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Theorem on π_{l2} Decays and Electron-Muon Universality*

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We show that the coefficient of the logarithmic lepton-mass singularity in the radiative corrections of order α to the total π_{l2} decay probability is not affected by the strong interactions and can therefore be rigorously computed. The phenomenological implications of this result and its connection with electron-muon universality are briefly discussed.

For a long time the study of π_{l2} decays has been one of the cornerstones of weak-interaction theory. Roughly at the time of the discovery of the electron decay mode, Berman and Kinoshita published their pioneering calculations of the order-α corrections to these processes, neglecting the effects of the strong interactions.[1,2] In particular Kinoshita calculated the corrections to the total decay probability and found a surprisingly large result. More recently, the calculation was repeated on the basis of a renormalizable model in which the π is treated as a member of a Higgs multiplet and essentially the same answer was obtained.[3] To this date it has remained unclear whether the results of the early calculations are merely properties of the peculiar models assumed or whether they apply to the real world. In this paper we answer this question to a considerable extent by means of the following theorem: "The coefficient of the logarithmic lepton-mass singularity (lms) in the radiative corrections of order α to the total π_{l2} decay probability is not affected by the strong interactions and can therefore be rigorously computed."[4] In proving this theorem we assume that the weak and electromagnetic interactions are described by a renormalizable gauge theory in which e-μ universality is natural and we neglect corrections of order G_F^2. As we will see, the theorem does not hold in general for partial decay probabilities.

In the gauge theories under consideration, the interaction of the W mesons with leptons and hadrons is described by

$$\mathcal{L}_W = -(g/\sqrt{2})(J^\lambda + L^\lambda)W_\lambda^\dagger + \text{H.c.}, \qquad (1)$$

where $L^\lambda = \bar{\nu}_e\gamma^\lambda a e + \bar{\nu}_\mu\gamma^\lambda a\mu + \ldots$ and $J^\lambda = \frac{1}{2}\cos\theta_C \times (V^\lambda - A^\lambda) + \ldots$ are the leptonic and hadronic currents, $a = \frac{1}{2}(1-\gamma_5)$, and ... indicates the possible contributions of heavy leptons and currents which carry strangeness, charm, and other possible flavors.

Virtual corrections.—We recall that the lms can arise because in the limit of zero lepton mass the invariants k^2 and $l\cdot k$ (k and l are the photon and lepton four-momenta) may vanish for quanta of nonzero frequency. A moment's thought tells us that aside from the usual field renormalization of the lepton, the only other virtual diagram that contributes to the lms is the one depicted in Fig. 1(a). (In nonphotonic diagrams,

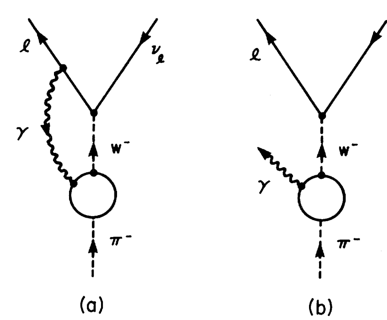

FIG. 1. Diagrams involving the strong interactions which contribute to lms.

lms does not arise because the heavy masses of the intermediate bosons and Higgs scalars act as effective infrared cutoffs; while diagrams in which the photon is exchanged between the W and l or in which the W is replaced by its associated Higgs scalar can only give lms of order G_F^2.)

In the 't Hooft–Feynman gauge the amplitude for Fig. 1(a) is

$$M_a=\frac{ig^2e^2}{4(2\pi)^4}\cos\theta_C\int\frac{d^4k}{k^2}\bar{u}_l\gamma^\mu\frac{1}{\not{l}+\not{k}-m_l}\gamma^\lambda a v_{\nu_l}\frac{1}{(P+k)^2-m_W^2}T_\mu(k), \tag{2a}$$

where

$$T_{\mu\lambda}(k)=\int d^4x e^{-ik\cdot x}\langle 0|T\{J_\mu^{\rm el}(x)[V_\lambda(0)-A_\lambda(0)]\}|\pi^-(P)\rangle. \tag{2b}$$

