are also grateful to Dr. H. W. den Hartog of this university for growing the Pr^{3+} in LaF_3 crystal and to P. de Bree for his help in some of the experiments. The expert technical assistance of H. J. Bruinenberg and W. Zevenberg in the design and construction of several mechanical parts in the setup were of vital importance. The investigations were supported by the Netherlands Foundation for Chemical Research (S.O.N.) with finicial aid from the Netherlands Organization for the Advancement of Pure Research (Z.W.O.).

[1]H. de Vries and D. A. Wiersma, Phys. Rev. Lett. 36, 91 (1976).

[2]I. D. Abella, N. A. Kurnit, and S. R. Hartmann, Phys. Rev. 141, 391 (1966).

[3]A. Bree and V. V. B. Vilkos, Spectrochim. Acta 27A, 2333 (1971).

[4]N. Takeuchi and A. Szabo, Phys. Lett. 50A, 361 (1974).

[5]T. W. Hänsch, Appl. Opt. 11, 895 (1968).

[6]J. Trotter, Acta Crystallogr. 14, 1135 (1961).

[7]T. J. Aartsma and D. A. Wiersma, unpublished.

[8]S. Chandra, N. Takeuchi, and S. R. Hartmann, Phys. Lett., 41A, 91 (1972).

[9]R. M. Hochstrasser and G. J. Small, J. Chem. Phys. 48, 3612 (1968).

[10]The fluorescent decay of pyrene is markedly nonexponential. Next to a fast decay component of 75 nsec there is a weak underlying slow component of about 400 nsec. This was observed for all three sites and is as yet unexplained.

Parametric Decay of "Kinetic Alfvén Wave" and Its Application to Plasma Heating

Akira Hasegawa
Bell Laboratories, Murray Hill, New Jersey 07974

and

Liu Chen*
Plasma Physics Laboratory, Princeton University, Princeton, New Jersey 08540

(Received 4 March 1976)

Because of its electrostatic properties, the rate of the parametric growth of the kinetic Alfvén wave [Alfvén wave with $\lambda \equiv k_\perp^2 c_s^2/\omega_{ci}^2 \sim O(1)$] is shown to be larger than that of the classic result by a factor of $\lambda\omega_{ci}/\omega_A$, where ω_{ci} and ω_A are the ion-cyclotron and the Alfvén frequencies, $k_\perp$ is the perpendicular wave number, and c_s is the ion sound speed. This fact can be utilized to heat a plasma with typical tokamak parameters.

In a previous report,[1] we have shown that a plasma with typical tokamak parameters can be heated by the kinetic Alfvén wave which can be excited in the plasma through resonant mode conversion. The Alfvén wave which is excited by the resonant mode conversion has a perpendicular wave length, $k_\perp^{-1}$, comparable to the ion gyroradius, ρ_i, and accompanies an electrostatic component. Hence it is called the kinetic Alfvén wave here. The dispersion relation of the kinetic Alfvén wave is given by[1]

$$\omega^2 = \begin{cases} k_\parallel^2 v_A^2 \left[1 + \left(\frac{3}{4} + \frac{T_e}{T_i}\right) k_\perp^2 \rho_i^2\right], & k_\perp \rho_i^2 \ll 1, \\ k_\parallel^2 v_A^2 \left(1 + \frac{T_e}{T_i}\right) k_\perp^2 \rho_i^2, & k_\perp^2 \rho_i^2 \gg 1. \end{cases} \quad (1)$$

It was shown in the linear analysis that both electrons and ions are heated in a collisional regime, while only electrons are heated in a collisionless regime by electron Landau damping.

In this report we show that, in a collisionless regime, ion heating is possible by utilizing either parametric decay to the ion acoustic wave if $T_e \gg T_i$, or ion nonlinear Landau damping if $T_e \lesssim 5T_i$.

Decay of a conventional Alfvén wave into another Alfvén wave and an ion acoustic wave has been studied by Sagdeev and Galeev[2] using the ideal magnetohydrodynamic (MHD) equations. Applications of this process for heating plasmas have been suggested by Vahala and Montgomery[3] and Rashmore-Davies and Ong.[4] However, because the growth rate is so small ($\sim \omega_0 B_p/B_0$; ω_0 is the pump frequency, B_p the pump amplitude, and B_0 the ambient magnetic field) the actual application is considered difficult. Furthermore, if $T_e \lesssim 5T_i$, the ion acoustic wave is heavily ion Landau damped and such a decay becomes practically absent.

