ed here.

[4]An x distribution given by $E\,d^3\sigma/dp^3 \propto \exp(-6|x|) \times \exp(-1.6p_t)$ for $|x| > 0.2$ and flat in x for $|x| < 0.2$ also seems to describe the data in Ref. 5. Assuming such a distribution would increase the total cross sections quoted here by 60%. The differential cross sections, however, are not significantly affected.

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Connection between Scale Breaking in Deep Inelastic Processes and Large-p_T Hadronic Reactions*

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(Received 15 March 1976)

Scaling violation in deep inelastic lepton-proton processes is shown to be related to the apparent p_T^{-8} behavior of the large-p_T pion inclusive distribution at $\sqrt{s} > 23.5$ GeV, within the framework of the quark-quark hard-collision model. Power-law scale-breaking fits of the structure functions as determined by the ep and μp data are used as inputs in our calculations. Excellent agreement with the high-energy data on large-p_T pions is obtained in both the energy and x_T dependences. No adjustable parameters are used except for one overall normalization.

The discovery of apparent scaling in deep inelastic scattering prompted the speculation that the production of particles with large transverse momenta in hadronic processes might proceed via the wide-angle scattering of pointlike constituents of the incident hadrons.[1] The prediction that at large p_T, inclusive cross sections should fall as $(1/p_T)^4$ was, however, contradicted by subsequent CERN intersecting-storage-rings (ISR) data[2] which suggested an exponent close to 8. Thus an alternative parton model,[3] in which the basic parton-parton interaction is discarded, leaving only those processes that can be pictured as involving the exchange of partons (rather than gluons), has been used to gain a phenomenological understanding of the existing data above 100 GeV.

The recent discoveries of scaling violation in lepton-induced processes imply that the hadronic constituents are probably not pointlike. This would, in turn, imply that in the Blankenbecler-Brodsky-Gunion (BBG) model[3] there is *no* subprocess that falls off slowly enough in p_T to account for the above-mentioned exponent of 8. If that is the case, the only way to rescue a constituent picture of high-p_T processes would be via the parton-parton subprocess, modified to include the effects of scale breaking. A previous attempt[4] has been made along these lines using the logarithmic violations of scaling that arise in asymptotically free theories. The results were negative. However, the observed pattern of scaling violation in no way demands asymptotic freedom; indeed, more conventional powerlike violations of scaling are equally consistent with the data.[5] It is this latter type of scale breaking which we shall investigate.

In this Letter we adopt the point of view that quarks are, in fact, not pointlike. Our aim is to relate the scaling violation in deep inelastic scattering to the large-p_T behavior in hadron-induced processes and to show that they are mutually consistent within the hard-collision model.

The differential probability $dP_{a/A}$ that a hadron A is seen by a probe with momentum transfer Q^2 to contain a constituent a with a fraction x_a of its longitudinal momentum is

$$dP_{a/A} \equiv f_{a/A}(x_a, Q^2)\,dx_a\,, \qquad (1)$$

where $f_{a/A}$ can be inferred from the deep inelastic lepton scattering data. Similarly, the probability that a constituent c yields a hadron C carrying a fraction y of the constituent's momentum is

$$dP_{C/c} \equiv g_{C/c}(y, Q^2)\,dy, \tag{2}$$

where $g_{C/c}$ can be inferred from the e^+e^- annihilation data. Operationally one may choose to adopt the view that for each range of Q^2 there is a set of approximately scaling structure functions.

