tioned above, preliminary indications are that it is considerably higher than the value observed at the lower bombarding energy (365 MeV).

The ^{63}Cu bombarding energy of 443 MeV is about 1.4 times the interaction barrier for ^{197}Au. This reaction can be compared with Th + Ar at 288 MeV and with Bi + Kr at 605 MeV also at ~ 1.4 times the interaction barrier.[4] Since the relative energy above the interaction barrier is similar in the three cases, our ^{63}Cu case is expected to be intermediate between the quasifission of Kr + Bi and the deep inelastic transfer from Ar + Th. The angular distribution in the Kr + Bi case is sharply peaked near the grazing angle, while it is more forward peaked in the Ar + Th case.[6] Thus our Cu + Au angular distribution with a broad maximum and a substantial contribution at 0° is ineed intermediate between the distributions obtained with Ar and Kr ions. These considerations indicate that quasifission and deep inelastic transfer processes are the same type of reaction, and that the observed angular distributions of products are determined by the magnitude of the repulsive Coulomb potential compared to the attractive nuclear potential.

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Simple Approximation for Multistep Amplitudes*

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A surface approximation to the multistep amplitude for direct reactions is presented. Within this approximation, the distorted-wave Born-approximation series is summed in closed form, and the result is tested by comparing with exact coupled-reaction-channels calculations for ^{17}O$(p,d,p)^{17}$O and ^{16}O$(d,p,d)^{16}$O. The approximation gives a good representation of the exact results, especially for the ^{16}O$(d,p,d)^{16}$O process which is dominated by strong absorption of the deuteron.

Multistep corrections to the distorted-wave Born approximation (DWBA) are currently of great interest,[1-9] especially in the light of recent observations[10] of direct reactions that cannot be adequately described by the DWBA. The present communication describes a new approximation[11] for the multistep transition amplitude which emphasizes the nuclear surface. For this reason our method may not be well adapted for the description of the dynamics of the interior, which is probably better represented by shell-model states than by asymptotic channels. However, for the rather large class of reactions involving strong absorption, the method appears promising. Conveniently, it requires only ordinary DWBA matrix elements which are easily provided by standard codes. Our method does not address questions of mathematical rigor. Rather, it aims to simplify realistic calculations of multistep contributions to direct reactions in which absorption is an important feature. Although the distorted-wave series has been criticized,[2,3,5,8,12] we use it to illustrate our method, which is also applicable to iterative versions of the coupled-reaction-channels[7] and coupled-integral-equations[2] approaches, as well as to the methods discussed in Refs. 3–5. We hope that our method will facilitate exploratory surveys of possible multistep processes and stimulate identification of dominant indirect mechanisms for any given reaction.

For the reaction $A(a,b)B$, the exact multistep correction to the partial-wave DWBA transition amplitude is given (in standard notation) by

$$T_{ba}{}^{M} = \langle \chi_b{}^{(-)} | \hat{V}_\beta G^{(+)} \hat{V}_\alpha | \chi_a{}^{(+)} \rangle . \tag{1}$$

Here, and throughout, Greek indices α, β, and γ refer to different mass partitions. The residual interactions in initial and final partitions are denoted by $\hat{V}_\alpha$ and $\hat{V}_\beta$, respectively, while $\chi_a{}^{(+)}$ and $\chi_b{}^{(-)}$ represent the initial- and final-channel distorted waves, including the internal states of the clusters and their relative orbital angular momenta L, M. For inelastic processes the total amplitude is obtained by adding $T_{ba}{}^{M}$ to the DWBA amplitude, while for elastic transitions ($a = b$) $T_{aa}{}^{M}$ is added to the amplitude generated by the optical potential that governs $\chi_a{}^{(+)}$. The complete Green's function, $G^{(+)}$, can be formally expressed in terms of the distorted-wave Green's function, $G_\gamma{}^{(+)}$, for the arrangement γ:

$$\begin{aligned} G^{(+)} &= G_\gamma{}^{(+)} (1 - \hat{V}_\gamma G_\gamma{}^{(+)})^{-1} \\ &= G_\gamma{}^{(+)} + G_\gamma{}^{(+)} \hat{V}_\gamma G_\gamma{}^{(+)} + \dots , \end{aligned} \tag{2}$$

where

$$G_\gamma{}^{(+)} = \sum_c | \Phi_c \rangle G_c (\vec{r}_\gamma , \vec{r}_\gamma{}') \langle \Phi_c | , \tag{3}$$

G_c being the Green's function corresponding to the channel (partial wave) c of arrangement γ. In principle, the sum in Eq. (3) is over any complete set of channels c belonging to a single arrangement γ. The selection of arrangement γ is arbitrary, as long as c is a complete set. In practice, it is necessary to restrict the channels c to a finite number N; the choice of γ and c then depends on physical insight. By making this truncation we can also avoid the mathematical difficulties[12] associated with three-body intermediate states, provided that we include only channels c in which both fragments are bound.