We show here that the parametric decay of the kinetic Alfvén wave to the ion acoustic wave has a growth rate larger than the Sagdeev-Galeev result by at least a factor of ω_{ci}/ω_A, where ω_A is the Alfvên wave frequency. This fact makes its application to the heating more realistic. We further show that even if $T_e \lesssim 5T_i$, where the decay is absent, ion nonlinear Landau damping which has a large growth rate $[\gtrsim \omega_A(B_p/B_0)^2(\omega_{ci}/\omega_A)^2\beta^{-1}]$ can be utilized to heat ions.

Such enhanced growth rates appear through the coupling due to the $\vec{E}_p\times\vec{B}_0$ and $v_\parallel\vec{B}_p/B_0$ drifts caused by the pump fields E_p and B_p. These drifts can produce a charge separation which cannot be canceled because of the effects of the finite ion gyroradius and finite electron inertia which are absent in the classic ideal MHD treatment.

We assume that the plasma has $\beta\sim(m_e/m_i)^{1/2}$. In this case the compressional perturbation of the magnetic field B_z may be negligible. Hence we use the classic two-potential field,[5] $E_z=-\partial\psi/\partial z$ and $\vec{E}_\perp=-\nabla_\perp\varphi$, where z and $\perp$ denote components parallel and perpendicular to the ambient magnetic field. The field equation can then be written as

$$n_i^{(1)}+n_i^{(2)}=n_e^{(1)}+n_e^{(2)}, \tag{2}$$

$$\frac{\partial}{\partial z}\nabla_\perp^2(\varphi-\psi)=\mu_0\frac{\partial}{\partial t}[J_{ze}^{(1)}+J_{zi}^{(1)}+J_{ze}^{(2)}+J_{zi}^{(2)}], \tag{3}$$

where subscripts i and e are for ions and electrons, and n and J are number-density and current-density perturbations with the superscripts (1) and (2) showing the first- and the second-order perturbations in the wave amplitude.

For the kinetic equation in this frequency range, the drift kinetic equation[6] is appropriate,

$$\frac{\partial f}{\partial t}+\nabla\cdot(\vec{v}_d f)+\frac{q}{m}[E_z+(\vec{v}_d\times\vec{B}_\perp)\cdot\vec{e}_z]\frac{\partial f}{\partial v_z}=0, \tag{4}$$

where

$$\vec{v}_d=v_z\vec{e}_z+\frac{\vec{E}_\perp\times\vec{B}_0}{B_0^2}+\frac{m}{qB_0^2}\frac{d\vec{E}_\perp}{dt}+v_z\frac{\vec{B}_\perp}{B_0}, \tag{5}$$

$f(\vec{x},v_z,t)$ is the drift distribution, and the rest of the notations are standard.

The linear response for the electron and ion distribution functions, $f_e^{(1)}$ and $f_i^{(1)}$, are obtained from Eq. (4) in their Fourier amplitudes:

$$f_{e\vec{k}}^{(1)}=f_e^{(0)}\bar{\psi}_{\vec{k}}, \tag{6}$$

and

$$f_{i\vec{k}}^{(1)}=-\lambda f_i^{(0)}\bar{\varphi}_{\vec{k}}+c_s^2\frac{\partial f_i^{(0)}/\partial v_z}{v_z-\omega/k_z}\bar{\psi}_{\vec{k}}, \tag{7}$$

where $\bar{\psi}=e\psi/T_e$, $\bar{\varphi}=e\varphi/T_e$, $c_s^2=T_e/m_i$, and $\lambda=k_\perp^2c_s^2/\omega_{ci}^2$. Substitution of these distribution functions into the field equations (2) and (3) yields the linear dispersion relations which can be decoupled; for the kinetic Alfvên wave,

$$\epsilon_A(\omega,\vec{k})\equiv 1-\frac{k_z^2v_A^2}{\omega^2}(1+\lambda_A)=0, \tag{8}$$

$$\bar{\psi}_{\vec{k}}=-\lambda_A\bar{\varphi}_{\vec{k}}, \tag{9}$$

and for the ion acoustic wave,

$$\epsilon_s(\omega,\vec{k})\equiv 1+\lambda_s+\chi_i=0, \tag{10}$$

$$\bar{\psi}_{\vec{k}}=\bar{\varphi}_{\vec{k}}, \tag{11}$$

and

$$\chi_i=-c_s^2\int\frac{\partial f_i^{(0)}/\partial v_z}{v_z-\omega/k_z}dv_z. \tag{12}$$

Note here that ϵ's and χ_i are normalized so that the leading terms in the ϵ's become unity.

