line is given by $E_x = (7.8\ \mathrm{MeV}^{-1})^{-1} J_c(J_c+1) + 12$ MeV. Assuming only that an equilibrated compound nucleus is formed in this reaction, the yrast line must lie at or below that indicated by the straight line in Fig. 2(b). The experimental data therefore suggest that the ^{26}Al nucleus (a) is deformed and undergoing rigid rotation for excitation energies E_x and angular momenta J_c given by the solid line in Fig. 2(b) and (b) has been formed at the highest bombarding energy with an angular momentum $[(25.6 \pm 1.4)\hbar]$ equal to the limit for a rotating liquid drop,[5] $\sim 26\hbar$.

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Physical Basis for Enhanced Forward Cross Sections in Heavy-Ion Reactions*

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Calculations show that indirect transitions can explain the forward cross section observed in some experiments, in particular ^{60}Ni(^{18}O, ^{16}O). Normal optical potentials which fit elastic and inelastic data are used in the analysis. An earlier analysis in terms of a surface transparent potential is shown to depend sensitively on a scaling factor that was introduced to simulate recoil effects. However when recoil is properly taken into account it is found that the surface transparent potential does not reproduce the data.

At moderate energies above the Coulomb barrier, quasi-elastic heavy-ion reactions are expected to exhibit a "grazing" peak in the differential cross section.[1] Brookhaven National Laboratory (BNL) experiments[2] produced the surprising result that several reactions of the type

$$^{60}\mathrm{Ni}(^{18}\mathrm{O}, ^{16}\mathrm{O})^{62}\mathrm{Ni}, \quad E = 65\ \mathrm{MeV}, \qquad (1)$$

had a large cross section forward of the grazing angle. Although indirect transfer can produce such effects,[3] this was not at first suspected to be the explanation because the experiment did not reveal any likely candidates as intermediate states. The BNL group proposed a surface transparent optical potential with the property that the edge of the absorptive part is very sharp and lies inside the real part.[4] These authors employed a scaling factor in the relationship between the coordinates in their distorted-wave Born approximation (DWBA) calculation which is different

from the one prescribed by the geometrical relationship between the coordinates.[5] This factor turns out to be an essential ingredient in their fit to the data. More precisely the DWBA amplitude can be written, with the neglect of recoil, as

$$T \propto \int \Psi_f^{(-)*}(\alpha\vec{R}) F(\vec{R}) \Psi_i^{(+)}(\vec{R})\, d^3R, \qquad \alpha = M/(M+X) \tag{2}$$

with M and $M+X$ being the masses of the target and residual heavy nuclei, and X being the mass of the transferred nucleon(s). While the factor α is less than unity, it is sometimes varied by a few percent in an attempt to simulate recoil effects.[6] The BNL group used a value of 1.05. Figure 1 shows the effect on the differential cross section of three choices for α, in each case employing the BNL surface transparent potential. It can be seen that, for this kind of potential, the cross section near the grazing angle depends very sensitively on the scaling factor. Therefore we need to consult a full recoil calculation. Delic[7] has calculated for us the cross section including recoil using the BNL potential. His results coincide in shape so closely with the curve labeled $\alpha = 1.02$ that they cannot be easily distinguished on the graph. We conclude therefore that with the proper inclusion of recoil effects, the surface transparent potential of Ref. 4 does not produce a sufficiently large forward cross section, compared to that at the grazing angle.

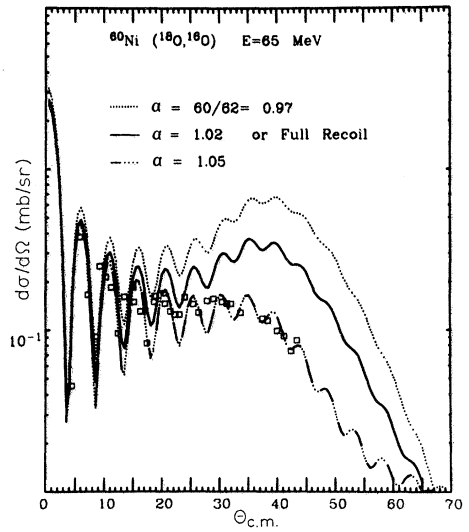

FIG. 1. Transfer cross sections in the no-recoil DWBA are shown for which the BNL transparent potential is used. The three cases correspond to different scaling factors [Eq. (2)]. The first is prescribed by the geometry; the last was employed by the BNL group, who intended that it should simulate recoil effects. The middle value reproduces with great accuracy a full recoil calculation (Ref. 7). Data are from Ref. 4.

