Spontaneous Symmetry Breaking of Gauge Supersymmetry*

R. Arnowitt†
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138

and

Pran Nath
Department of Physics, Northeastern University, Boston, Massachusetts 02115
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Spontaneous symmetry breaking of gauge supersymmetry to global supersymmetry is seen to determine in part the internal-symmetry gauge group through the equations $R_{AB} = \lambda g_{AB}$. For the case $\lambda \neq 0$ the determination is complete, the resultant group being the U(1) Maxwell-Einstein coordinate group. For $\lambda = 0$, spontaneous breaking automatically requires parity breakdown in the vacuum, and is consistent with SU(2)$\otimes$U(1)$\otimes$color.

Theories based on supersymmetry[1] possess the interesting feature of placing Fermi and Bose fields in the same multiplet. When supersymmetry is combined with the principle of local gauge invariance, all fields, Fermi and Bose, become gauge fields.[2] Further, the non-Abelian gauge invariance determines the self-interactions of the gauge fields, greatly reducing the arbitrariness of such theories. Thus gauge supersymmetry forms an interesting framework for unified gauge theories of interactions. Previous work has shown[2] that if in fact a spontaneous breakdown of gauge supersymmetry occurs, then a unification of different interaction structures could arise. The purpose of this note is to show that a breakdown of gauge supersymmetry does indeed occur spontaneously, and to discuss some of its consequences. The resultant vacuum state then possesses a generalized global supersymmetry, and rigorous Maxwell, Einstein, and (perhaps) color local gauge invariance remains.

The nature of the internal symmetry group is at present an unresolved problem in unified gauge theories. While there are several very interesting suggestions, no fundamental principles exist for determining this group. The internal symmetry enters gauge supersymmetry in the following manner. The total group of gauge supersymmetry is the group of general coordinate transformations $z^A = z^{A'} + \xi^A(z)$ in the Bose-Fermi supersymmetry space $z^A = (x^\mu, \theta^{\alpha a})$. Here $\theta^{\alpha a}$ are a set of anticommuting c-number Majorana spinor coordinates; $\alpha = 1, \ldots, 4$ is the Dirac index and $a = 1, \ldots, N$ is the internal-space index. (We will suppress the latter when no ambiguity results.) Einstein gauge invariance corresponds to the case $\xi^\mu(z) = \xi^\mu(x)$, $\xi^\alpha = 0$, while the internal-symmetry gauge group is generated by Fermi transformations on the internal-symmetry index only, e.g., $\xi^\mu = 0$, $\xi^{\alpha a} = \lambda^r(x)(M_r)^a{}_b\theta^{\alpha b}$. [$M_r$ is a constant matrix and $\lambda^r(x)$ is the gauge function.] Thus, *prior to spontaneous breakdown,* the internal gauge group of the theory is GL(N; R) and is uniquely determined by the dimensionality of the Fermi space. Spontaneous symmetry breaking arises as follows. In gauge supersymmetry all fields are components of a single gauge supermultiplet $g_{AB}(z)$, the metric tensor of supersymmetry space: $ds^2 = dz^A g_{AB} dz^B$. Gauge supersymmetry uniquely determines the field equations to be[3]

$$R_{AB}[g_{CD}] = \lambda g_{AB}, \quad \lambda = \text{const} \tag{1}$$

(which explicitly shows how the gauge invariance determines the field interactions). At the tree level, spontaneous breaking is determined by looking for nonvanishing solutions of Eq. (1) with g_{AB} replaced by its vacuum expectation value $g_{AB}^{(0)}(\theta) = \langle 0|g_{AB}|0\rangle$. Under an arbitrary transformation, the gauge change of g_{AB} is $\delta g_{AB} = g_{AB}'(z) - g_{AB}(z)$, where $g_{AB}'(z')$ is the tensor transform of g_{AB}. *After spontaneous breakdown,* then, the internal-symmetry group reduces to those transformations $\xi^{\alpha a}$ which leave the "vacuum metric" invariant: $\delta g_{AB}^{(0)} = 0$.

We restrict our considerations here to those vacuum solutions invariant under a (generalized) global supersymmetry[4] group (as well as a remaining local gauge group). The possibility arises, however, that vacuum solutions may exist only for certain choices of the internal symmetry space (particularly since supersymmetry does not break easily). Thus the requirement that gauge supersymmetry spontaneously break to global supersymmetry may determine, at least in part, the nature of the internal symmetry group. *This possibility appears to us to be a unique feature of gauge supersymmetry.* There are two independent cases: $\lambda \neq 0$ and $\lambda = 0$. In the former, spontaneous breaking occurs only for $N = 2$ (Dirac spinor coordinates) and the theory completely determines its own symmetry group. In the latter case the dimensionality of the space does not appear to be determined, but spontaneous breaking occurs *only if parity (P) and charge conjugation (C) are not conserved.* The $N = 4$ possibility accommodates broken $SU(2) \otimes U(1)$ and maximal P and C nonconservation. It is also possible to include a conserved color group, and so one may begin to build realistic looking models based on gauge supersymmetry.