We now introduce the parametrically coupled equations between these two modes through the pump potential $\bar{\varphi}_0$, where $\bar{\varphi}_0$ is related to the transverse component of the wave magnetic field $\vec{B}_{\perp p}$ of the pump through

$$\frac{\vec{B}_{\perp p}}{B_0}=\frac{i\vec{k}_{\perp 0}\times\vec{k}_{\parallel 0}}{\omega_0\omega_{ci}}c_s^2(\bar{\varphi}_0-\bar{\psi}_0)=\frac{i\vec{k}_{\perp 0}\times\vec{k}_{\parallel 0}}{\omega_0\omega_{ci}}c_s^2(1+\lambda_0)\bar{\varphi}_0, \tag{13}$$

where, on the right-hand side, the subscript 0 refers to the pump quantities.

There are many nonlinear terms that contribute to the coupling and the derivation of the coupling coefficient is nontrivial. However, we leave the derivation to the extended paper to be published elsewhere.[7] Instead, we state the important terms which contribute to the coupling coefficient when λ is large (larger than ω/ω_{ci}). These are $\vec{v}_E$ $(=\vec{E}_\perp\times\vec{B}_0/B_0^2)$ times the second term in Eq. (7) [the product of $\vec{v}_E$ and the first term in Eq. (7) cancels with the $\vec{v}_E\cdot\nabla E_\perp$ term which originates from the ambipolar drift term] and $(\vec{v}_E\times\vec{B}_\perp)\cdot\partial f_i^{(0)}/\partial\vec{v}_z$ for the ions. For the electrons, in addition to the corresponding terms the term $\vec{v}_z\vec{B}_\perp/B_0 f_e^{(1)}$ turns out to be crucial.

The coupled equations then read

$$\begin{aligned}\epsilon_A(\omega,\vec{k})\bar{\varphi}_{\vec{k}}&=\Lambda_A\bar{\varphi}_0\bar{\varphi}_{\vec{k}_0-\vec{k}}^*,\\ \epsilon_s(\omega_0-\omega,\vec{k}_0-\vec{k})\bar{\varphi}_{\vec{k}_0-\vec{k}}&=\Lambda_s\bar{\varphi}_0\bar{\varphi}_{\vec{k}}^*,\end{aligned} \tag{14}$$

where

$$\Lambda_A = \Lambda \frac{1}{\omega_A \lambda_A (1+\lambda_A)}, \qquad (15)$$

$$\Lambda_s = \Lambda \frac{1}{k_{zs}}\left(\frac{k_{z0}}{\omega_0} - \frac{k_{zA}}{\omega_A}\right), \qquad (16)$$

and

$$\Lambda = i c_s^2 \frac{(\vec{k}_A \times \vec{k}_s)\cdot \vec{e}_z}{2\omega_{ci}}[(1+\lambda_0)(1+\lambda_A)+\chi_i]. \qquad (17)$$

In these expressions subscripts A and s refer to the kinetic Alfvén and the ion acoustic waves, respectively, and $\vec{e}_z$ is the unit vector in the direction of the ambient magnetic field $\vec{B}_0$.