These distribution functions enter into the quark-quark[6] hard-collision model for large-momentum-transfer hadronic reactions $A+B\to C+X$ in the way shown in Fig. 1. It is in the central blob that the large-angle scattering of the quarks takes place. For hadron C observed at 90° in the c.m. system of A and B, the invariant inclusive cross section is[7]

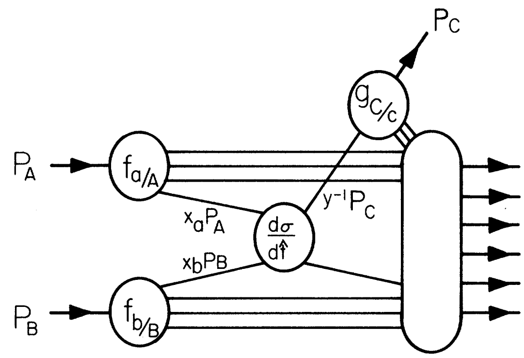

FIG. 1. Hard-collision model for $A+B\to C+X$.

$$E_C\frac{d^3\sigma}{dp_C^{\,3}} = \frac{1}{\pi}\sum_{a,b,c}\int_{x_1}^{1}dx_a\int_{x_2}^{1}dx_b\, f_{a/A}(x_a, Q^2)\, f_{b/B}(x_b, Q^2)\,\frac{d\sigma}{d\hat{t}}\,\frac{g_{C/c}(y, Q^2)}{y}, \tag{3}$$

where

$$x_T = 2p_T/\sqrt{s}\,,\quad x_1 = x_T/(2-x_T),$$

$$x_2 = x_a x_T/(2x_a - x_T),\quad y = \tfrac{1}{2}x_T(x_a^{-1}+x_b^{-1}),$$

$$\hat{t} = -Q^2 = -s x_a x_T/2y,$$

and $d\sigma/d\hat{t}$ is the differential cross section for the hard-collision subprocess.

For comparison later, let us first indicate the conventional results of the hard-collision model.[1] In that model the structure functions $f(x)$ and $g(y)$ are scale invariant, corresponding to constituents which are pointlike for all (large) Q^2. Using spin-$\frac{1}{2}$ partons with vector interaction, (3) can be reduced to

$$E_C\, d^3\sigma/dp_C^{\,3} = s^{-2}h_1(x_T) = p_T^{\,-4}h_2(x_T), \tag{4}$$

where h_1 and h_2 are some scaling functions. Equation (4) is, of course, the obvious result from dimensional analysis and is known to be in conflict with the present experimental data up to 53-GeV c.m. energy. More explicitly, in order to compare with data the p_T dependences for fixed s values, we use $f(x)\propto \nu W_2(x)/x$ as given by the so-called "precocious-scaling" ep data[8] and $g(y)\propto(1-y)/y$, which is the conventional parametrization for pion production. The results are shown in Fig. 2 by the dashed lines, arbitrarily normalized to facilitate comparison with the π^0 inclusive cross sections at 90°.[2]

Since the data falls in p_T faster than the above prediction based on pointlike quarks, it is then reasonable to suppose that quarks are not pointlike, at least up to 53-GeV c.m. energy. This point of view is in accord with the phenomenological fact that the structure functions for electroproduction exhibit significant breaking of Bjorken scaling even for $q^2 < 20$ GeV^2. We now connect quantitatively these two features of the hadronic

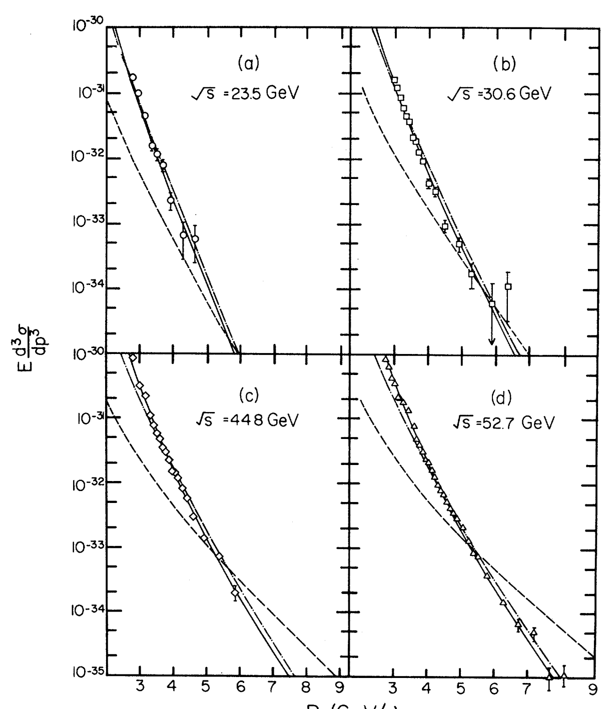

FIG. 2. Theoretical results for the π^0 spectra at 90°: scaling (dashed line); nonscaling, Eq. (5) (dot-dashed line); nonscaling, Eq. (6) (solid line). Data are taken from Ref. 2.