Separating out the Born term, the most general expression for $T_{\mu\lambda}(k)$ is

$$T_{\mu\lambda}(k)=-f_\pi(2P+k)_\mu(P+k)_\lambda\frac{F_\pi(k^2)}{(P+k)^2-m_\pi^2}+H_1g_{\mu\lambda}+H_2P_\mu k_\lambda+H_3P_\mu P_\lambda+H_4k_\mu P_\lambda+H_5k_\mu k_\lambda+iH_6\epsilon_{\mu\lambda\alpha\beta}k^\alpha P^\beta, \tag{2c}$$

where $H_i=H_i(k^2,s)$ $(i=1,2\dots6)$, $s=(P+k)^2$, and $F_\pi(k^2)$ is the electromagnetic form factor of the pion normalized to $F_\pi(0)=1$. We first note that because $\bar{u}_l\not{k}[\not{l}+\not{k}-m_l]^{-1}=\bar{u}_l$, H_4 and H_5 do not contribute to the lms. Next we observe that we can replace $H_i(k^2,s)$ with $H_i(0,s)$ because $H_i(k^2,s)-H_i(0,s)\sim k^2$ for small k^2 and the additional k^2 factor prevents the lms [similarly $F_\pi(k^2)\to F_\pi(0)$]. Gauge invariance tells us that

$$H_3(0,s)=0, \tag{3a}$$

$$H_1(0,s)+H_2(0,s)P\cdot k=f_\pi, \tag{3b}$$

so that

$$T_{\mu\lambda}(k)=-f_\pi(2P+k)_\mu(P+k)_\lambda[(P+k)^2-m_\pi^2]^{-1}+f_\pi g_{\mu\lambda}+H_2(P_\mu k_\lambda-g_{\mu\lambda}P\cdot k)+iH_6\epsilon_{\mu\lambda\alpha\beta}k^\alpha P^\beta+\dots, \tag{4}$$

where H_2 and H_6 are now evaluated at $k^2=0$ and, from this point on, ... represents terms which do not contribute to the lms or terms of higher order in G_F. The contribution to M_a of the first two terms in Eq. (4) can be explicitly computed: When combined with the lepton field renormalization, they are found to give the same lms as obtained in Ref. 2. After some elementary algebra, the contribution to M_a of the structure-dependent terms involving H_2 and H_6 can be written as

$$M_a^{(H_2,H_6)}=-M_0\frac{im_W^2\alpha}{\pi^3f_\pi m_\pi^2}\int\frac{d^4k}{k^2}\frac{(P\cdot k)^2[\hat{H}_2-\hat{H}_6]}{(k^2+2l\cdot k)}+\dots, \tag{5}$$

where $M_0=-f_\pi\cos\theta_C(g^2/4m_W^2)m_l\bar{u}_l a v_{\nu_l}$ is the zeroth-order amplitude, $\hat{H}_i=H_i/(s-m_W^2)$ $(i=2,6)$, and we have neglected terms proportional to m_l^2, k^2, and $l\cdot k$ since they do not contribute to the lms. To extract the lms from Eq. (5) we perform a series of steps. When we combine the two denominators in Eq. (5), shift variables $k'=k+lx$, and perform a Wick rotation (the required analyticity properties in k_0 and k_0' are obtained from the Low expansion of $T_{\mu\lambda}$), Eq. (5) becomes

$$M_a^{(H_2,H_6)}=M_0\frac{\alpha m_W^2}{\pi^3f_\pi m_\pi^2}\int_0^1dx\int d^4k''\frac{[P\cdot(k'-lx)]^2[\hat{H}_2-\hat{H}_6]}{(k''^2+m_l^2x^2)^2}+\dots, \tag{6}$$

where $k_0'=-ik_0''$, $k_i'=k_i''$, and the d^4k'' integration is over Euclidean four-dimensional space. The next step is to divide the integration into two regions: (a) $k''^2\equiv k_0''^2+\vec{k}''^2\le m_\pi^2$ and (b) $k''^2\ge m_\pi^2$. Region (b) cannot contribute to the lms because k''^2 in the denominator of Eq. (6) acts as an infrared cutoff and the corresponding integral is finite as $m_l\to 0$. To compute the contribution of region (a) we note that the restriction $k_0''^2+\vec{k}''^2\le m_\pi^2$ implies that we are in the region of analyticity of $\hat{H}_2$ and $\hat{H}_6$ with respect to the variable s. Therefore, we can expand $\hat{H}_2$ and $\hat{H}_6$ in a power series in s. The numerator of Eq. (6) becomes now a sum of terms involving odd and even powers of k'. Because of the symmetry of the integration, the odd powers cancel while the even powers generate factors of the form $(k''^2)^n$, $n\ge 0$, in the numerator. Inspection of Eq. (6) shows that only terms with $n=0$ can contribute to the lms. Therefore, the corresponding lms is simply obtained by setting $k'=0$ in the numerator of Eq. (6) and considering only the contribution of region (a). Performing the d^4k'' integration, using $2P$

