Here the equation of motion was used and

$$Y \equiv E + 2(n-1)(k + GZ_{m}^{-1}) + (1 - \frac{1}{2}n)Z_{m}^{-1}Z.$$
(31)

Use of Eqs. (9), (10), (15), and (26) gives

$$\gamma_m Y = -\overline{\beta}(2\zeta/g_R + \frac{1}{2}\gamma_m/g_R + 6k') + O(n-4).$$

So at a fixed point Y = 0 if $\gamma_m \neq 0$. Hence $\theta_{\mu}{}^{\mu}$ is soft there, when n = 4.

Fuller details of this work will appear elsewhere.

I am grateful to C. G. Callan for his interest in this work, and to C. Lovelace for reminding me about Schroer's paper. I would also like to thank H. S. Tsao and N. J. Woodhouse for useful discussions.

¹C. G. Callan, S. Coleman, and R. Jackiw, Ann. Phys. (N.Y.) 59, 42 (1970).

²S. Coleman and R. Jackiw, Ann. Phys. (N.Y.) 67, 552 (1971).

³D. Z. Freedman, I. J. Muzinich, and E. J. Weinberg, Ann. Phys. (N.Y.) 87, 95 (1974).

⁴D. Z. Freedman and E. J. Weinberg, Ann. Phys. (N.Y.) <u>87</u>, 354 (1974).

⁵J. H. Lowenstein, Phys. Rev. D <u>4</u>, 2281 (1971).

⁶B. Schroer, Lett. Nuovo Cimento 2, 867 (1971). ⁷E.g., J. D. Bjorken and S. D. Drell, *Relativistic Quantum Fields* (McGraw-Hill, New York, 1965).

⁸G. 't Hooft and M. Veltman, Nucl. Phys. <u>B44</u>, 189 (1972).

⁹The factor of Z is for later convenience; the operator $Z_m \varphi^2$ is finite, where $\varphi \equiv Z^{-1/2} \varphi_0$ is the renormalized field.

¹⁰C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation* (W. H. Freeman, San Francisco, 1973).

¹¹R. Penrose, in *Relativity*, *Groups and Topology*, edited by C. de Witt and B. de Witt (Gordon and Breach, New York, 1964); S. Deser, Ann. Phys. (N.Y.) 59, 248 (1970).

¹²G. 't Hooft, Nucl. Phys. B61, 455 (1973).

¹³H. S. Tsao (private communication) has also made the observation of finiteness up to three loops. Note that Freedman and Weinberg (Ref. 4) incorrectly assume that H_0 is independent of n.

¹⁴J. C. Collins and A. J. Macfarlane, Phys. Rev. D <u>10</u>, 1201 (1974); M. J. Holwerda, W. L. Van Neerven, and R. P. Van Royen, Nucl. Phys. B75, 302 (1974).

¹⁵J. C. Collins, Nucl. Phys. <u>B80</u>, 341 (1974).

¹⁶J. C. Collins, Phys. Rev. D <u>10</u>, 1213 (1974).

¹⁷J. C. Collins, Nucl. Phys. <u>B92</u>, 477 (1975); P. Breitenlohner and D. Maison, Max-Planck-Institut, München, Report No. MPI-PAE/PTh 25 (to be published). ¹⁸Note that $ZZ_m^{-1} \sim (n-4)^{-1/3}$, by Eq. (16).

Exact Equivalence of Chromodynamics to a String Theory*

Itzhak Bars

Department of Physics, Yale University, New Haven, Connecticut 06520 (Received 22 March 1976)

A previously proposed model of hadrons constructed out of quarks and strings is further developed into a fully interacting theory in any dimension. It is shown that in two dimensions chromodynamics is equivalent to this theory in the sense that the hadronic spectrum and matrix elements for the strong, weak, and electromagnetic interactions are *identical* in both theories.