(1) *Case* $\lambda \neq 0$.—The generalized global supersymmetry transformation is generated by $\xi^\mu = i\bar{\delta}\Gamma^\mu\theta$, $\xi^{\alpha a} = \delta^{\alpha a}$, where $\delta^{\alpha a}$ is an infinitesimal which anticommutes with all Fermi quantities and $\Gamma^\mu = \Gamma\gamma^\mu$ is a matrix in the combined Majorana and internal symmetry space such that $\eta\Gamma^\mu$ is symmetric.[5] We conventionally assume that Fermi and Bose coordinates have the same dimension and so Γ has dimensions of mass. The form of the metric preserved by this ξ^A is $g_{\mu\nu}^{(0)} = \eta_{\mu\nu}$, $g_{\mu\alpha}^{(0)} = -i(\bar{\theta}\Gamma_\mu)_\alpha$, and $g_{\alpha\beta}^{(0)} = \eta_{\alpha\beta} + (\bar{\theta}\Gamma_\mu)_\alpha(\bar{\theta}\Gamma^\mu)_\beta$. Substitution of this vacuum metric into Eq. (1) yields to conditions on Γ_μ:

$$-2\Gamma_\mu\Gamma^\mu = \lambda\,, \quad -\mathrm{Tr}\Gamma_\mu\Gamma_\nu = \lambda\eta_{\mu\nu}\,, \tag{2}$$

where the trace is in the combined Majorana and internal-symmetry space. These relations are consistent only if the internal-symmetry index N equals 2, i.e., the internal space possesses an O(2) [or U(1)] symmetry. The only parity-preserving solution of Eq. (2) is $\Gamma^\mu = \beta\gamma^\mu$, where β is a (real) constant of dimensions of mass. Equation (2) yields $8\beta^2 = \lambda$. The physical fields are then deviations from the vacuum expectation value of the metric: $g_{AB}(z) = g_{AB}^{(0)} + h_{AB}(z)$. Thus writing[6]

$$\begin{aligned} h_{\mu\nu}(z) &= h_{\mu\nu}(x) + \bar{\theta}\theta p_{\mu\nu}(x) + \dots\,, \\ h_{\mu\alpha}(z) &= \bar{\theta}_\alpha S_\mu(x) + (\bar{\theta}\epsilon)_\alpha[eA_\mu(x) + \bar{\psi}(x)\gamma_\mu\theta] \\ &\qquad + \dots\,, \\ h_{\alpha\beta}(z) &= \eta_{\alpha\beta}\varphi_0(x) + \dots\,, \end{aligned} \tag{3}$$

one may identify $A_\mu(x)$ as the Maxwell field and $g_{\mu\nu}(x) \equiv \eta_{\mu\nu} + h_{\mu\nu}(x)$ as the Einstein field; $\bar{\psi}(x)$ is a charged spinor field while $S_\mu(x)$ is the gauge meson for scale transformations.[2] Electromagnetic gauge transformations are generated by $\xi^\alpha = \lambda(x)(\epsilon\theta)^\alpha$, $\xi^\mu = 0$. Only this gauge invariance (along with the Einstein gauge invariance) is preserved by the vacuum metric $g_{AB}^{(0)}$. [This is the U(1) gauge invariance of the theory.] Thus, for example, the scale transformation gauge, $\xi^\alpha = \lambda_s(x)\theta^\alpha$, of S_μ is spontaneously broken. This breakdown of scale invariance allows mass growth to arise at the tree level for fields other than the photon and graviton.[7] Thus a mass term of the type $\Gamma_\lambda\Gamma^\lambda p_{\mu\nu} \sim \beta^2 p_{\mu\nu}$ appears in Eq. (1) in the field equations for $p_{\mu\nu}$. As pointed out previously,[2] this is sufficient to achieve a unification of electromagnetism and gravity with (correctly positive) Einstein constant $G_E = e^2/(8\pi\beta^2)$ since Eq. (1) has the form[8] $G_{\mu\nu}[g_{\lambda\rho}(x)] = -8p_{\mu\nu} + \dots$, and $K^{\mu\nu}[p_{\lambda\rho}] - 4\beta^2 p^{\mu\nu} = \frac{1}{2}e^2 T_M{}^{\mu\nu} + \dots$. Mass growths in the θ^3 sectors of $h_{\mu\alpha}$ similarly produce a unification of the Maxwell-Dirac interactions.