and leptonic data using (3) as the basis for our calculation. As mentioned earlier, we assume the scale breaking to be powerlike and adopt the experimentalists'[9] parametrization based on the Chanowitz-Drell form of the quark form factor.[10] The best fit that they obtained (with $x=1/\omega$) is

$$\nu W_2(x, Q^2) = (1 + Q^2/\Lambda^2)^{-2} F_2(x), \qquad (5)$$

with $\Lambda^{-2} = 0.0204 \pm 0.0017$ for $0.1 < x < 0.8$. In our calculation we used[11] $\Lambda^2 = 50$, and assumed as a first approximation $f(x, Q^2) \propto \nu W_2(x, Q^2)/x$. For $g(y)$ we note that $s\,d\sigma/dx$ for $e^+e^- \to$ hadrons shows[12] no scaling violation up to $E_{\text{c.m.}} \sim 7$ GeV for $y > 0.5$, so we continue to use $g(y) \propto (1-y)/y$. We remark that the important region in our calculation which yields results that can be compared with the existing data[2] is for $0.1 < x < 0.8$ and $0.5 < y < 1$. In that region scaling violation is observed in $f(x, Q^2)$ but not in $g(y)$. The results of our calculations for various fixed energies are shown by the dot-dashed lines in Fig. 2. Evidently, agreement with the data is dramatically improved.

One may object to the naive way of accounting for the scaling violation as given in (5), since there exist both theoretical and experimental reasons to expect that a factorizable form is inadequate. Theoretically,[5,13] certain models suggest that the area under the νW_2 curve should be independent of Q^2 and its first moment should decrease with increasing Q^2. Experimentally, the μp data[14] from Fermilab reveal indications roughly compatible with this theoretical view although the errors are too large to be conclusive.[15] In order to see the effects of these large-Q^2 features, we modified νW_2 by adding to (5) a term which is important only at small x, the damping at higher values of x being more severe at higher values of Q^2. The modified form is

$$\nu W_2(x, Q^2) = (1 + Q^2/\Lambda^2)^{-2} F_2(x) + b(Q^2)(1-x)^{\alpha Q^2}, \qquad (6)$$

where the first term on the right-hand side is the nonscaling structure function of (5). The factor $b(Q^2)$ is determined by the condition that the integral over x of νW_2 as given by (6) is independent of Q^2. The parameter α is adjusted so that (6) is consistent with the μp data[14]; a value of 0.5 is then obtained. A repetition of our previous calculation using (6) in place of (5) yields the results as shown by the solid lines in Fig. 2. The near-perfect agreement with the data is striking.

We emphasize that apart from an overall normalization, it is a no-parameter fit: The two parameters Λ^2 and α in (6) are fixed by the ep and μp data, respectively. In fact, even the normalization is reasonable in that the corresponding α_{eff} for the parton-gluon coupling turns out to be of order 1. Defining A and B by $f = A\,\nu W_2/x$ and $g = B(1-y)/y$, we have used the approximate values $A = 2.5$ as inferred from the ep data and $B = 0.4$ from the e^+e^- data for $0.5 < y < 1$. The overall normalization N, as defined by $N = A^2 B \alpha_{\text{eff}}^2$, is determined by our fit in Fig. 2 to be 5.2 (dot-dashed line) and 3.7 (solid line) for all energies. Thus, α_{eff} is 1.4 and 1.2, respectively.