According to some clues,¹ in an exact color SU(3) gauge theory (chromodynamics), the dynamics of confinement may be similar to the dynamics of the string model. These clues, combined with the intuition conveyed by duality diagrams, were the main motivations of previous work,² where the first principles for constructing a detailed model of hadrons out of quarks and strings were studied.

In this paper the model is further developed by adding a new term to the action which permits the hadrons to interact via a local interaction of the constituents. The resulting fully interacting model de-

(32)

^{*}Research supported by the National Science Foundation under Grant No. MPS 75-22514.

scribes the strong, weak, and electromagnetic interactions of the bound-state hadrons as well as their spectrum.

By specialization to two space-time dimensions, and introduction of a new Hamiltonian formalism for interacting strings, the meson and the baryon spectra, the three-meson strong interaction vertex, and the timelike as well as spacelike weak-electromagnetic form factors are calculated. The results are identical to those obtained in chromodynamics. The details and techniques of these calculations will be presented elsewhere, while here the main results will be given.

An interacting string model of hadrons.—The general structure of the model is outlined by the total action

$$S = \sum_{\text{mesons}} S_M + \sum_{\text{baryons}} S_B + \sum_{\text{quarks}} (S_{\text{str}}^{\text{int}} + S_W^{\text{int}}).$$
(1)

The meson action S_M is²

$$S_{M} = \int d\tau \left\{ L_{0}(x_{1}(\tau)) + L_{0}(x_{2}(\tau)) - \gamma_{M} \int_{1}^{2} d\sigma \sqrt{-g} \right\},$$
(2)

where $\sqrt{-g}$ is the usual Nambu string Lagrangian and $L_0(x_I(\tau))$, I = 1, 2, describe spin- $\frac{1}{2}$ Dirac quarks² following the world lines defined by the ends of the string $x_1^{\mu}(\tau) = x^{\mu}(\tau, \sigma = 0)$ and $x_2^{\mu}(\tau) = x^{\mu}(\tau, \sigma = \pi)$:

$$L_{0}(x_{I}) = \overline{\psi}_{I} \gamma_{\mu} \overline{\vartheta}_{\tau} \psi_{I} x_{I\tau}^{\mu} / 2(x_{I\tau}^{2})^{1/2} - (-x_{I\tau}^{2})^{1/2} \overline{\psi}_{I} m \psi_{I}.$$
(3)

The fields $\psi_I^{ai}(\tau)$ carry flavor (a) as well as color (i) indices which will be suppressed except as noted.

The action S_B describes either a Y-shaped or a Δ -shaped baryon or both. In this paper we will consider the Δ -shaped baryon whose action is given by

$$S_{B} = \int d\tau \{ L_{0}(x_{1}) + L_{0}(x_{2}) + L_{0}(x_{3}) - \gamma_{B}(\int_{1}^{2} + \int_{2}^{3} + \int_{3}^{1}) d\sigma \sqrt{-g} \},$$
(4)

where the meaning of each term is analogous to the meson case.

The interaction among hadrons will proceed as a *local interaction* between quarks: When their corresponding world lines meet at a point in space-time a quark and an antiquark can annihilate each other (or can get created), thus allowing strings to join and split:

$$\sum_{\text{quarks}} S_{\text{str}}^{\text{int}} = -\frac{1}{4} f \sum_{I \neq J} \int d\tau \int d\tau' \,\delta^{(d)} (x_I^{\mu}(\tau) - x_J^{\mu}(\tau')) (-x_{I\tau}^2)^{-1/2} (-x_{J\tau'}^2)^{1/2} \\ \times \left\{ \overline{\psi}_I(\tau) (\overleftarrow{\partial}_{\tau} \gamma \cdot x_{I\tau} + m x_{I\tau}^2) \psi_J(\tau') + \overline{\psi}_J(\tau') (\gamma \cdot x_{I\tau} \overrightarrow{\partial}_{\tau} + m x_{I\tau}^2) \psi_I(\tau) \right\}.$$
(5)

This action is Poincaré and reparametrization invariant with respect to both τ and τ' which parametrize the two colliding world lines. The constant f will be fixed by crossing symmetry.