Let us first look at the resonant decay process. This applies to a case with $T_e \gg T_i$ whereby the ion acoustic wave can exist. Then $\chi_i = -k_z^2 c_s^2/\omega^2 = -(1+\lambda_s)$. Unlike the MHD case in which only backscattering is allowed, three different types of decay are possible here as shown in the ω-k_z diagram in Fig. 1. The growth rate γ_D is obtained from the coupled equation (14),

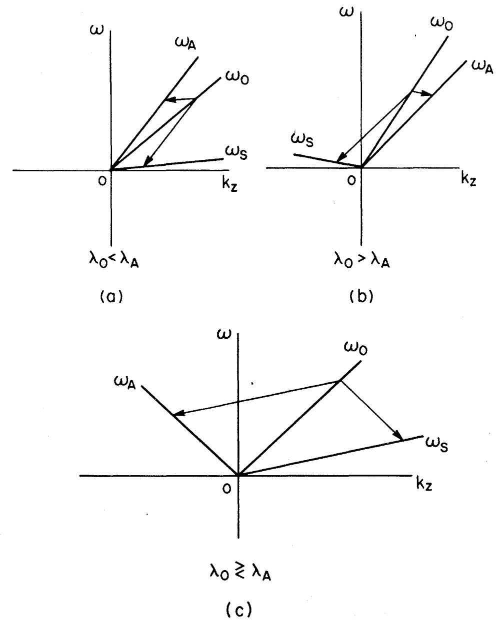

FIG. 1. Three types of decay are possible from ω_0 to ω_A depending on the signs of k_{zs} and k_{zA} as well as the size of λ_0 $(=k_{\perp 0}^2 c_s^2/\omega_{ci}^2)$ with respect to λ_A $(=k_{\perp A}^2 \times c_s^2/\omega_{ci}^2)$. In the MHD limit, where $\lambda \to 0$, only case (c) is acceptable. In case (a), $|k_\perp|$ is decreased, while in case (b), $|k_\perp|$ is increased as a consequence of the delay.

$$\gamma_D = \left[\frac{\omega_A \omega_s \Lambda_A \Lambda_s^* |\overline{\varphi}_0|^2}{4(1+\lambda_s)}\right]^{1/2}$$

$$= \frac{\omega_{ci}}{4}\left[\frac{\omega_s \lambda_s}{\omega_A \lambda_0 (1+\lambda_0)(1+\lambda_s)(1+\lambda_A)}\right]^{1/2} \frac{v_A}{c_s} |\sin\theta| \, |(1+\lambda_A)(1+\lambda_0) - (1+\lambda_s)| \left|\frac{B_{\perp p}}{B_0}\right|, \qquad (18)$$

where θ is the angle between $k_{\perp A}$ and $k_{\perp s}$. The growth rate obtained above is larger than that derived by Sagdeev and Galeev[2] by a factor of $\lambda\omega_{ci}/\omega_A$ as is expected.

Let us now consider the nonlinear Landau damping process by the ions. This corresponds to a case with $T_e \lesssim 5T_i$. Note that the ion acoustic wave considered here has a phase velocity $c_s(1+\lambda_s)^{-1/2}$ which is smaller than c_s and hence is more likely subject to ion Landau damping. To obtain the growth rate for nonlinear Landau damping one normally must include a higher-order response, $f^{(3)}$. However, because of the combined nonlinear contribution of electrons to the coupling, it turns out that this higher-order response is unimportant. The growth rate of the kinetic Alfvén wave through the nonlinear Landau damping γ_N is then obtained again from Eq. (14), using $\omega_0 - \omega_A \sim (k_{z0} - k_{zA})v_{Ti}$ (v_{Ti} is the ion thermal speed),

$$\gamma_N = \omega_A \Lambda_A \Lambda_s^* |\overline{\varphi}_0|^2 \frac{\mathrm{Im}\chi_i}{2|\epsilon_s|^2} = \frac{\omega_{ci}\lambda_s \sin^2\theta}{8\lambda_0(1+\lambda_0)(1+\lambda_A)} \frac{\omega_{ci}}{\omega_0} \frac{v_A^2}{c_s^2} \frac{T_e/T_i[(1+\lambda_0)^2(1+\lambda_A)^2 + T_e^2/T_i^2]}{(1+\lambda_s)^2 + T_e^2/T_i^2} \left|\frac{B_{\perp p}}{B_0}\right|^2. \qquad (19)$$

Note here that the use of the drift kinetic equation does not allow the finite-ion-gyroradius effect which would reduce the coupling coefficient by a factor of $e^{-b}I_0(b)$, $b = k_\perp^2 \rho_i^2$. Hence for a large value of λ in the above expression, γ_N is maximized roughly at $\lambda \sim T_e/T_i$.