We now turn to what we consider to be the physical basis for the large forward cross section seen in the experiment. The clue was provided in a previous paper which took into account the fact that for the reactions Sn(^{16}O, ^{18}O), the ejectile, ^{18}O, is produced in its excited 2^+ state more frequently than in its ground state.[8] Its subsequent de-excitation through an inelastic interaction with the residual nucleus has the effect on the ground-state cross section of (1) shifting the grazing peak forward by a few degrees and (2) producing a forward-angle yield that is 10 times larger than calculated from the direct process alone. The analogous process in the entrance channel of Reaction (1) above can explain the observed forward distribution when all relevant cross sections are determined to the extent possible by the available experiments.

The relevant experimental data needed to determine the important indirect processes and to assess their effect on the ground-state cross section of Reaction (1) are the following:

(i) The cross sections for producing the low excited states of ^{16}O and ^{62}Ni in Reaction (1). The lowest lying collective state is the 2^+ in Ni. To affect the ground state it would have to be not only strongly coupled, but have a significantly larger cross section than the ground state itself. Since it does not,[2] it is unimportant as an intermediate state.

(ii) The cross sections for excited states of ^{18}O and ^{60}Ni in the reaction inverse to (1). These data are not available. However, the cross section for a similar reaction was measured at Berkeley,[9] namely ^{60}Ni(^{16}O, ^{18}O)^{58}Ni. It was found that ^{18}O is produced in its excited state with a cross section 3.4 times larger than for the ground state at $\theta_{c.m.} = 32.6°$. Therefore the ^{18}O(2^+) state is a possibly important intermediate state in Reaction (1).

(iii) The elastic and inelastic cross sections for producing ^{18}O(2^+) by scattering from nickel determine the optical model parameters and the deformation parameter β which characterizes the strength of the inelastic transition.

As for the elastic and inelastic data,[10] these are shown for the neighboring target in Fig. 2 together with our calculation and the optical potential parameters. The inelastic scattering is calculated on the assumption of a macroscopic vibrational form factor for the 2^+ transition in ^{18}O.

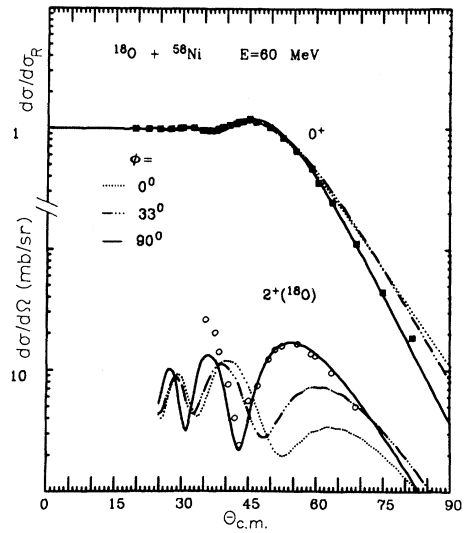

FIG. 2. Elastic and inelastic cross sections (coupled-channel Born approximation) are compared with the data (Ref. 10) for several values of the phase of the inelastic form factor [Eq. (3)] with deformation lengths $\beta_N R = 1.01$ fm and $\beta_C R = 0.94$ fm. The middle one corresponds to the formal collective model prescription specified by the optical potential. Optical potential parameters are $V = 70$, $W = 45$, $r = 1.19$, and $a = 0.56$. For $^{16}O + ^{58}Ni$, $W = 18.5$, $a = 0.54$, taken from Ref. 11.

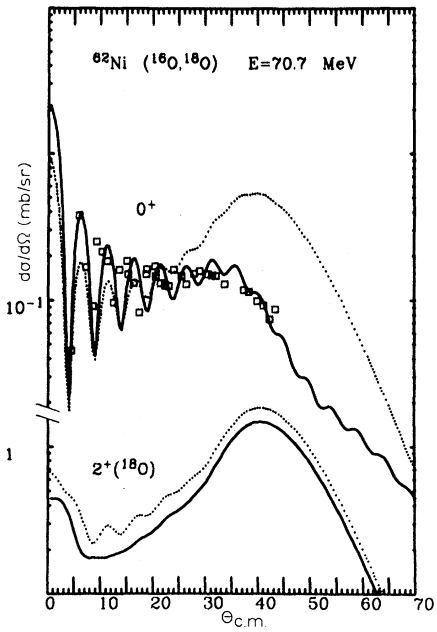

FIG. 3. The solid lines are cross sections based on a coupled-channel calculation in which both states of oxygen, the 0^+ and 2^+, are fed both by direct transfer and by indirect transfer through the other state. Dotted lines show cross sections computed for each state when only the direct transfer contributes (but the inelastic coupling is still included). Normal optical potentials were employed. All cross sections are normalized by the same factor. The ground-state data (Ref. 2) are for the time-reversed reaction at the corresponding energy of $E = 65$ MeV (in the lab).