Combining Eqs. (1)–(3) gives the distorted-wave series

$$\begin{aligned} T_{ba}{}^{M} = &\sum_c \langle F_b | K_{bc} G_c K_{ca} | F_a \rangle \\ &+ \sum_{cc'} \langle F_b | K_{bc} G_c K_{cc'} G_{c'} K_{c'a} | F_a \rangle + \dots . \end{aligned} \tag{4}$$

Here $F_c(r_\gamma)$ is the radial distorted wave for channel c, and K_{bc} is the form factor that connects channels b and c.

We can express $K_{ca} F_a$ in terms of a new, local effective potential $\hat{U}_c$, for each channel c, as

$$\begin{aligned} &\int K_{ca}(r_\gamma , r_\alpha) F_a (r_\alpha) dr_\alpha \\ &= \frac{\langle F_c | K_{ca} | F_a \rangle}{\langle F_c | \hat{U}_c | F_c \rangle} \hat{U}_c (r_\gamma) F_c (r_\gamma) . \end{aligned} \tag{5}$$

This equation, which formally defines the auxiliary potential $\hat{U}_c$, specifies only its shape, but leaves its normalization at our disposal. Although $\hat{U}_c$ is in general complicated, we expect these complications to simplify if the form factor is surface localized. The surface approximation (SA) exploits this feature, replacing $\hat{U}_c$ by $\tilde{U}_c$, a smooth, surface-peaked potential. Since the magnitude of the auxiliary potential is arbitrary, we choose it small, so that $\tilde{U}_c$ can be treated as a "perturbation."

Let $\tilde{F}_c$ denote the perturbed scattering function obtained when $\tilde{U}_c$ is added to the potential generating F_c. Then, correct through first order in $\tilde{U}_c$, we have

$$\tilde{F}_c = F_c + G_c \tilde{U}_c F_c . \tag{6}$$

Equations (5) and (6) allow one to calculate $G_c K_{ca} F_a$ by solving the radial equations for $\tilde{F}_c$ and F_c, i.e., with and without the "perturbation" $\tilde{U}_c$. The amplitudes in Eq. (5) can be calculated by means of a DWBA program, because the DWBA amplitude for $a \to c$ is

$$T_{ca} = \langle F_c | K_{ca} | F_a \rangle , \tag{7}$$

and the perturbation of the optical elastic-scattering amplitude due to $\tilde{U}_c$ is

$$\tilde{t}_c - t_c = \langle F_c | \tilde{U}_c | F_c \rangle . \tag{8}$$

Combining Eqs. (5)–(8) yields

$$G_c K_{ca} F_a = (\tilde{F}_c - F_c) T_{ca} / (\tilde{t}_c - t_c) , \tag{9}$$

and substituting this (and analogous results) into Eq. (4) finally gives

$$T_{ba}{}^{M} = \sum_c D_{bc} T_{ca} + \sum_{cc'} D_{bc} D_{cc'} T_{c'a} + \dots , \tag{10}$$

where the quantities D are defined by

$$D_{bc} = \frac{\tilde{T}_{bc} - T_{bc}}{\tilde{t}_c - t_c} \text{ and } D_{cc'} = \frac{\tilde{T}_{cc'} - T_{cc'}}{\tilde{t}_{c'} - t_{c'}} . \tag{11}$$

Here $\tilde{T}$ is a DWBA amplitude calculated with a perturbed wave function on the right-hand side, e.g.,

$$\tilde{T}_{bc} = \langle F_b | K_{bc} | \tilde{F}_c \rangle . \tag{12}$$

The series (10) can be summed to yield a result in closed form:

$$T_{ba}{}^{M} = \sum_c \sum_{c'} D_{bc} [(1 - D)^{-1}]_{cc'} T_{c'a} , \tag{13}$$

where D is an $N \times N$ matrix in channel space. Equation (13) is our main result. Very conveniently, it involves only quantities which can be routinely calculated with a DWBA code. According to Eq. (13), the important indirect routes are not necessarily those involving intermediate states with large cross sections. Instead, the ratios D are influential. They are large for those steps where the DWBA amplitude is sensitive to changes in the optical potential.