(2) *Case* $\lambda = 0$.—The spontaneous symmetry-breaking condition here is Eq. (2) with $\lambda = 0$. The most general Γ^μ consistent with proper Lorentz invariance has the form $\Gamma^\mu = M_{(s)}\gamma^\mu + M_{(a)}i\gamma^\mu\gamma^5$, where $M_{(s,a)}$ are real, respectively symmetric and antisymmetric matrices in the internal-symmetry space. $M_{(a)}$ clearly measures the amount of parity nonconservation in the vacuum metric. The trace condition now requires that

$$\mathrm{Tr}[M_{(s)}]^2 + \mathrm{Tr}[M_{(a)}]^2 = 0\,. \tag{4}$$

If $M_{(a)}$ were zero (and the vacuum preserved parity) then Eq. (4) would also imply $M_{(s)}$ vanished (since $M_{(s)}$ is symmetric). Thus for $\lambda = 0$, *spontaneous symmetry breaking of gauge supersymmetry implies a vacuum state with parity nonconservation.*

To illustrate the above result, we consider a doublet of Dirac spinor Fermi coordinates (the simplest nontrivial possibility). In Majorana notation we write the Fermi coordinates as $\theta^{\alpha qa}$, where $q = 1, 2$ is the charge degree of freedom $(\theta_{\mathrm{Dirac}}{}^{\alpha q} \equiv \theta^{\alpha 1a} - i\theta^{\alpha 2a})$ and $a = 1, 2$ is a U(2)-symmetry space index. The $\Gamma^\mu \equiv \Gamma\gamma^\mu$ which conserves charge can be written in chiral components: $\Gamma = \beta^{r\sigma}(-\epsilon\mu_r)P_\sigma$, $\sigma = \pm$, where $P_\pm = \frac{1}{2}(1 \pm i\epsilon\gamma^5)$ are the chiral right and left projection operators, $\epsilon_{qq'}$ is the charge matrix,[6] and μ_r are the real antisymmetric matrices of U(2).[9] Equation (2) (with $\lambda = 0$) now yields two types of solutions: (i) $\Gamma_\pm = (\beta^0 + \beta^3\tau_3)P_\pm$, and (ii) $\Gamma_\pm = \beta^+(1 + \tau_3)P_\pm + \beta^-(1 - \tau_3)P_\mp$.[10] The β's are arbitrary constants of dimension of mass.

We discuss here in general terms the Γ_- solution of case (i). The U(2) Fermi coordinates admit a set of chiral $U(2) \otimes U(2)$ gauge transformations with corresponding gauge vector mesons in the metric:

$$h_{\mu\alpha} = (\bar{\theta}t_A)_\alpha v_\mu{}^A(x) + \ldots, \quad t_A = \mu_r P_\sigma. \tag{5}$$

Those transformations $\xi^\alpha = \lambda^A(x)(t_A\theta)^\alpha$ whose $t_A = \mu_r P_{-\sigma}$ do not commute with Γ_- are spontaneously broken; the corresponding vector mesons will acquire masses scaled by the β's. This is the case for the charged right chiral W mesons. On the other hand, the entire subgroup $SU(2)_L \otimes U(1)_Y$ commutes with Γ_-, and so Γ_- produces no mass growth for these mesons. However, $SU(2)_L \otimes U(1)_Y$ does not preserve the $\eta_{\alpha\beta}$ part of $g_{AB}{}^{(0)}$, and this accounts for mass growth of these mesons.[11] In particular, only the $W_\mu{}^\pm$ and Z_μ become massive, while the photon combination remains correctly massless.[12] Three Goldstone bosons arise in $h_{\alpha\beta}$,

$$h_{\alpha\beta} = [\eta i\epsilon\gamma^5(\mu_1\varphi_w{}^1 + \mu_2\varphi_w{}^2 + \mu_3\varphi_z)]_{\alpha\beta} + \ldots, \tag{6}$$

and may be absorbed by the vector mesons into the gauge-invariant combination $W_\mu{}^{1,2} + \partial_\mu\varphi_w{}^{1,2}$, $Z_\mu + \partial_\mu\varphi_z$ (e.g., in the unitary gauge).