Invoking the gauge principle discussed in Ref. 2, one couples external weak and electromagnetic fields W_{α}^{μ} to the flavor indices of the quarks by the minimal substitution

$$x_{I\tau}^{\mu}\partial_{\tau} \rightarrow x_{I\tau}^{\mu}\partial_{\tau} + \frac{1}{2}iex_{I\tau}^{2}\frac{1}{2}\lambda W^{\mu}(x_{I}).$$
(6)

This generates a new piece in the action through Eqs. (3) and (5) from which one can obtain the physical currents of the theory:

$$\sum_{\text{quarks}} S_{W}^{\text{int}} = -ie \sum_{I} \int d\tau (-x_{I\tau}^{2})^{1/2} \overline{\psi}_{I}(\tau) \frac{1}{2} \lambda_{Y} \cdot W[x_{I}(\tau)] \psi_{I}(\tau) - \frac{1}{4} ief \sum_{I \neq J} \int d\tau \int d\tau' \, \delta^{(d)} (x_{I}^{\mu}(\tau) - x_{J}^{\mu}(\tau')) (-x_{I\tau}^{2})^{1/2} (-x_{J\tau}^{2})^{1/2} \times \{ \overline{\psi}_{I}(\tau) \frac{1}{2} \lambda_{Y} \cdot W(x_{I}) \psi_{J}(\tau') + \overline{\psi}_{J}(\tau') \frac{1}{2} \lambda_{Y} \cdot W(x_{I}) \psi_{I}(\tau) \}.$$
(7)

The model is written in any number of dimensions.

Results in two dimensions. The string.—A full analysis of the new longitudinal motions of the string was given by Bardeen *et al.* and by Patrascioiu.³ The *n*th normal mode corresponds to a string which is folded on itself *n* times. Quantum Poincaré invariance can be easily demonstrated in the light-cone gauge.²

Dirac particle.—In the absence of interactions (5) and (7) but in the presence of the string interactions (2) and (4), the Dirac particle described by Eq. (3) has the general solution

$$\psi_I = b_I(\tau) \mu(\mu_I, p_I(\tau)) + d_I^{\dagger}(\tau) \nu(\mu_I, p_I(\tau)).$$
(8)

Here $p_I^{\ \mu} = \mu_I x_{I\tau}^{\ \mu} / (-x_{I\tau}^{\ 2})^{1/2}$ is the canonical momentum, and $\mu_I = \overline{\psi}_I m \psi_I$ is a constant of motion. They satisfy the relations $p^2 + \mu^2 = 0$, $\gamma \cdot pu = i\mu u$, $\gamma \cdot pv = -i\mu v$, $\overline{u}u = -\overline{v}v = 1$.

In the light-cone gauge $x^+(\tau, \sigma) = \tau$, canonical quantization is given by $[x_I^-, p_I^+] = ig^{-+} = -i$, $\{b_I, b_I^+\} = I = \{d_I, d_I^+\}$. After normal ordering, μ_I takes the form $\mu_I = b_I^+ m b_I + d_I^+ m d_I$. Note that $p_I^+ = \mu_I (-x_I \tau^2)^{-1/2}$ is positive.

If the string interaction is removed $(\gamma \rightarrow 0)$ the quarks become free and satisfy $\partial_{\tau} p_I^{\mu} = 0$. Then the interaction of Eq. (5) also becomes identically zero for the solution (8). The weak-electromagnetic interactions of Eq. (7), when operators are quantized and properly ordered, reproduce vertices identical to field theory in lowest order. Crossing symmetry is then satisfied provided $f = 4\pi$. This free-quark result is the first indication of a deep connection between our approach and *quantum* field theory.