We have applied our method to $^{17}\mathrm{O}(p,p)^{17}\mathrm{O}$ and $^{16}\mathrm{O}(d,d)^{16}\mathrm{O}$ elastic scattering in a model that includes a single internal state of $^{16}\mathrm{O}$, $^{17}\mathrm{O}$, and d. The model assumes that all bound states are s wave, and neglects nucleon spin and isospin effects. The nucleon-induced process, which does not involve strong absorption, should provide a stringent test of the SA method, since this reaction may involve multistep contributions from the interior of the nucleus. For these simple cases only one intermediate state, $c=c'$, connects the given initial and final channels a and b. For $^{17}\mathrm{O}(p,p)^{17}\mathrm{O}$ Eq. (13) now simplifies so that for each partial wave L we have

$$T_{pp}{}^{M}=T_{dp}D_{pd}/(1-D_{dd}),\tag{14}$$

with

$$D_{pd}=\frac{\tilde{T}_{pd}-T_{pd}}{\tilde{t}_{d}-t_{d}},\quad D_{dd}=\frac{\tilde{T}_{dd}-T_{dd}}{\tilde{t}_{d}-t_{d}}.\tag{15}$$

Similarly, for $^{16}\mathrm{O}(d,d)^{16}\mathrm{O}$,

$$T_{dd}{}^{M}=T_{pd}D_{dp}/(1-D_{pp}),\tag{16}$$

where D_{dp} and D_{pp} are defined by Eq. (15) with d and p interchanged throughout. In our calculations[1] folded optical potentials are used and consequently D_{dd} and D_{pp} vanish. Thus, for this application folded potentials eliminate third and higher-order contributions[1] and Eqs. (14) and (16) reduce to

$$T_{pp}{}^{M}=D_{pd}T_{dp},\quad T_{dd}{}^{M}=D_{dp}T_{pd}.\tag{17}$$

We have calculated $T_{pp}{}^{M}$ and $T_{dd}{}^{M}$ exactly, by applying[1] a coupled-reaction-channels code to the coupling of p and d channels. The respective incident laboratory energies are $E_p=10.98$ MeV and $E_d=10.49$ MeV. The Q value for the (p,d) transition to the intermediate state is -1.045 MeV. The nucleon-target interaction utilized in the folded-optical-potential calculation was taken from Wilmore and Hodgson.[13] Further details of this calculation can be found in Ref. 1.

Tables I and II show the magnitudes and phases of the exact multistep amplitudes for (p,p) and (d,d), compared with several approximations. The uncorrected optical-model amplitudes (denoted "optical" in Tables I and II) are also shown, to indicate the relative importance of different partial waves, and to show the phase relations between optical and multistep amplitudes. It is important to stress the insensitivity of our re-

TABLE I. Partial-wave transition amplitudes, $(2L+1)T_L$, for $^{17}\mathrm{O}(p,p)^{17}\mathrm{O}$.

a) MAGNITUDES

	OPTICAL	MULTISTEP			
L		EXACT	DERIVATIVE	VOLUME	ON-SHELL
0	.687	.065	.047	.065	.068
1	.961	.214	.102	.080	.070
2	1.650	.250	.218	.151	.430
3	2.229	.302	.061	.186	.060
4	.516	.289	.239	.210	.500
5	.106	.117	.107	.107	.103
6	.024	.038	.038	.037	.032
7	.006	.011	.010	.011	.010
8	.001	.003	.003	.003	.003
9	.000	.001	.001	.001	.001

b) PHASES (degrees)

	OPTICAL	MULTISTEP			
L		EXACT	DERIVATIVE	VOLUME	ON-SHELL
0	70	-97	-152	-99	-149
1	73	165	90	151	- 77
2	144	106	48	136	73
3	74	-123	35	143	83
4	19	161	99	113	173
5	7	137	105	92	147
6	3	123	81	73	114
7	1	122	84	67	96
8	0	123	105	65	92
9	0	124	110	64	91

