Properties of Inclusive Hadron Spectra in Muon-Nucleon Scattering at 150 GeV/c*

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(Received 22 March 1976)

We have studied muon-produced hadrons from a deuterium target. The structure functions and the charge ratios are reported for neutrons; the transverse momentum and azimuthal distributions are reported for deuterons. The structure function for the neutron is similar to that of the proton. The charge ratio of produced hadrons follows the expectation of a simple spin-$\frac{1}{2}$ quark model. Transverse-momentum results agree with those at lower energy and are similar to those from hadron-hadron interactions. No azimuthal anisotropy is seen.

In a previous paper[1] we reported measurements of the hadron spectra from virtual-photon interactions with protons. In this paper we extend these measurements to neutrons. The data were obtained using the muon scattering facility at Fermilab and comprise 2.1×10^{10} muons incident on 8.4 g/cm^2 of liquid hydrogen and 1.84×10^{10} muons incident on 20.1 g/cm^2 of liquid deuterium. Approximately 3800 and 7300 muon scatters from hydrogen and 7300 muon scatters from deuterium, having $q^2>0.5$, form the data base.

The hadron invariant structure function for the neutron is

$$F_n(x') = \int dp_T^2 \frac{1}{\pi} \frac{E^*}{(p_{\max}^{*2}-p_T^2)^{1/2}} \frac{1}{\sigma} \frac{d^2\sigma}{dp_T^2 dx'}, \quad (1)$$

where $x' = p_\parallel^*/(p_{\max}^{*2}-p_T^2)^{1/2}$ is a Feynman scaling variable, and p_T, $p_\parallel^*$, E^*, $p_{\max}^*$ are respectively the transverse momentum, longitudinal momentum, energy, and maximum momentum of a hadron in the center-of-mass system for the virtual photon and neutron. The square of the center-of-mass energy is $s=2M_n\nu+M_n^2-q^2$ where M_n is the neutron mass, ν is the lab energy lost by the scattered muon, and $-q^2$ is the square of the mass of the virtual photon. $F_n(x')$ is derived from the corresponding deuteron and proton structure functions as follows.

We assume that the deuteron-muon inclusive cross section $\sigma_d(q^2, s)$ is the sum of the neutron and proton cross sections. We also assume the hadrons per deuteron interaction are the sum of those from the neutron and the proton. This leads to

$$F_n(x') = \frac{\sigma_d}{\sigma_n} F_d(x') - \frac{\sigma_p}{\sigma_n} F_p(x'). \quad (2)$$

The ratios σ_d/σ_n and σ_p/σ_n are functions of $\omega = 2m\nu/q^2$ and are taken from inelastic electron scattering experiments.[2] The muon inclusive data are consistent with these but have less accuracy. The hadron charge ratio is obtained in an analogous way.

The apparatus and methods were described in Ref. 1. The corrections applied to the data are the same except for the deuteron radiative correction and the increased rescattering in the denser deuterium. Detailed radiative corrections, beyond the subtraction of the elastic tail, have negligible effect on the results. Corrections for the Fermi motion of the nucleons in the deuteron or for the shadowing of one nucleon by the other are also negligible. Only statistical errors are shown on the graphs.

Figure 1 shows $F_n(x')$ for the neutron. The data show no significant variation with q^2 or s, and the structure functions are equal, within statistics, to those for the proton. Figure 2 shows the forward charge ratio N^+/N^- for the neutron plotted as a function of ω. Also shown are the proton data from this experiment and other proton and neutron data.[3] The value of ω used for our data is the average value for the events in the q^2-s ranges of Fig. 1. The solid lines in Fig. 2 are fits to the data of Dakin *et al.*[3] based on a simple quark model[4] of the proton and neutron.

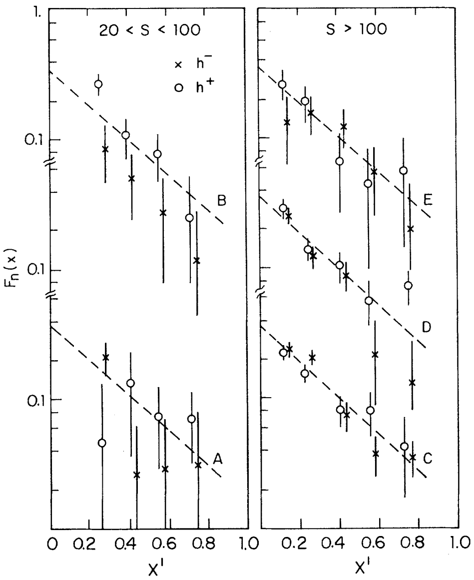

FIG. 1. $F_n(x')$ for the neutron for various q^2-s ranges. The line is $0.35\exp(-3.25x')$. Curves A and C, $0.5<q^2<3$ GeV^2; curves B and D, $3<q^2<10$ GeV^2; curve E, $q^2>10$ GeV^2.

