Exact Calculation of *n-d* Scattering at 14.1 MeV with a Local Realistic Interaction: Elastic-Channel Results

J. J. Benayoun, J. Chauvin, C. Gignoux, and A. Laverne
Institut des Sciences Nucléaires, Centre de Tri, 38044 Grenoble-Cedex, France
(Received 31 March 1976)

The Faddeev differential equations in configuration space are solved numerically for the nucleon-nucleon interaction of de Tourreil and Sprung effective in the states ${}^1S_0$, ${}^3S_1$, ${}^3D_1$, ${}^1P_1$, and ${}^3P_{0,1,2}$. Differential cross section, polarizations P_n, iT_{11}, Q, and R, and spin correlation coefficients C_{xx} and C_{yy} are computed here. Contributions from other interaction terms are evaluated perturbatively. Agreement with experimental data is found, except for the vector polarization iT_{11}, and C_{xx}.

The exact solution of the three-body quantum scattering problem has now become numerically possible for local two-body potentials, because of the Faddeev equations. In this Letter, we report the extension to a realistic case of the configuration-space approach briefly described by Gignoux, Laverne, and Merkuriev.[1]

We solve the Faddeev equations describing the scattering of a neutron on a deuteron. The results given here concern elastic scattering at $E_n^{\text{lab}} = 14.1$ MeV and apply certainly as well to the scattering of protons by deuterons.[2-5] We have considered the nucleon-nucleon (N-N) interaction of de Tourreil and Sprung, type C (SSC)[6] as an input in our computation and have solved "exactly" the Faddeev equations for this interaction taken to be effective only in the two-nucleon states ${}^1S_0$, ${}^3S_1$, ${}^3D_1$, ${}^1P_1$, and ${}^3P_{0,1,2}$. The contributions of the N-N interaction in states ${}^1D_2$ and ${}^3D_2$ to the elastic scattering amplitude are estimated through the generalized Born approximation.[7] Our treatment is an appreciable improvement on the work of Stolk and Tjon,[8] who solve exactly the Faddeev integral equations in momentum space for the Reid soft-core interaction (RSC)[9] acting in the two-nucleon S states only and make a perturbative treatment of the rest of the force. Moreover, while being fitted on the same two-nucleon phase shifts, the SSC potential gives a lower χ^2 per data point than the RSC potential. The observable quantities reported here have been obtained by solving the radial Faddeev differential equations in configuration space in each subspace of good parity and total angular momentum $J \leq \frac{11}{2}$ (we resorted to the Born approximation for $\frac{13}{2} \leq J \leq \frac{19}{2}$). The two-nucleon interaction states taken into account make the number of coupled radial equations in each subspace less than or equal to 19.

The details concerning the uniqueness of the solution of the Faddeev differential equations in configuration space, the angular momentum reduction, and the numerical formulation of the problem are given at length by Merkuriev, Gignoux, and Laverne.[10] Here, the number of coupled equations make the rank of the linear set to solve much larger than for the example in Ref. 10 (since the matrix of the set stems from discretizing differential equations, it still retains a band structure, and more than 98% of its coefficients are zeros). Rather than using the direct Gauss elimination method to solve numerically the linear set, we then have looked for an iterative solution of the linear set, accelerated by means of Padé techniques. Numerous tests and comparison with direct solution have convinced us of the very good convergence of the iterations (less than six Padé approximants were necessary).

For each subspace, the solution of the linear set yields both the elastic S matrix and the breakup amplitude. Evidence of the accuracy of our solution is the smallness of (i) the asymmetry of the elastic S matrix (less than 0.2%) and (ii) the nonconservation of total elastic plus breakup flux (less than 2%).

We present our results in Fig. 1 for the differential cross section (experiment from Ref. 2); in Figs. 2 and 3 for the polarizations P_n and iT_{11} of the neutron and deuteron, respectively (experiment from Ref. 5 and Fiore *et al.*[11]); in Figs. 4 and 5 for the tensor polarizations Q and R of the deuteron (definition and experiment from Ref. 11); and in Fig. 6 for the vector spin correlations C_{xx} and C_{yy} (definition and experiment by Chauvin, Garreta, and Fruneau[12]).