TABLE II. Partial-wave transition amplitudes, $(2L+1)T_L$, for $^{16}\mathrm{O}(d,d)^{16}\mathrm{O}$.

a) MAGNITUDES

	OPTICAL	MULTISTEP			
L		EXACT	DERIVATIVE	VOLUME	ON-SHELL
0	.574	.041	.030	.021	.020
1	1.681	.095	.058	.060	.034
2	2.282	.183	.152	.050	.093
3	3.103	.032	.083	.063	.031
4	3.905	.241	.248	.270	.076
5	2.594	.251	.242	.322	.065
6	1.224	.153	.137	.211	.030
7	.379	.053	.051	.142	.009
8	.123	.020	.020	.085	.003
9	.041	.008	.008	.003	.001

b) PHASES (degrees)

	OPTICAL	MULTISTEP			
L		EXACT	DERIVATIVE	VOLUME	ON-SHELL
0	93	-151	-152	-155	118
1	81	118	110	68	193
2	86	41	34	53	73
3	102	13	119	102	160
4	86	117	103	107	158
5	61	75	68	76	127
6	28	34	31	28	104
7	15	15	15	15	93
8	12	10	10	11	91
9	11	7	6	- 25	90

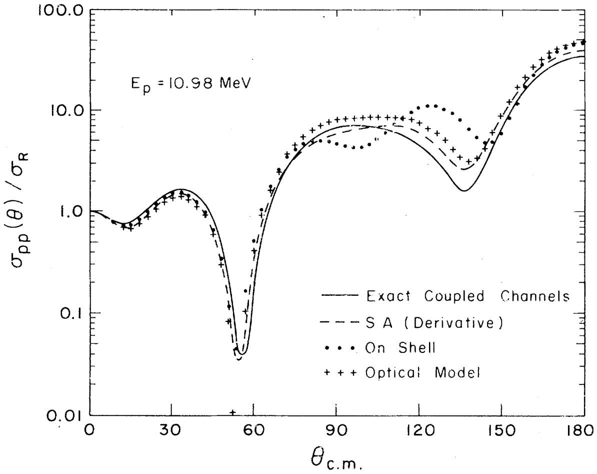

FIG. 1. Ratio of elastic differential cross section to Rutherford for $^{17}O(p,p)^{17}O$. Comparison of exact coupled-channels calculation (solid curve) with the on-shell (dotted curve) and derivative surface approximation (dashed curve).

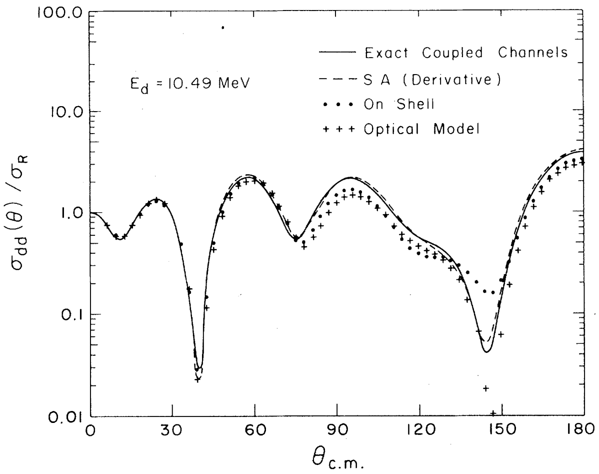

FIG. 2. Ratio of elastic differential cross section to Rutherford for $^{16}O(d,d)^{16}O$. Comparison of exact coupled-channels calculations (solid curve) with the on-shell (dotted curve) and derivative surface approximations (dashed curve).

sults to $\tilde{U}_c$. Two different choices of $\tilde{U}_c$ were used: the derivative of a real Woods-Saxon shape with diffuseness $a=1.0$ fm (denoted "derivative"), and a complex Woods-Saxon shape with $a=0.8$ fm (denoted "volume"). The radius parameter was assigned a value of $R=3.3$ fm. Unlike a previously described[9] surface-delta approximation, our method permits a realistic width for $\tilde{U}_c$. Use of values in the ranges $0.6 \leq a \leq 1.2$ and $2.7 \leq R \leq 3.9$ revealed little sensitivity to either a or R. The strength of the interactions also was varied from 0.25 to 10 MeV with negligible effect. Even the extreme difference between the real derivative and complex volume interactions only mildly affects the multistep amplitudes, again indicating an insensitivity to the choice of $\tilde{U}_c$. These results should be compared with the column labeled "on-shell," which represents the usual on-shell approximation.[14] This approximation also leads to the closed-form expression, Eq. (13), but with