Our data are consistent with these fits. In this quark model the neutron and proton are composed of a sea of quark-antiquark pairs as well as valance quarks. At high ω most of the virtual-photon interactions are expected to be with the quark sea which is charge symmetric. The similarity of the neutron and proton data at high ω and the fact that the charge ratios tend to 1 as ω increases is evidence for this sea.

The neutron and proton data are similar. From here on we treat the deuteron as if composed of two independent and equivalent nucleons. The rest of the results are from the deuterium data only.

It is of interest to compare the hadronic structure function for muon-nucleon scattering with that from electron-positron collisions. Figure 3 shows $F(x')$ for the deuterium data compared to the analogous structure function for electron-positron annihilation.[5] The variable and structure function for annihilation is different in detail from $F(x')$ used here but for $x'>0.2$ it is essentially the same. Table I describes the functions plotted. The similarity of the photon and annihilation data is striking. This feature is predicted by a quark model.[4]

Hadron production by scalar and transverse virtual photons can produce interference terms in the azimuthal distributions of hadrons about

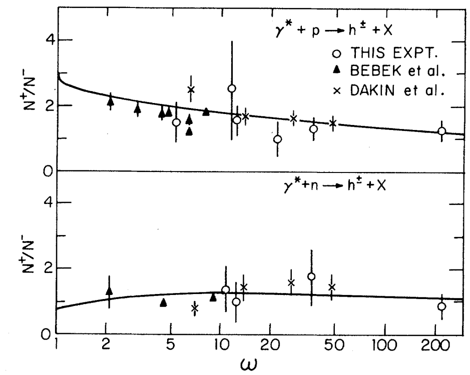

FIG. 2. Ratio of positive to negative charged hadrons as a function of average ω for this and other experiments. The solid lines are the fits mentioned in the text. The x' cuts are $0.4<x'<0.85$ for this experiment and Dakin *et al.*, $0.3<x'<0.7$ for Bebek *et al.*

the direction of the virtual photon. Most generally we may write[6]

$$\frac{d\sigma}{d\varphi_h}=A+\epsilon B-\epsilon C\cos2\varphi_h - 2(\epsilon+\epsilon^2)^{1/2}D\cos\varphi_h, \qquad (3)$$

where φ_h is the azimuthal angle of the hadron about the virtual photon referenced to the azimuthal angle of the leptons and ϵ is the transverse polarization of the virtual photon. The data are binned as a function of p_T, x', q^2, and ϵ. No significant anisotropy is shown by the azimuthal distributions. Figure 4 shows $d\sigma/d\varphi_h$ for two ranges of q^2 and with x' cut to be clearly in the region of photon fragmentation. The data

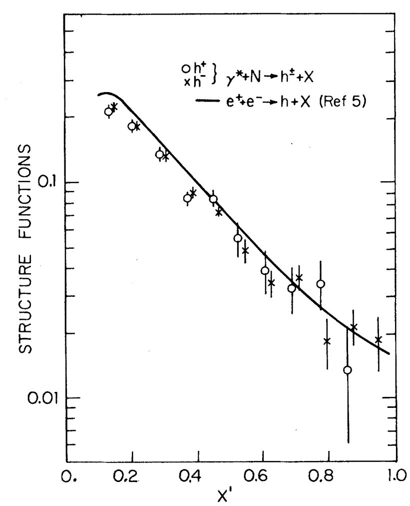

FIG. 3. Structure functions for processes described in Table I.

TABLE I. Description of structure functions.