$\cdot l = m_\pi^2 + m_l^2$ appropriate to the two-body decay, and introducing the new integration variable $s = m_\pi^2(1-x)$ in place of x, we find

$$M_a^{(H_2,H_6)} = -M_0(\alpha/2\pi)\{\ln(m_\pi/m_l)\,(1/f_\pi)\int_0^{m_\pi^2} ds(1-s/m_\pi^2)^2[H_2(0,s)-H_6(0,s)]+\ldots\}. \tag{7}$$

Combining Eq. (7) with the contribution of the first two terms in Eq. (4) and adding the field renormalization of the lepton and the photon-infrared divergent part of the π field renormalization we finally obtain for the virtual corrections to the transition probability

$$\Delta P_V = P_0\frac{\alpha}{\pi}\left\{\frac{1+\mu^2}{1-\mu^2}\left[2\ln\mu\ln\left(\frac{m_\pi}{\lambda_{\min}}\right)+(\ln\mu)^2\right]+2\ln\left(\frac{m_\pi}{\lambda_{\min}}\right)-\tfrac{1}{2}\ln\mu\right.$$
$$\left.+\ln\mu\left[3+\frac{1}{f_\pi}\int_0^{m_\pi^2} ds\left(1-\frac{s}{m_\pi^2}\right)^2[H_2(0,s)-H_6(0,s)]\right]+\ldots\right\}, \tag{8}$$

where $\mu = m_l/m_\pi$ and P_0 is the uncorrected decay rate. The terms involving H_2 and H_6 in Eq. (8) represent the structure-dependent contributions to lms induced by the strong interactions. The remainder agrees with Kinoshita's result.[2]

Inner bremsstrahlung.—We consider the contribution of diagram 1(b) and the additional graph in which the photon is emitted by the lepton. The total amplitude can be written as $M_{\text{I.B.}} = M_{\text{I.B.}}^{(1)} + M_{\text{I.B.}}^{(2)}$, where $M_{\text{I.B.}}^{(1)}$ is independent of the strong interaction form factors and $M_{\text{I.B.}}^{(2)}$ involves only terms proportional to H_2 and H_6. The contribution of $|M_{\text{I.B.}}^{(1)}|^2$ to the total transition probability is given in Eq. (4) of Ref. 2. The only additional lms arises from the interference of $M_{\text{I.B.}}^{(1)}$ and $M_{\text{I.B.}}^{(2)}$. This can be readily computed by summing over photon and lepton polarizations and integrating over the neutrino momenta, photon-lepton angle, and electron energy, in that order. For this structure-dependent contribution we find

$$\Delta P_{\text{I.B.}}^{(H_2,H_6)} = -P_0(\alpha/\pi)\ln\mu\,(1/f_\pi)\int_0^{m_\pi^2} ds(1-s/m_\pi^2)^2[H_2(0,s)-H_6(0,s)]+\ldots. \tag{9}$$

Compare Eqs. (8) and (9): The structure-dependent terms cancel in the total decay probability thus establishing the theorem! To obtain the model-independent answer we add Eq. (8), Eq. (9), and Kinoshita's calculation of the contributions from $|M_{\text{I.B.}}^{(1)}|^2$ and find that all the lms cancel in the total decay probability except for the term $3(\alpha/\pi)\ln\mu$. This surviving contribution arises from the presence of the $f_\pi g_{\mu\lambda}$ term in Eq. (4): a consequence of gauge invariance! [See Eq. 3(b)]. As pointed out by Kinoshita, because M_0 is proportional to m_l this result does not contradict the theorem on the cancellation of mass singularities.[5]

The cancellation of structure-dependent terms does not generally occur in partial decay probabilities. For example, if the lepton energy is restricted to the configuration $E_m - \Delta E \leq E \leq E_m$ with $\Delta E \ll E_m$ (E_m is the lepton energy in the two-body decay), the interference of $M_{\text{I.B.}}^{(1)}$ and $M_{\text{I.B.}}^{(2)}$ can be neglected and we are left with the structure-dependent terms of Eq. (8).