The parameter φ referred to in the figure corresponds to the phase of the nuclear form factor,

$$F(r) = e^{i\varphi}(-\beta R)(V^2 + W^2)^{1/2}\,\partial f/\partial r, \qquad f = \{1 + \exp[(r - R)/a]\}^{-1}, \tag{3}$$

which according to frequent practice is the phase prescribed by the monopole part of the potential $V + iW$, namely,

$$\varphi = \tan^{-1}(W/V). \tag{4}$$

It is recognized that this need not be the case.[12] The value of $\varphi = 33°$ is prescribed by a strict interpretation of the vibrational model [Eq. (4)], but $\varphi = 90°$, which corresponds to a purely imaginary nuclear form factor, reproduces the data best of those shown, and is used in the subsequent calculations of transfer reactions. Actually it is probably relevant to observe that rather than rotating the nuclear form factor by the difference $90° - 33° = 57°$, a rotation of the Coulomb form factor by this amount produces the same result since the relative phase is the same in either case. Such a complex Coulomb form factor can arise because of virtual Coulomb excitations, just as the imaginary part of the nuclear optical potential arises, in part, from virtual inelastic excitations. While this would be an interesting point to pursue, for our purposes here it is sufficient to regard the above phase as a convenient means, together with the deformation constant β and the optical potential, of parametrizing the inelastic amplitude which enters as an intermediate step in the calculation of Reaction (1). We stress that the optical potential which fits the data as shown in Fig. 2 is of a normal strong absorbing type, unlike the BNL surface transparent potential.

In Fig. 3 our complete calculation is shown for the inverse reaction to (1) which includes the direct transition to the ground state of both final nuclei and the coherent indirect transition corresponding to particle transfer to the 2^+ state of ^{18}O followed by inelastic de-excitation to the ground state. We neglect the explicit calculation of recoil effects but employ the scaling factor $\alpha = 1.02$ which as far as the direct transition is concerned reproduces the angular distribution of the full recoil calculation, as discussed in connection

with Fig. 1. This point is not crucial, however, since with "normal" optical potentials the sensitivity to α is much less than indicated in Fig. 1. Also shown in Fig. 3 is the cross section for direct transfer alone, which by comparison allows us to see the large effect of the indirect transition on the ground-state cross section. In an earlier publication we gave a simple classical explanation of why the two-step process is forward biased compared to a single-step process.[8] In addition it is broader because each scattering process introduces its own dispersion on the previous one. (Thus a δ-function angular distribution centered at θ_0 scatters to θ_0 for a single scattering, and all angles between 0 and $2\theta_0$ for a double scattering.) Both effects are apparent in the figure. The ratio at the nuclear surface of the two transfer form factors involving, respectively, the ground and excited states of ^{18}O is consistent with the data of the neighboring reaction, in accordance with point (ii) above, as concerns the relative cross sections, although the forward peaking of the 2^+ cross section is not reproduced.[9] It is quite possible that its angular distribution would be modified by higher states just as transitions through it modify the ground-state cross section. Since, however, neither the experimental data for the 2^+ nor those for any higher state are available for the reaction inverse to (1), we leave this matter as it stands.

In conclusion, we have exhibited a physical process that is capable of accounting for the large forward cross section in the BNL experiment ^{60}Ni(^{18}O, ^{16}O)^{62}Ni at an energy where one would normally expect a grazing peak with sharply falling cross section on either side. This process consists of the inelastic excitation of the projectile ^{18}O followed by the transfer of two nucleons to form the ground states of ^{16}O and ^{62}Ni. The destructive interference of this process with the direct transfer causes a decrease of cross section in the region of the grazing angle where the two have comparable amplitudes. At angles forward of the grazing angle, the cross section is dominated by the second-order process, since, as discussed above, its distribution is more forward biased and broader.

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