Mechanism for 180° Proton Production in Energetic Proton-Nucleus Collisions*

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The energetic protons seen at 180° in 600- and 800-MeV proton-nucleus collisions by Frankel *et al.* are accounted for by a single-scattering mechanism that incorporates a new phenomenological form for the momentum distribution in the nucleus.

Frankel *et al.*[1] have reported detecting high-energy (100–400 MeV) protons at 180° from 600- and 800-MeV protons incident on a variety of nuclear targets. The high energies and therefore short time involved suggest that this is a direct reaction rather than a statistical process involving distribution of the incident energy among many particles. The simplest direct process is a single scattering and that is the mechanism we consider here. There seems little point in turning to multiple-scattering mechanisms that maintain the direct reaction features until the simplest is explored. This single scattering requires that the struck nucleon be moving backward with high virtual momentum before the collision. The rapid fall-off with detected proton energy observed by Frankel *et al.* from this point of view is a direct manifestation of the rapidly falling momentum distribution in the nucleus. In this Letter we introduce a new parametrization of that distribution, and using the single-scattering mechanism, calculate the inclusive cross sections of Frankel *et al.* The parameters of our momentum distribution are obtained by fitting the quasielastic electron scattering data which are dependent on lower momentum components[2,3] and thus there are no free parameters in our calculation of the proton inclusive cross section. We obtain the magnitude of the cross section as well as the general dependence on detected proton energy, bombarding energy, and target atomic number.

Our mechanism for the process is represented by the Feynman graph of Fig. 1. The incident proton with momentum p (in the lab frame) strikes a virtual nucleon of momentum k; the residual nucleus recoils with momentum k and in the state S_k. After the collision the observed proton has momentum q and the unobserved nucleon has momentum p'. The nucleon-nucleon collision is described by an (off-shell) two-body scattering amplitude. The experiment measures $d\sigma/d^3q$ at 180° and hence sums over p' and S_k subject to energy and momentum conservation. If we assume that most of the states S_k are at low excitation energies compared with the other energies of the problem, we can neglect the S_k dependence of the argument of the energy conserving δ function. The sum over S_k can then be done by closure[3] so that in $d\sigma/d^3q$ only $n(k)$, the probability for finding a particle of momentum k in the target ground state, appears. Making this approximation we obtain for the cross section associated with the mechanism of Fig. 1

$$\frac{d\sigma}{d^3q}=\frac{M^4}{pE(\vec{q})}\frac{1}{2(2\pi)^5}\int\frac{d^3k}{E(\vec{k})E(\vec{p}+\vec{k}-\vec{q})}\left[n_p(k)\sum|m_{pp}|^2+n_n(k)\sum|m_{pn}|^2\right]$$

$$\times\,\delta(E(\vec{p})+M-E(\vec{q})-E(\vec{p}+\vec{k}-\vec{q})-\bar{\epsilon}),\qquad(1)$$

where $E(\vec{p})=(\vec{p}^2+M^2)^{1/2}$, $\sum|m|^2$ is the square of the nucleon-nucleon scattering amplitude summed over spins, and $\bar{\epsilon}$ is the average nucleon interaction energy. The quantity $n_p(k)$ $[n_n(k)]$ is the probability density of finding a proton [neutron] of momentum k in the target ground state. The normalization is

$$2\int\frac{d^3k}{(2\pi)^3}\frac{M}{E(\vec{k})}n_p(k)=Z\qquad(2a)$$

and

$$2\int\frac{d^3k}{(2\pi)^3}\frac{M}{E(\vec{k})}n_n(k)=A-Z.\qquad(2b)$$

The argument of the energy conserving δ function

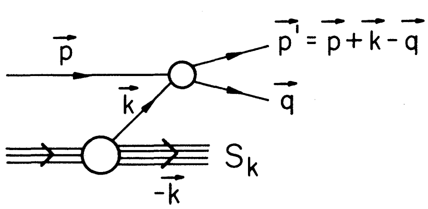

FIG. 1. Single-scattering mechanism for proton inclusive scattering.