Meson and baryon spectra.—Specializing to strings with no folds, and following the ideas and gauge choices of Refs. 2 and 3, one obtains the light-cone Hamiltonians for noninteracting mesons and baryons from Eqs. (2) and (4):

$$P^{-} = \mu_{1}^{2}/2p_{1}^{+} + \mu_{2}^{2}/2p_{2}^{+} + \gamma_{M}|x_{1}^{-} - x_{2}^{-}|, \qquad (9a)$$

$$P^{-} = \sum_{I=1}^{N} (\mu_{I}^{2}/2p_{I}^{+}) + \gamma_{B} \sum_{I>J}^{N} |x_{I}^{-} - x_{J}^{-}|, \qquad (9b)$$

where N=3 for color SU(3), and $p_I^+>0$, x_I^- , and μ_I are quantum operators.

We define normalized, color-singlet, hadron states which also have definite transformation properties under the flavor charges

$$Q_{\alpha} = \sum_{I} \left[b_{I}^{\dagger} \frac{1}{2} \lambda_{\alpha} b_{I} - d_{I}^{\dagger} \frac{1}{2} \lambda_{\alpha}^{T} d_{r} \right].$$

They are simultaneous eigenstates of the appropriate *total* momentum P^+ and the mass operator $M^2 = 2P^+P^-$, etc., for mesons and baryons:

$$N^{-1/2}b_{1}^{ai\dagger}d_{2}^{bi\dagger}|0;n,r^{+}\rangle; \quad (N!)^{-1/2}\epsilon_{ij\dots k}b_{1}^{ai\dagger}b_{2}^{bj\dagger}\dots b_{N}^{ck\dagger}|0;n_{1},\dots,n_{N-1},r^{+}\rangle, \tag{10}$$

where r^+ is the eigenvalue of P^+ , and 0 denotes the vacuum with respect to the oscillators. These eigenstates can be expanded in terms of states labeled by eigenvalues of quark momenta p_I^+ :

$$(N)^{-1/2} b_1^{ai\dagger} d_2^{bi\dagger} |0;k^+,l^+\rangle; \quad (N!)^{-1/2} \epsilon_{ij_{***}k} b_1^{ai\dagger} b_2^{bj\dagger} \dots b_N^{ck\dagger} |0;k^+,l^+,\dots,q^+\rangle.$$
(11)

The states in Eq. (11) are normalized noncovariantly to δ functions in momenta, while the states in Eq. (10) are normalized covariantly to $(2\pi)(2r^+)\delta(r^+ - r^{+\prime})\delta_{nn\prime}$, etc. The expansion coefficients define the wave function φ for mesons:

$$\langle k^{+}, l^{+} | n, r^{+} \rangle = 2\sqrt{\pi} \,\delta(r^{+} - k^{+} - l^{+}) \varphi_{n}^{ab}(k^{+}, l^{+}), \tag{12}$$

and similarly for baryons. By sandwiching P^- between states of type (10) and (11), one derives an integral equation for φ which determines the spectrum of mesons^{2,3}:

$$\left(\frac{M_n^2}{2r^+} - \frac{m_a^2}{2k^+} - \frac{m_b^2}{2l^+}\right) \varphi_n(k^+, l^+) = \frac{\gamma_M}{\pi} \operatorname{P} \int \frac{ds^+}{(s^+)^2} \varphi_n(k^+ + s^+, l^+ - s^+);$$
(13)

and a similar one for baryons (for N=3):

$$\left(\frac{M_n^2}{2r^+} - \frac{m_a^2}{2k^+} - \frac{m_b^2}{2l^+} - \frac{m_c^2}{2q^+}\right) \varphi_n(k^+, l^+, q^+) \\ = \frac{\gamma_B}{\pi} \operatorname{P} \int \frac{ds^+}{(s^+)^2} \left\{ \varphi_n(k^+ + s^+, l^+ - s^+, q^+) + \varphi_n(k^+ + s^+, l^+, q^+ - s^+) + \varphi_n(k^+, l^+ + s^+, q^+ - s^+) \right\}.$$
(14)

