesky, G.W. Butler, C.J. Orth, R.A. Williams, G. Friedlander, M. A. Yates, and S. B. Kaufman, Phys. Rev. Lett. 34, 821 (1975).

 3 R. R. Silbar, Phys. Rev. C 12, 341 (1975).

⁴Yu. A. Batusov, S. A. Bunyatov, V. M. Siderov, and V. A. Yarba, Yad. Fiz. 6, 998 (1967) [Sov. J. Nucl. Phys. 6, 727 (1968)].

5D. T. Chivers, E. M. Rimmer, B.W. Allardyce, R. C. Witcomb, J.J. Domingo, and N. W. Tanner, Nucl. Phys. A126, 129 (1969).

 ${}^{6}P$. J. Karol and J. M. Miller, Phys. Rev. 166, 1089 (1968); W. J. Nieckarz, Jr., and A. A. Caretto, Jr., Phys. Rev. C 2, 1917 (1970).

 ${}^{7}P$. J. Karol, M. V. Yester, R. L. Klobuchar, and A. A. Caretto, Jr., Phys. Lett. 58B, 489 (1975); N. Jacob and S. S. Markowitz, LASL Report No. LA-5959-PR (unpublished); M. V. Yester, A. A. Caretto, Jr., M. Kaplan, P.J. Karol, and R. L. Klobuchar, Phys. Lett. 45B, 327 (1973); P. L. Reeder and S. S. Markowitz, Phys. Rev. 133, 8689 (1964); H. S. Plendl, D. Burch, K. A. Eberhard, M. Hamm, A. Richter, C.J. Umbarger, and W. P. Trower, Nucl. Phys. B44, 413 (1972).

 ${}^{8}P$. J. Karol, A. A. Caretto, Jr., R. L. Klobuchar, D. M. Montgomery, R. A. Williams, and M. V. Yester, Phys. Lett. 44B, 459 (1978).

⁹G. D. Harp, K. Chen, G. Friedlander, Z. Fraenkel, and J. M. Miller, Phys. Rev. ^C 8, ⁵⁸¹ (1978).

 10 G. F. Chew and G. C. Wick, Phys. Rev. 85, 636 (1952).

 ${}^{11}C$. B. Dover and R. H. Lemmer, Phys. Rev. C 7, 2312 (1973).