reflects the assumption that the excitation energy and recoil energy of the residual nucleus may be neglected. We interpret Fig. 1 as a regular Feynman diagram and hence the "energy" of the exchanged particle k does not appear in the δ function.[4] We use the δ function in (1) to do the angular integral. The remaining k integral runs between allowed kinematic limits given by

$$k_{\min} = Q - (2M\bar{\omega} + \bar{\omega}^2)^{1/2},$$
$$k_{\max} = Q + (2M\bar{\omega} + \bar{\omega}^2)^{1/2},$$

where $Q = |\vec{p} - \vec{q}|$ and $\bar{\omega} = E(\vec{p}) - E(\vec{q}) - \bar{\epsilon}$. The values of $k_{\min}$ in this problem are in the range 700 to 1400 MeV/c. These are very large momenta compared with typical nuclear values and since, in this region, $n(k)$ falls rapidly with increasing k, the k integral in (1) will be dominated by k's near $k_{\min}$. The cross section will therefore be approximately proportional to $n(k_{\min})$.

For $|m|^2$ in (1) strictly we need the off-shell amplitude. We assume we can take $|m|^2$ out of the integral in (1), take $k = k_{\min}$, and take the on-shell $|m|^2$ appropriate to scattering from momentum p to p' (a relatively small momentum transfer) at the final center-of-mass energy. Other treatments of the off-shell amplitude introduce uncertainties of about a factor of 2 in the cross section, but short of a full off-shell theory, the choice remains arbitrary to this extent.

It remains to find a model for $n(k)$. We recall that quasielastic electron scattering has been used to investigate the momentum distribution for low momentum[2,3] and it is natural to try to fit both electron and proton scattering with a common distribution. Quasielastic (e, e') scattering has been fit with a momentum distribution corresponding to a zero-temperature noninteracting Fermi gas.[2] This is not suitable here as it has no high-momentum components at all. We have tried to generalize the distribution to that of a finite-temperature Fermi gas, but as we shall see, it is not possible to incorporate enough high-momentum component to fit the Frankel experiments and still be able to fit the (e, e') data. This is essentially due to the extremely fast (Gaussian) falloff of the Fermi-gas distribution at high k. It should be noted that the Gaussian falloff in q seen by Frankel *et al.* does not imply a Gaussian falloff for $n(k)$. In searching for a better form, we found two that are phenomenologically equivalent, and which are motivated by our study of a one-dimensional many-body problem.[5] These are

$$n_s(k) = N_s k \gamma_s / \sinh \gamma_s k, \qquad \text{(3a)}$$

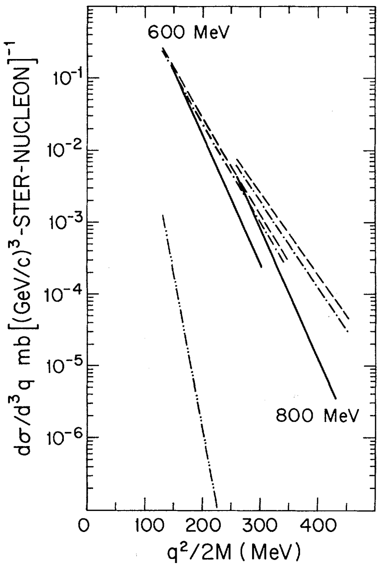

FIG. 2. Backward inclusive proton spectrum for 600- and 800-MeV protons on Ta. The solid line is a fit to the data of Ref. 1. The dashed and dash-dotted lines are our calculations with momentum distribution (3a) and (3b). The —·· line uses the finite-temperature Fermi-gas momentum distribution.

and

$$n_c(k) = N_c / \cosh^2 \gamma_c k. \qquad \text{(3b)}$$

N is a normalization constant while γ is a momentum scale. These $n(k)$ are functions of k^2, but for large k fall only like $\exp(-\gamma_s k)$ or $\exp(-2\gamma_c k)$. In fact for very large k, $n(k)$ should fall like a power,[6] but this presumably does not happen until k/A is large, a regime not attained here. We fix N and γ (and $\bar{\epsilon}$) for each case again by fitting the (e, e') quasielastic data.[7] For example, for Ta we find that $\bar{\epsilon} = 35$ MeV, $\gamma_s = 2.5$ fm, and $\gamma_c = 1.0$ fm.