The limits of each integral are determined by the positivity of each argument of φ . It is convenient to scale the quark momenta by r^+ and define $\varphi_n^{\ ab}(x) \equiv \varphi_n^{\ ab}(k^+, r^+ - k^+)$, where $0 \le k^+/r^+ \le 1$, etc.⁴ Identical results have been obtained in chromodynamics by considering only *planar diagrams*, by 't Hooft in the case of mesons and by Durgut together with the present author in the case of baryons.⁶ We can now make the identification $2\gamma_M = g^2(N - N^{-1})$, $2\gamma_B = g^2(1 + N^{-1})$ for any N,⁶ including N = 3. Note that chromodynamics chooses the Δ -shaped baryon in two dimensions.

Three meson vertex.—This process is represented by a duality diagram in which a string splits by

VOLUME 36, NUMBER 26

 $q\bar{q}$ creation to form two new strings. To first order, the interaction Hamiltonian is obtained from Eq. (5) by changing the sign and replacing Eq. (8). The result is proportional to $\partial_{\tau}p_{I}^{+}$ which is determined by the zeroth-order meson Hamiltonian (9a), $\partial_{\tau}p_{I}^{+}=\gamma_{M}\epsilon(x_{I}^{-}-x_{I'}^{-})$, where *I'* denotes an (anti)quark in the same meson as quark *I*. Thus, with a careful ordering of the light-cone gauge operators one has

$$P_{\text{int}}^{-} = \frac{1}{8} i f \gamma_{M} \sum_{I \neq J} \left\{ \delta(x_{I}^{-} - x_{J}^{-}) (\mu_{I} \mu_{J} / p_{I}^{+} p_{J}^{+})^{1/2} \overline{u}(\mu_{I}, p_{I}) \gamma_{5} v(\mu_{J}, p_{J}) \times \left[\mu_{J}^{-1} \epsilon (\frac{1}{2} (x_{I}^{-} + x_{J}^{-}) - x_{J}^{-}) - \mu_{I}^{-1} \epsilon (\frac{1}{2} (x_{I}^{-} + x_{J}^{-}) - x_{I}^{-}) \right] b_{I}^{+} d_{J}^{+} (\mu_{I} \mu_{J} / p_{I}^{+} p_{J}^{+})^{1/2} + \dots \right\}.$$
(15)

Using standard *time-dependent* perturbation theory with the initial meson state labeled by (r_1^{μ}, n_1) and the final two-meson state labeled by $(r_2^{\mu}, n_2; r_3^{\mu}, n_3)$ as in Eq. (10), we calculate the transition amplitude $\int_{-\infty}^{\infty} d\tau \langle \operatorname{out} | P_{\operatorname{int}}^{-} | \operatorname{in} \rangle$. The calculation is performed by introducing the intermediate states (11) and using Eq. (12). The result has the form $(2\pi)^2 \delta^{(2)} (r_1^{\mu} - r_2^{\mu} - r_3^{\mu}) A(r_1, r_2, r_3)$, where

$$A = \frac{\gamma_M f \sqrt{\pi}}{2\pi^2 \sqrt{N}} \int \frac{dl_2^+ dk_3^+}{(l_2^+ + k_3^+)^2} \varphi_{n_2}(r_2^+ - l_2^+, l_2^+) \varphi_{n_3}(k_3^+, r_3^+ - k_3^+) \\ \times [\varphi_{n_1}(r_2^+ - l_2^+, r_3^+ + l_2^+) - \varphi_{n_1}(r_2^+ + k_3^+, r_3^+ - k_3^+)].$$
(16)

This is identical (up to a change of variables) with the result of Callan. Coote, and $Gross^7$ obtained in chromodynamics.