The measured cross sections, tensor polarizations, and coefficients C_{yy} are well reproduced by our computation. In spite of having its maximum at $\theta_{\text{c.m.}} = 125°$ too low by 20%, our neutron polarization (Fig. 2) is in better agreement with experiment than the one reported by Stolk and Tjon.[8] Computations with separable potentials have disclosed significant differences between neutron

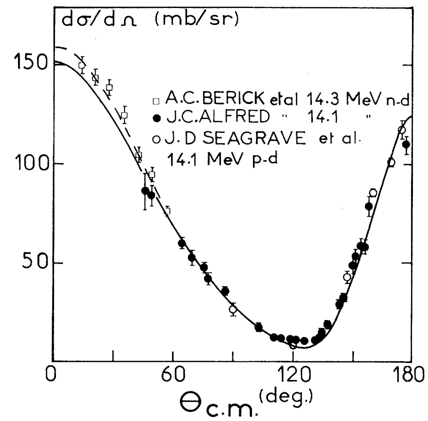

FIG. 1. Differential cross section for n-d scattering calculated with SSC interaction: Full line corresponds to interaction in $^1S_0$, $^3S_1$, $^3D_1$, $^1P_1$, and $^3P_{0,1,2}$ states; dashed line represents the same calculation with $^1D_2$ and $^3D_2$ added perturbatively.

polarizations computed exactly or perturbatively[13]; the differences between Stolk and Tjon's result and ours is then more likely due to their perturbative treatment than to the different (but realistic) N-N interactions used. From Ref. 10, the same can be said about the variations in the forward cross section.

The major disagreement with experiment lies in the deuteron vector polarization (Fig. 3) whose maximum at 135° is too low by 20% and whose dip at 100° is especially poor. We do not think that this disagreement can be attributed to the reaction energy since our computed values around 100° do not even lie within the experimental values at 15 and 12.5 MeV.[11] The similar disagreement found in computations with separable potentials can no longer be attributed only to the influence of poor phase shifts $\delta(^3D_1)$ and/or ϵ_1,[14] since we have the same disagreement here, while these phase shifts are realistically reproduced by the SSC potential. In place of the SSC potential in the $^1P_1$ state, we have substituted the potential of

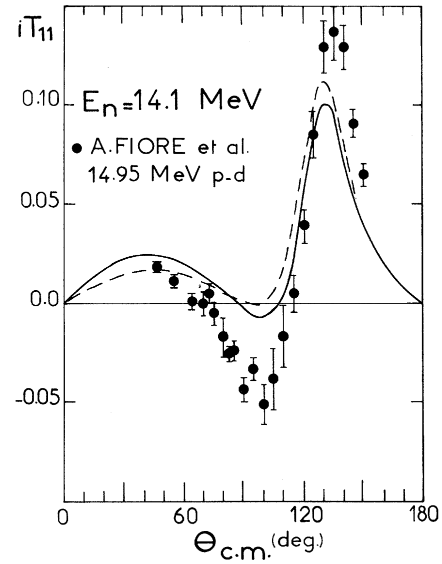

FIG. 3. Deuteron vector polarization; curves as in Fig. 1.

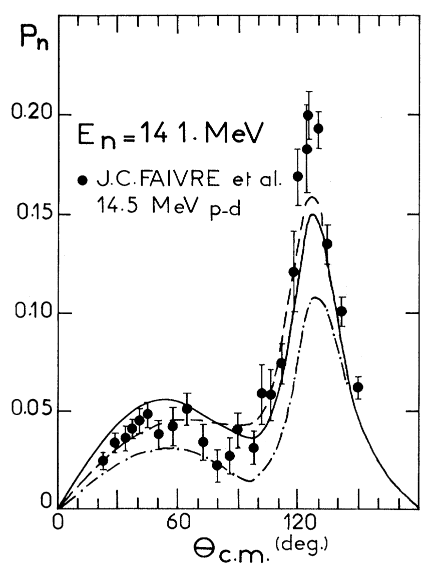

FIG. 2. Neutron polarization: Dash-dotted line recalls the result of Ref. 8 for the Reid soft core potential, other curves as in Fig. 1.