$$D_{bc}=iT_{bc}/S_c \text{ and } D_{cc'}=iT_{cc'}/S_{c'}.$$

In general, the multistep amplitudes are large for the important (surface) partial waves and the overall multistep contribution produces a sizable effect in the angular distributions (presented in Ref. 1). For (p,p), the derivative and volume prescriptions both reproduce the exact $T_L{}^M$ better than the on-shell prescription. However, the considerable differences between derivative and volume results for (p,p) indicate that, in reactions with weak absorption in initial and final channels, the SA method is only good for estimates of magnitude, and for these reactions the volume choice may provide the better estimate.

For (d,d) the SA method is encouragingly successful, probably because strong absorption ensures that the surface dominates the reaction. The exact results are well reproduced both in magnitude and in phase, and the surface choice of $\tilde{U}_c$ is clearly superior. The on-shell approximation fails to predict either the magnitude or the phase.[15]

In Figs. 1 and 2 we present the exact and approximate angular distributions for the respective processes $^{17}O(p,d,p)^{17}O$ and $^{16}O(d,p,d)^{16}O$. These cross sections do not provide as sensitive a test as the partial-wave amplitudes do. However, they clearly reflect the superiority of the SA to the on-shell approximation.

We conclude that the SA method is a promising technique for exploratory calculations of multistep amplitudes, especially when there is strong absorption. Fortunately, strong absorption is typical of all reactions with composite projectiles, and is actually dominant in heavy-ion processes.[16]

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Extraction of Resonance Parameters from Nuclear Scattering Data*

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A procedure is described for extracting the pole position of a resonant amplitude from a knowledge of the phase shift versus energy. Pole positions are determined for the four p-wave resonances in the nucleon-α system with high precision. The use of the pole position to define the mass of unstable nuclear states is discussed.

A recent nuclear-energy-level compilation[1] indicates rather large uncertainties in the quoted masses and widths of the $p_{3/2}$ and $p_{1/2}$ resonances in ^{5}He and ^{5}Li (see Table I). It is customary to define the masses of these states in terms of the observed central energy of an experimental resonance peak and in the $p_{1/2}$ cases it is not surprising that large uncertainties are present, since the corresponding N-α phase shifts do not pass through $\pi/2$ and no sharp group appears in the experimental spectra. It could be argued that although the position of a prominent peak may be useful for characterizing a resonance experimentally, this position does not necessarily have a compelling theoretical significance as the definition of the mass of the level.

From the viewpoint of the analytic properties of the scattering amplitude, a resonance is characterized by its pole position and residue, and the pole position could serve as a suitable definition of the mass of an unstable level. In this paper we will study the utility of this alternative definition by extracting the pole positions of the N-α resonances from published energy-dependent phase-shift analyses. Since the pole occurs at an energy obviously not accessible to experimentation, our procedure involves a continuation of measured quantities to complex energies, but as we will see in the cases studied, the pole positions can be determined with much greater precision than that quoted in Table I. Similar results have been obtained in the π-N system[2] where the pole position of the 3-3 resonance has been accurately determined by a variety of methods.

TABLE I. Resonance parameters from Ref. 1.

Level	$E_{\text{c.m.}}$ (MeV)	$\Gamma_{\text{c.m.}}$ (MeV)
$n\alpha, p_{3/2}$	0.89 ± 0.05	0.30 ± 0.01
$n\alpha, p_{1/2}$	4.9 ± 1.5	2.0 ± 0.8
$p\alpha, p_{3/2}$	1.97 ± 0.05	≈ 0.8
$p\alpha, p_{1/2}$	7–12	2.5 ± 1.0

We first treat n-α scattering, adapting a method developed by Ball and Goble[3] to the chargeless case. We write the p-wave amplitude for either $J=\frac{3}{2}$ or $\frac{1}{2}$ in the usual way,

$$f(k)=\exp(i\delta)\sin\delta/k=k^2/(k^3\cot\delta-ik^3), \tag{1}$$

where k is the center-of-mass wave number and δ is the phase shift. Since $k^3\cot\delta$ is analytic[4] at $k=0$, the usual effective-range expansion can be