Timelike form factor.—In this process a photon or W boson creates a $q\bar{q}$ pair bound by a string (mesons). To first order, the light-cone gauge interaction Hamiltonian is obtained by replacing Eq. (8) in the second term of Eq. (7):

$$P_{\text{int}}^{-} = \frac{1}{2} ie f \sum_{I \neq J} \left(\frac{\mu_{I} \mu_{J}}{p_{I}^{+} p_{J}^{+}} \right)^{1/2} \left\{ \overline{u}(\mu_{I}, p_{I}) \gamma^{\mu} v(\mu_{J}, p_{J}) b_{I}^{\dagger} \frac{\lambda^{\alpha}}{2} d_{J}^{\dagger} W_{\mu}^{\alpha} (\frac{1}{2}(x_{I} + x_{J})) \delta(x_{I}^{-} - x_{J}^{-}) + \dots \right\} \times (\mu_{I} \mu_{J} / p_{I}^{+} p_{J}^{+})^{1/2},$$

where operators are carefully ordered. The external field is a plane wave with momentum q^{μ} , $W_{\mu}^{\alpha}(x) = \epsilon_{\mu}^{\alpha} \exp[-i(q^{-}\tau + q^{+}x^{-})]$. The "in" state is $|0;0,0\rangle$, and the "out" state represents a meson as in Eq. (10). The transition amplitude has the form

$$\int_{-\infty}^{\infty} d\tau \langle \operatorname{out} | P_{\operatorname{int}}^{-} | \operatorname{in} \rangle = \frac{1}{2} e \lambda_{ab}^{\alpha} (2\pi)^2 \delta^{(2)} (q^{\mu} - r^{\mu}) \{ A_{+} \epsilon^{\mu\nu} q_{\nu} + A_{-} q^{\mu} \} \epsilon_{\mu}^{\alpha}$$

The calculation is performed by introducing the intermediate states (11), giving the result

$$A_{\pm}(q^{2}) = \frac{f\sqrt{N}}{8\pi\sqrt{\pi}} \frac{m_{a} \pm m_{b}}{(-q^{2})} \int_{0}^{1} dx \, \varphi_{n}^{ab}(x) \left(\frac{m_{a}}{x} \pm \frac{m_{b}}{1-x}\right).$$

An identical expression is obtained in chromodynamics.⁷

Spacelike form factor.—For the purpose of this paper it is sufficient to calculate only the string diagram (Fig. 1, part a or b) since it directly tests the first term of Eq. (7) in comparison to chromodynamics. The other diagrams involve the timelike form factor, the three-meson vertex, and the meson propagators (spectrum) which have already been compared to chromodynamics in the previous paragraphs.

The relevant first-order Hamiltonian with ordered operators is

$$P_{\text{int}} = ie \sum_{I} (\mu_{I}/p_{I}^{+})^{1/2} \{ \overline{u}(\mu_{I}, p_{I}) \gamma \circ W_{\alpha}(x_{I}) b_{I}^{\dagger} \frac{1}{2} \lambda^{\alpha} b_{I} u(\mu_{I}, p_{I}) + \dots \} (\mu_{I}/p_{I}^{+})^{1/2} ,$$

where $W_{\mu}^{\alpha}(x)$ is again a plane wave with momentum q^{μ} . The initial and final meson states are as in Eq. (10). The calculation proceeds similarly to the previous paragraphs yielding a transition amplitude of the form

$$e^{\frac{1}{2}\lambda_{da}}\delta_{bc}(2\pi)^{2}\delta^{(2)}(r'^{\mu}-r^{\mu}-q^{\mu})[2\epsilon_{+}^{\alpha}r^{+}B(q^{2})+2\epsilon_{-}^{\alpha}r^{-}C(q^{2})],$$

where

$$B(q^{2}) = \int_{y}^{1} dx \, \varphi_{n}^{ab}(x) \varphi_{m}^{cd} \frac{x - y}{1 - y} , \quad C(q^{2}) = \frac{m_{a}m_{d}}{(-r^{2})} \int_{y}^{1} \frac{dx}{x(x - y)} \varphi_{n}^{ab}(x) \varphi_{M}^{cd}\left(\frac{x - y}{1 - y}\right),$$