The actual detailed way in which the mass growth for $v_\mu{}^A$, $p_{\mu\nu}$, etc., occurs is more complicated than in the $\lambda \neq 0$ theory. Thus even though scale invariance is spontaneously broken and the β parameters of dimension of mass enter the metric, the tree contributions to the masses vanish since by Eq. (2), $\Gamma_\mu\Gamma_\nu = 0$ for $\lambda = 0$ (e.g., mass terms such as $\Gamma_\lambda\Gamma^\lambda p_{\mu\nu}$ are now zero). Mass growth does occur dynamically though. Thus the scalar field $f(x)$ appears in $h_{\alpha\beta}$:

$$h_{\alpha\beta} = M_0(\bar{\theta}\epsilon)_\alpha(\bar{\theta}\epsilon)_\beta f(x) + \ldots, \tag{7}$$

where the arbitrary mass M_0 has been factored out so that $f(x)$ has canonical Bose dimension. $f(x)$ cannot be gauged away, and the term $M_0 f p_{\mu\nu}$ appears, for example, in the $p_{\mu\nu}$ equations. From interactions with a set of axial vector mesons, $h_{\mu\alpha} = (\bar{\theta}i\epsilon\gamma^5)_\alpha a_\mu(x) + \ldots$, we find that at the one-loop level $\langle 0|f|0\rangle \sim M_0$ and is finite. (At higher order, the mass will also depend on the β's.) Similar mass growth occurs for other bosons.

(3) *Concluding remarks.*—The above discussion shows that spontaneously broken gauge supersymmetry with $\lambda = 0$ accommodates the weak, electromagnetic, and gravitational interactions in a unified fashion. The theory automatically supplies several mass scales, β^0, β^3, and M_0, as is needed for such diverse interactions. The simplest way of including the strong interactions would be to give a color index to the Fermi coordinates: $\theta^{\alpha a c}$, $c = 1,2,3$. One then has an additional massless SU(3) octet of color gluons which could generate the strong interactions. (This possibility will be discussed in more detail elsewhere.) One of the features of gauge supersymmetry is that it is a very tightly constrained structure so that the additional condition of spontaneous breaking greatly reduces the allowed possibilities. There are other constraints not yet examined which may further limit the internal symmetry group possibilities, e.g., one must choose only the absolute minimum of the effective potential, and also the resultant theory must be renormalizable. (The fact that the one-loop calculation of $\langle 0|f|0\rangle$ gives finite results is a hopeful sign for the latter.) It also remains to be examined whether a renormalizable theory which includes gravity can be constructed without ghosts being present. Preliminary investigations at the classical tree level indicate the possible existence of tachyons, but with superheavy masses $\sim\beta$. If such fields do indeed exist, one might expect that in the Appelquist-Carrazone limit $\beta \to \infty$, these tachyons would disappear in the effective Lagrangian which governs the "low"-energy ($\ll 10^{38}$ GeV) dynamics. This important question is currently under investigation.

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[1]D. V. Volkov and V. P. Akulov, Phys. Lett. 46B, 109 (1973); J. Wess and B. Zumino, Nucl. Phys. B70, 39 (1974); A. Salam and J. Strathdee, Nucl. Phys. B76, 477 (1974). For a recent review see S. Ferrara, Laboratori Nazionali di Frascati Report No. LNF-75/46(P) (to be published).

[2]P. Nath and R. Arnowitt, Phys. Lett. 56B, 171 (1975); R. Arnowitt, P. Nath, and B. Zumino, Phys. Lett. 56B, 81 (1975). See also P. Nath and R. Arnowitt, J. Phys. (Paris) 37, C2-75 (1976); R. Arnowitt and P. Nath, in Proceedings of a Conference on the Riddle of Gravitation, Syracuse, N. Y., 1975 (to be published); P. Nath, in *Proceedings of a Conference on Gauge Theories and Modern Field Theory, Boston, 1975,* edited by R. Arnowitt and P. Nath (MIT Press, Cambridge, Mass., 1976), p. 281.

[3]R_{AB} is the contracted curvature tensor in supersymmetry space. Thus Eq. (1) is a set of second-order differential equations for the fields g_{AB}. All notation is as in Ref. 2.

[4]This remaining global supersymmetry (of Ref. 1) would later be broken dynamically.

[5]We use the notation $\eta_{\alpha\beta}=(-C^{-1})_{\alpha\beta}$ where C is the charge conjugation matrix. For Majorana spinors $\overline{\theta}_\alpha \equiv (\theta^\dagger\gamma^0)_\alpha = \theta^\beta\eta_{\beta\alpha}$. The choice $\Gamma^\mu=\gamma^\mu$ corresponds to the global supersymmetry of Ref. 1.