All of the quantities in (1) are now fixed and we calculate the cross section. In Fig. 2 we show the calculation for tantalum compared with the Frankel data for 600- and 800-MeV bombarding energy. We see that (3a) and (3b) give the magnitudes and general trends equally well, while the Fermi-gas fit is three to four decades too small. In Fig. 3 we compare our calculated cross sections with the data of Ref. 1 for 600-MeV protons incident on C, Cu, and Ta. We only show the calculation for case (3a) to keep the figure simple. Here (3b) would do about as well. The absolute magnitude, slopes, and dependence on A and bombarding energy are all qualitatively correct. It should be stressed again that there are no free parameters, since the parameters in $n(k)$ have

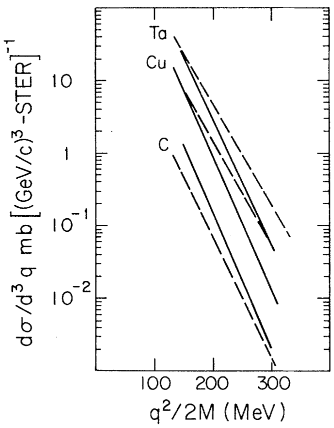

FIG. 3. Comparison of calculated (– – –) and experimental (—) inclusive proton spectra for 600-MeV protons incident on C, Cu, and Ta.

been fixed[8] from another and very different set of experiments and that we have made no attempt to search for more sophisticated or complicated forms for $n(k)$ or the off-shell amplitude since this would be inappropriate in terms of the direct, simple account of the data we are trying to give.

There are many questions raised by our calculation. Can the microscopic theory of $n(k)$ be improved? Can the single-scattering assumption be further checked or justified?[9] Can the ideas used here explain the deuterons or tritons also observed by Frankel *et al.*? What are the implications of the large high-momentum tail in our model for other medium-energy processes? We do not have complete answers to these questions. Rather, our purpose is to show that the proton cross sections can be simply explained and to relate that explanation to another set of data which depend on internal nuclear motion in a completely different physical region.

In summary, we propose a simple direct mechanism that accounts for the energetic backward protons seen by Frankel *et al.* in high-energy proton-nucleus collisions. The explanation is based on $n(k)$, the momentum distribution in the nuclear target, for very large momenta. We have a new parametrization of $n(k)$ that accounts for the data, which is gratifying, but at the same time points out that these remarkable experiments are giving information on $n(k)$ in a new and largely uncharted domain.

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[1]S. Frankel *et al.*, Phys. Rev. Lett. 36, 642 (1976).

[2]E. J. Moniz *et al.*, Phys. Rev. Lett. 26, 445 (1971); R. R. Whitney *et al.*, Phys. Rev. C 9, 2230 (1974).

[3]W. Czyż, Phys. Rev. 131, 2141 (1963); T. deForest and J. D. Walecka, Adv. Phys. 15, 1 (1966).

[4]In the Fermi-gas model used in Ref. 2 the argument of the energy conserving δ function is $E(\vec{p})+E(\vec{k})-E(\vec{q})-E(\vec{p}+\vec{k}-\vec{q})-\overline{\epsilon}$.

[5]F. Calogero and A. Degasperis, Phys. Rev. A 11, 265 (1975); R. D. Amado and R. M. Woloshyn, to be published.

[6]W. Czyż and K. Gottfried, Nucl. Phys. 21, 676 (1961).

[7]We use the same parameter γ for both neutron and proton momentum distributions. In Ref. 2 proton and neutron Fermi momenta are weighted so that proton and neutron densities are equal. Choosing such a weighting for γ has only a small effect on the final results.

[8]To gauge the sensitivity of the fit we note that a 5% change in γ leads to a 5% change in the height of the quasielastic (e,e') peak and a change of 25% (at the lowest observed q) or 60% (at the highest) in the proton cross section.

[9]For example, the single-scattering mechanism makes definite predictions for the angular dependence of the proton inclusive cross section which S. Frankel has proposed to measure.