The above theorem has interesting phenomenological implications. In fact, aside from the contribution of $|M_{\text{I.B.}}^{(2)}|^2$ which will be discussed later, the theoretical expression for the fractional correction $\Delta P/P_0$ to the total π_{l2} decay rate can be expanded in a power series in m_l and $\ln m_l$. We have shown that the terms of order $\ln m_l$ are structure independent. Their contribution to the ratio R of the total decay probabilities for $\pi \to e + \bar{\nu}_e$ and $\pi \to \mu + \bar{\nu}_\mu$ is $-3(\alpha/\pi)\ln(m_\mu/m_e) \times R_0$ which amounts to a very large correction: -3.7%. Aside from this, the strong interactions may affect the terms of zeroth order in m_l and of order $m_l^2\ln m_l$, m_l^2, etc. The terms of zeroth order in m_l cancel in the ratio R while the contributions of order m_l^2 can be potentially significant only for the muon decay mode. As structure-dependent contributions induced by the strong interactions are expected to be of relative order m_μ^2/m^2, where m is a typical hadronic mass, and there are no large logarithms available, it is difficult to see how such terms can give rise to large unaccounted corrections. In addition to these effects, one must also consider the contribution of $|M_{\text{I.B.}}^{(2)}|^2$ which is potentially large for the electron decay mode because, unlike P_0 and the other contributions to ΔP, it is not suppressed by a factor m_e^2. Fortunately, these terms can be analyzed by a combination of theoretical and experimental arguments: Neglecting the slight energy dependence of $H_2(0,s)$

and $H_6(0, s)$, applying conserved vector current to relate $|H_6(0, 0)|$ to $\Gamma_{\pi^0\rightarrow 2\gamma}$,[6] and using the experimental value[7] for $\gamma \equiv H_2(0, 0)/H_6(0, 0)$, we find that the contribution of $|M_{\mathrm{I.B.}}^{(2)}|^2$ to the total decay rate into the electron mode amounts to only $\simeq +0.06\%$ for the solution $\gamma_{\mathrm{exp}} = 0.15$ and $\simeq +0.28\%$ for the solution $\gamma_{\mathrm{exp}} = -2.07$.

In summary, our results give considerable support to the main features of the early calculations.[8] A recent re-examination of an old experiment indicates that the theoretical prediction for R given in Ref. 2 lies within 2 standard deviations of the experimental value.[9] In view of our results and the inconclusive nature of the present experimental data, we believe that a precise measurement of the ratio R is needed to test more accurately electron-muon universality in weak interactions.

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Elastic Photoproduction of ω Mesons from Hydrogen, Deuterium, and Complex Nuclei*

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We have studied ω photoproduction using 7.5- to 10.5-GeV tagged photons. Cross sections from hydrogen lie 22% below the bubble chamber results of Ballam *et al.*, but have a similar slope. Density matrices indicate approximate s-channel helicity conservation. The proton-neutron cross-section difference is midway between zero and the value suggested by the γp-γn total-cross-section difference. Fits to the cross sections for D, Be, and Cu and those for Be, C, Al, Cu, and Pb from a previous experiment yield $\sigma_{\omega N} = 25.4 \pm 2.7$ mb and $\gamma_\omega^2/4\pi = 7.6 \pm 1.2$.

Photoproduction of ω mesons is of interest for several reasons. Among them are these: (1) The γ-ω direct coupling constant $\gamma_\omega^2/4\pi$ and the ω-nucleon total cross section $\sigma_{\omega N}$ can be determined[1-3] from the A dependence of forward photoproduction. (2) A_2 exchange contributes to ω photoproduction, giving rise to a difference in cross section from neutrons and protons. The difference in the slope of the cross sections provides information about the A_2 exchange trajectory.[4] The difference in the forward cross sections is related, through vector dominance, to the difference in total hadronic photoproduction cross sections on protons and neutrons.[5] (3) As an example of a dominantly diffractive process, the extent to which helicity is conserved is of interest.

Two counter experiments[1,2] have studied ω photoproduction from complex nuclei. A series of bubble-chamber experiments[6] have studied photoproduction from hydrogen. One counter experi-