1524



FIG. 1. Spacelike form factor.

with $y = -q^{+}/r^{+}$, and $x = k^{+}/r^{+}$ as before. The full form factor of Fig. 1 has been calculated in chromodynamics by Einhorn.⁸ The results for Fig. 1, part *a*, are identical in the two theories. As $q^{+} \rightarrow -\infty$ the form factor decreases with a power law,⁸ in contrast to previous different attempts in the string theory.

Discussion.—Every term of our total action has now been tested relative to chromodynamics by comparing selected matrix elements. Our theory, taken in the no-fold string sector, fully agrees with the planar diagram results of chromodynamics. Corrections to the planar diagrams are in one-to-one correspondence with string splitting and joining diagrams. Such corrections, in both theories, are built up perturbatively from the propagators and vertices discussed in this paper. Therefore, the two theories are expected to agree to all orders. This, however, needs a more rigorous proof.

The present theory predicts additional states involving folded strings.³ Folded strings are a feature of any dimension, and are already present in the standard treatment of the string. A fold always moves with the *speed of light*. On the basis of the agreement already found, I would conjecture that chromodynamics also may contain corresponding bound states. For example, colorsinglet bound states of quark, antiquark, and gluons are possible in principle, with the gluons representing the folds. The picture is completed by associating the string with the three types of flux lines that must exist between the SU(3)-colored quarks and gluons. The gluon must have two flux lines attached to it while the quark has only one. We conjecture that the *dependent* degrees of freedom of the gluon field are responsible for a "potential" associated with the flux lines, while the *canonical* degrees of freedom are responsible for the folds.

The *independent* longitudinal modes³ were clearly essential for agreement with chromodynamics, so they must be retained in higher dimensions in the string approach, if such agreement is desired. The theory is then nonlinear and nontrivial to solve. Its low-energy spectrum will differ from dual models.

Will the exact equivalence persist in four dimensions? This is likely¹ and it would be very interesting to develop both theories hand in hand in four dimensions.

*Research (Yale Report No. COO-3075-140) supported in part by the U. S. Energy Research and Development Administration.

¹G. 't Hooft, Nucl. Phys. <u>B72</u>, 461 (1974); K. Wilson, Phys. Rev. D <u>10</u>, 2445 (1974); J. Kogut and L. Susskind, Phys. Rev. D <u>11</u>, 395 (1975); H. B. Nielsen and P. Olsen, Nucl. Phys. <u>B61</u>, 47 (1973).

²I. Bars and A. J. Hanson, Phys. Rev. D <u>13</u>, 1744 (1976).

³W. A. Bardeen, I. Bars, A. J. Hanson, and R. D. Peccei, Phys. Rev. D <u>13</u>, 2364 (1976). See also A. Pa-trascioiu, Nucl. Phys. <u>B81</u>, 525 (1974).

⁴Because of the interaction (5), a zigzag motion of the quarks inside the hadrons should, just like quark selfenergy diagrams, change the effective mass of the quarks. Accordingly, in Eq. (9) (and the integral equations that follow) we should use a dressed mass $\tilde{\mu}_I^2 = \mu_I^2 + C$. We have not yet calculated C; for agreement with chromodynamics we must have $C = -2\gamma_M/\pi$.

⁵G. 't Hooft, Nucl. Phys. B75, 461 (1974).

⁶M. Durgut, to be published.

⁷C. G. Callan, N. Coote, and D. Gross, Phys. Rev. D 13, 1649 (1976).

⁸M. Einhorn, to be published.