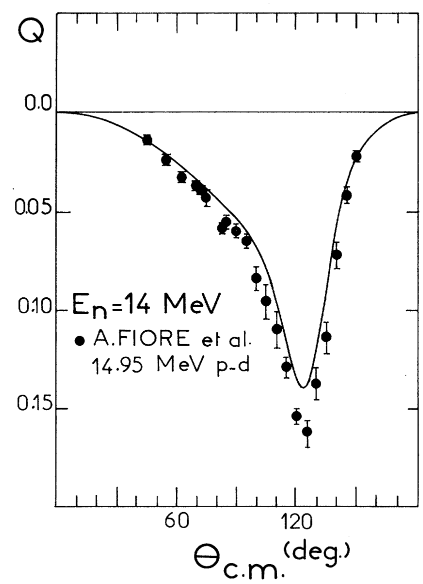

FIG. 4. Deuteron tensor polarization Q for the effective SSC interaction. Perturbed result is not distinct.

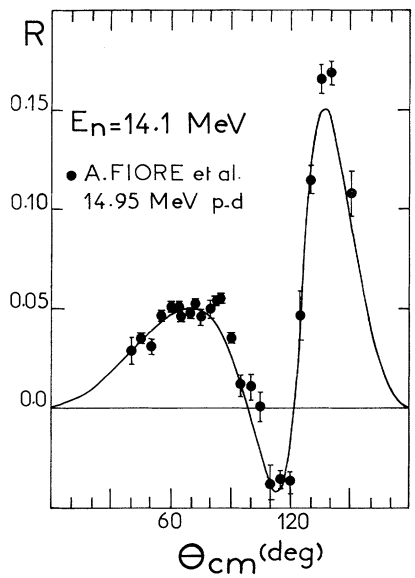

FIG. 5. Deuteron tensor polarization R. Same remark as in Fig. 4.

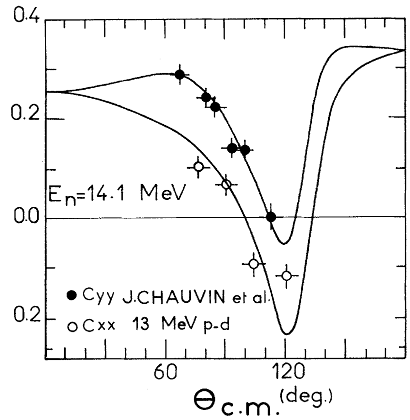

FIG. 6. Spin correlation coefficients: curves as in Fig. 1.

Gogny, Pires, and de Tourreil,[15] which gives different phase shifts $\delta(^1P_1)$ but the same N-N measurable data. The result was that the 125° maximum was increased by 10% but no other variation was found, especially around 100°. Other observables remained unchanged.

The computed coefficient C_{xx} (Fig. 6) at 120° is half the experimental value. It would be advisable to make further measurements before investigating the reasons for this considerable disagreement with only one datum point. However, in this geometry, the doublet n-d amplitude, which is especially sensitive to the type of N-N interaction, contributes dominantly.[16]

The exact solution of the Faddeev equations with a local realistic N-N potential provides a good description of elastic n-d scattering with no free parameter. We are now studying the possible relationship between the remaining disagreements and the ambiguities in N-N phase shifts, especially in the subspace $T=0$. The computations of breakup cross sections that we are doing should tell us if a realistic N-N interaction can resolve the disagreement with experiment previously observed[17] and if breakup processes could provide the same type of information.

We are indebted to C. Fayard, G. H. Lamot, C. Stolk, and R. de Tourreil for interesting discussions and correspondence.

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