Interaction	Structure function	Variable	c.m. energy
$\gamma^* + N \rightarrow h^\pm + x$	$F(x')$	$x' = \dfrac{p_\parallel^*}{(p_{\max}^{*2} - p_T^2)^{1/2}}$	$s > 100\ \mathrm{GeV}^2$
$e^+ + e^- \rightarrow h + x$	$\dfrac{1}{4\pi\sigma}\dfrac{d\sigma}{dx}$	$x = \dfrac{2p}{E_{\mathrm{c.m.}}}$	$s = 55\ \mathrm{GeV}^2$

have $s > 20\ (\mathrm{GeV})^2$ and are summed over all p_T and ϵ. Both are consistent with a constant distribution. Ravndal[6] showed that the absence of polarizations is consistent with a spin-$\frac{1}{2}$ parton model.

To complete the description of the inclusive hadron distributions we have fitted the p_T^2 dependence of the differential structure function $(E_h/\sigma) \times (d^3\sigma/dp_h^3)$ as a function of x'. We have used the form

$$\frac{E_h}{\sigma}\frac{d^3\sigma}{dp_h^3} = \left(\frac{E_h}{\sigma}\frac{d^3\sigma}{dp_h^3}\bigg|_{p_T^2=0}\right) \times \exp\left(\frac{-2bp_T^2}{1+(1+p_T^2/M^2)^{1/2}}\right). \qquad (4)$$

This is the transverse-momentum distribution used in Ref. 1 but it is parametrized differently. M is mainly determined by the low-x' data. We have used the value of M determined at $x' = 0.15$ as a fixed parameter in fits to the data at other x'. M is found to be 0.45 ± 0.05 GeV.

Some of the data and the results of the fits are shown in Fig. 5. The χ^2 per degree of freedom for the fits are consistent with statistical fluctuations only. Also plotted is the directly measured average transverse momentum $\langle p_T \rangle$. Both b and $\langle p_T \rangle$ have a strong x' dependence; this behavior and the magnitudes of b and $\langle p_T \rangle$ are typical of hadron-hadron collisions.[7] The value of $(E_h/\sigma) \times (d^3\sigma/dp_h^3)$ at $p_T^2 = 0$ is in good agreement with the lower-energy measurements of Bebek *et al.*[3]

We conclude the following. (1) The photon-produced hadron-inclusive distributions are consistent with a simple spin-$\frac{1}{2}$ quark model. This is shown by the charge ratios, the similarity of $F(x')$ for both neutron and proton, the apparent lack of azimuthal hadron polarization, and the agreement of our measurements with those at lower energy. (2) Hadronic transverse-momentum distributions produced by virtual photons are similar to those produced in hadron-hadron interactions.

We thank the staffs of Fermilab, of our home institutions, and of the Rutherford Laboratory whose help made this work possible.

*Work supported by the U. S. Energy Research and Development Administration Contracts No. E(11-1)3064 and No. 1195, National Science Foundation Contract No.

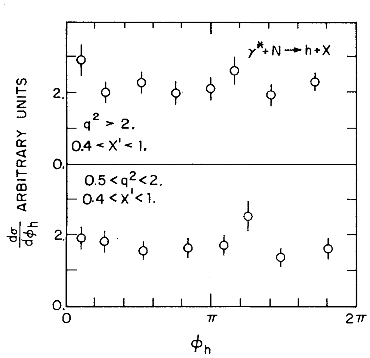

FIG. 4. Azimuthal cross section for hadrons.

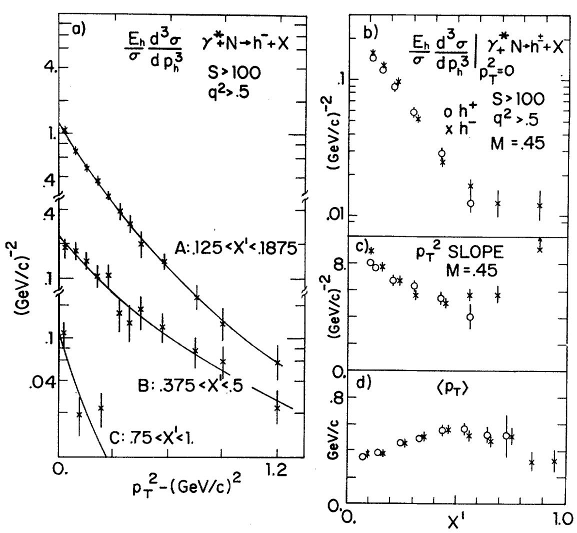

FIG. 5. The differential structure function $(E_h/\sigma) \times (d^3\sigma/dp_h^3)$: (a) typical data with fits; (b), (c) fitted parameters as functions of x'; (d) average transverse momentum measured directly.