[6]$\epsilon_{aa'}$ is the real antisymmetric charge matrix in the O(2) internal-symmetry space: $\epsilon_{12}=1=-\epsilon_{21}$. The contribution to $h_{\mu\alpha}(z)$ which is independent of θ^α, i.e., $\overline{\psi}_{\mu\alpha}(x)$, has been omitted in Eq. (3) as it is pure gauge and can be eliminated by the gauge transformation $\xi^\mu = \overline{\lambda}^\mu(x)\theta$, $\xi^\alpha=0$.

[7]As pointed out recently by S. Weinberg (to be published), it is the *formal* scale invariance of certain gloglobal supersymmetric models that prevents the breakdown of supersymmetry at the dynamical (loop) level. (The scale anomalies apparently do not modify this result.) Thus the appearance of explicit breakdown of scale invariance in gauge supersymmetry implies that the further dynamical breakdown of the remaining global supersymmetry is to be expected.

[8]As pointed out by R. Jackiw [in *Laws of Hadronic Matter*, International School of Subnuclear Physics "Ettore Majorana," Erice, 1973, edited by A. Zichichi (Academic, New York, 1975), p. 225], the breakdown of scale invariance should lead to a Goldstone boson whose subsequent Higgs absorption might result in a mass growth for the graviton. Such a mass growth does indeed occur here, but for $p_{\mu\nu}$ and not $g_{\mu\nu}(x)$, and so the graviton remains correctly massless. φ_0 and S_μ in Eq. (3) are the Goldstone bosons absorbed by $p_{\mu\nu}$.

[9]One has $\mu_r = \frac{1}{2}\{\epsilon, \epsilon\tau_1, i\tau_2, \epsilon\tau_3\}$, where $[\mu_r, \mu_s] = -\epsilon_{rst}\mu_t$.

[10]Γ_+ and Γ_- differ only by chiral interchange. Spontaneous breaking, of course, can only say that the theory prefers one chiral state over the other, but not which one.

[11]The chiral right mesons also grow additional masses by this mechanism.

[12]There is also a second massless vector meson in the $U(2)\otimes U(2)$ set corresponding to heavy-particle number (or lepton number). There exists the possibility that this meson acquires a mass dynamically.

Fusion of ^{14}N + ^{12}C at Energies up to 178 MeV*

R. G. Stokstad, J. Gomez del Campo,† J. A. Biggerstaff, A. H. Snell, and P. H. Stelson
Oak Ridge National Laboratory, Oak Ridge, Tennessee 37830
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The cross section for the fusion of $^{14}N+^{12}C$ has been determined at bombarding energies covering the range $E_{^{14}N}=43$–178 MeV (3 to 12 times the interaction barrier height). The cross section decreases slowly with increasing energy and, at the highest energy, suggests that ^{26}Al has been formed with an angular momentum equal to the liquid-drop limit.

Although there is considerable experimental information on fusion cross sections, σ_{fus}, for heavier systems,[1] $A_1+A_2 \gtrsim 40$, little information is available at high energies for lighter systems with $A_1+A_2 \lesssim 30$. This may be due to the serious difficulty of separating the products of direct inelastic reactions and of fusion when the evaporation residues have masses comparable to or less than that of the projectile. Experimental information in this mass region is especially desirable because of the microscopic,[2] time-dependent Hartree-Fock[3] calculations which are now becoming available for reactions involving light targets and projectiles. We have therefore undertaken to measure the reaction products for the system $^{14}N+^{12}C$ for a wide range of incident energies. An important contribution to our ability to deduce fusion cross sections from these measurements has been the development of a Hauser-Feshbach computer code[4] which predicts the laboratory energies, angular distributions, and relative intensities of the evaporation residues. The main result of our measurements is that σ_{fus} decreases slowly over an energy range extending from about 3 times to 12 times the interaction barrier; at the highest energy the deduced critical angular momentum equals the limit predicted for a rotating liquid drop.[5]

Beams of ^{14}N produced by the Oak Ridge isochronous cyclotron at seven energies in the range 43.9 to 178.1 MeV were used to bombard carbon foils of 156 and 257 $\mu g/cm^2$. The target thickness was determined by weighing the foils and by Rutherford scattering at 19 MeV. The two methods agreed within their errors of ~4% and ~6%, respectively. The principal contaminants in the target were ^{13}C (1%), ^{16}O (1.3%), and hydrogen ($\lesssim 2\%$). Reaction products with $Z=3$ to 12 were identified with a ΔE(ionization-chamber)-E counter telescope. Angular distributions were measured in the range 4° to 40° (lab). The accuracy of the absolute normalization is estimated to be about ± 8%.

The yields of neon, sodium, and magnesium