In our calculations above we have used the usual approximation $f(x, Q^2) \propto \nu W_2(x, Q^2)/x$. To be more precise, we should distinguish to contributions from the various different types of quarks. To see what difference that would make, we have gone to the other extreme by identifying $f(x, Q^2)$ with the u quark distribution as determined by Barger and Phillips.[8] Calculations have been made both with and without the modification factor in (6). The results are essentially the same as the solid and dot-dashed curves shown in Fig. 2 with insignificant deviations. We therefore conclude that in the calculation of the π^0 cross section further attention to the other quark types would be pointless: Our results show that what is crucial here is the space-time structure of the quarks, and not their individual quantum numbers.

We have displayed the results of our calculations as fixed-energy plots. However, the usual parton models predict that the cross sections should have the factorizable form

$$E_C\, d^3\sigma/dp_C^3 = p_T^{-n} h(x_T), \qquad (7)$$

which fits the constant-charge-ratio data[2] with $n = 8.24$. The success of (7) has been taken to render significant support for the pointlike-quark models. To see how our results compare with the data which have been taken to confirm (7), we have plotted in Fig. 3 $p_T^{8.24} E\, d^3\sigma/dp^3$ versus x_T for the same p_T range as the data. The theoretical curves in Fig. 3, groups a and b, correspond, respectively, to the dot-dashed and solid lines in Fig. 2, i.e., to using (5) and (6) as inputs for $f(x, Q^2)$. Clearly, the agreement is astonishingly good. It should be emphasized, therefore, that the factorizable form of (7) is *not* implied by the data: The universal behavior in x_T is only apparent and cannot be expected to hold in the presence of scale breaking for all s and x_T.

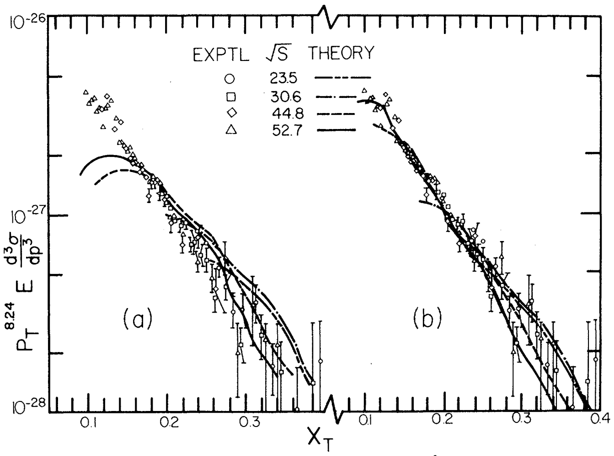

FIG. 3. Theoretical results for the π^0 spectra at 90°: group *a*, nonscaling, Eq. (5); group *b*, nonscaling, Eq. (6). Again data are taken from Ref. 2.

In conclusion, we stress that our attempt has been to establish a *phenomenological* connection between the scaling violation in the deep inelastic lepton-induced processes on the one hand, and the large-transverse-momentum behavior of the hadron-induced inclusive reactions on the other, within the framework of the quark-quark hard-collision model. As input to the calculation of the pion spectrum, we use nonscaling experimental data in a power-law parametrization, but we do not speculate on the theoretical origin of the scaling violation. There are no adjustable parameters in our calculation apart from the overall normalization, which turns out to be very reasonable in that it corresponds to $\alpha_{eff} \sim 1$. The result clearly indicates that the quark-quark hard-collision model is a viable picture, contrary to earlier conclusions. Consequently, the least that one can infer is that quark-quark scattering should be included as an important subprocess among other possible ones. Also, quark counting rules taken together with scaling violation predict for all subprocesses except quark-quark scattering too rapid a fall off in p_T to account for the ISR data.[2] Hence, the only way to understand the large-p_T phenomena in the hard-collision scattering picture is through the quark-quark subprocess. Our results lend support to this view. We believe that this work suggests strongly the existence of quark structure with the quark size about one tenth that of the proton.

We thank Professor J. Gunion for helpful discussions.

*Work supported in part by the U. S. Energy Research and Development Administration under Contract No. AT(45-1)2230.

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