MPS 71-03-186, and the Science Research Council, (U.K.).
[1]W. A. Loomis *et al.*, Phys. Rev. Lett. 35, 1483 (1975).
[2]A. Bodek *et al.*, Phys. Rev. Lett. 30, 1087 (1973).
[3]J. T. Dakin *et al.*, Phys. Rev. D 10, 1401 (1974); C. J. Bebek *et al.*, Phys. Rev. Lett. 34, 759 (1975).
[4]J. T. Dakin and G. J. Feldman, Phys. Rev. D 8, 2862 (1973); J. D. Bjorken, in Proceeding of the SLAC Summer Institute in Particle Physics, Stanford, California, 21–31 July 1975 (unpublished).
[5]R. Schwitters, in *Proceedings of the International Symposium on Lepton and Photon Interactions at High Energies, Stanford, California, 1975*, edited by W. T. Kirk (Stanford Linear Accelerator Center, Stanford, Calif., 1975).
[6]F. Ravndal, Phys. Lett. 43B, 301 (1973).
[7]P. Bosetti *et al.*, Nucl. Phys. B54, 141 (1973).

Theorem on π_{l2} Decays and Electron-Muon Universality*

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(Received 12 April 1976)

We show that the coefficient of the logarithmic lepton-mass singularity in the radiative corrections of order α to the total π_{l2} decay probability is not affected by the strong interactions and can therefore be rigorously computed. The phenomenological implications of this result and its connection with electron-muon universality are briefly discussed.

For a long time the study of π_{l2} decays has been one of the cornerstones of weak-interaction theory. Roughly at the time of the discovery of the electron decay mode, Berman and Kinoshita published their pioneering calculations of the order-α corrections to these processes, neglecting the effects of the strong interactions.[1,2] In particular Kinoshita calculated the corrections to the total decay probability and found a surprisingly large result. More recently, the calculation was repeated on the basis of a renormalizable model in which the π is treated as a member of a Higgs multiplet and essentially the same answer was obtained.[3] To this date it has remained unclear whether the results of the early calculations are merely properties of the peculiar models assumed or whether they apply to the real world. In this paper we answer this question to a considerable extent by means of the following theorem: "The coefficient of the logarithmic lepton-mass singularity (lms) in the radiative corrections of order α to the total π_{l2} decay probability is not affected by the strong interactions and can therefore be rigorously computed."[4] In proving this theorem we assume that the weak and electromagnetic interactions are described by a renormalizable gauge theory in which $e-\mu$ universality is natural and we neglect corrections of order G_F^2. As we will see, the theorem does not hold in general for partial decay probabilities.

In the gauge theories under consideration, the interaction of the W mesons with leptons and hadrons is described by

$$\mathcal{L}_W = -(g/\sqrt{2})(J^\lambda + L^\lambda) W_\lambda^\dagger + \text{H.c.}, \qquad (1)$$

where $L^\lambda = \bar{\nu}_e \gamma^\lambda a e + \bar{\nu}_\mu \gamma^\lambda a \mu + \ldots$ and $J^\lambda = \frac{1}{2}\cos\theta_C \times (V^\lambda - A^\lambda) + \ldots$ are the leptonic and hadronic currents, $a = \frac{1}{2}(1-\gamma_5)$, and ... indicates the possible contributions of heavy leptons and currents which carry strangeness, charm, and other possible flavors.

Virtual corrections.—We recall that the lms can arise because in the limit of zero lepton mass the invariants k^2 and $l \cdot k$ (k and l are the photon and lepton four-momenta) may vanish for quanta of nonzero frequency. A moment's thought tells us that aside from the usual field renormalization of the lepton, the only other virtual diagram that contributes to the lms is the one depicted in Fig. 1(a). (In nonphotonic diagrams,

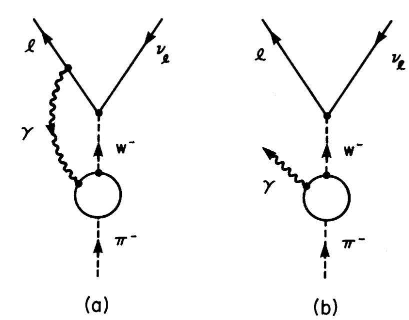

FIG. 1. Diagrams involving the strong interactions which contribute to lms.