For these growth processes to occur the pump field B_p/B_0 should exceed a threshold. For the case of nonlinear Landau damping, the threshold value depends on the damping rate of the kinetic Alfvén wave δ_A (due to linear electron Landau damping) which is on the order of 1%. Then for a reasonable choice of other parameters, the threshold amplitude of the magnetic field can be reduced to $\lesssim 10^{-2}B_0$, and the corresponding growth rate approaches the wave frequency. As was shown,[1] if the kinetic Alfvén wave is excited by resonant mode conversion, its amplitude is enhanced by a factor $(\kappa\rho)^{-2/3}$ compared with the externally applied field; hence even with a field of several tens of gauss, the kinetic Alfvên wave will have an amplitude of several hundred gauss which can exceed the threshold of the nonlinear ion Landau damping.

The ion heating occurs because of the cascading of the nonlinear Landau damping. As in other cases of parametric heating,[8] the heating rate of ions is roughly given by $\gamma(B_p^2/2\mu_0)\omega_s/\omega_A$ sec^{-1} per unit volume.

In summary, we have shown that when $k_\perp c_s/\omega_{ci}\sim O(1)$, the parametric excitation of the kinetic Alfvên wave has a growth rate larger by a factor of ω_{ci}/ω_A than the MHD result. The larger growth rate is due to the noncanceling charge separation created by the $\vec{E}\times\vec{B}$ drift of the pump field. Such enhanced growth rate and hence reduced threshold level can be utilized to heat ions in tokamak-type plasmas even with $T_e\sim T_i$. If the pump field is applied through resonant mode conversion the enhanced amplitude of the converted kinetic wave eases such a process.

*Research supported by U. S. Energy Research and Development Administration Contract No. E(11-1) 3073.

[1]A. Hasegawa and L. Chen, Phys. Rev. Lett. 35, 370 (1975).

[2]R. Z. Sagdeev and A. A. Galeev, in *Nonlinear Plasma Theory*, edited by T. O'Neil and D. Book (Benjamin, New York, 1969), p. 7.

[3]G. Vahala and D. Montgomery, Phys. Fluids 14, 1137 (1971).

[4]C. N. Rashmore-Davies and R. S. B. Ong, Phys. Rev. Lett. 32, 1172 (1974).

[5]B. B. Kadomtsev, *Plasma Turbulence*, translated by L. C. Ronson (Academic, New York, 1965), p. 82.

[6]A. Hasegawa, *Plasma Instabilities and Nonlinear Effects* (Springer, Berlin, 1975), p. 84.

[7]A. Hasegawa and L. Chen, to be published.

[8]E. Valeo, C. Oberman, and F. W. Perkins, Phys. Rev. Lett. 28, 340 (1972); D. F. DuBois and M. V. Goldman, Phys. Rev. Lett. 28, 218 (1972); Y. Y. Kuo and J. A. Fejer, Phys. Rev. Lett. 29, 1667 (1972); A. Hasegawa and L. Chen, Phys. Fluids 18, 1321 (1975); A. Rogister, Phys. Rev. Lett. 34, 80 (1975).

Time Evolution of Velocity-Space Instabilities on Counterstreaming, Magnetically Confined Electron Beams*

Ady Hershcovitch and Peter A. Politzer
Department of Nuclear Engineering and Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
(Received 3 December 1975)

A theoretical and experimental investigation of the time evolution of instabilities in counterpenetrating, low-density, nonrelativistic electron beams is reported. The dominant nonlinear effect is the dispersion in energy of the beam electrons by wave-particle interactions. A large change in the frequency of the fastest growing mode is predicted at moderate values of beam temperature (corresponding to $k_\perp\lambda_D\sim 1$). An experiment designed to detect the new mode is performed. The observations confirm the theoretical model.

We report here the results of a theoretical and experimental investigation of the evolution in time of an instability occurring in a counterstreaming electron beam system confined by a uniform magnetic field. The instability of this system, with monoenergetic beams, has been studied by several workers during the past few years.[1-6] Instabilities which have been predicted, and observed, are the two-stream instability[1] and the one-half-cyclotron-frequency instability.[2-6] The half-cyclotron-frequency instability occurs as the result of interaction between a space-charge wave on one beam and the backward cyclotron wave on the oppositely directed beam.[2-6] This interaction takes place only when the beams have finite cross section, or for short perpendicular wavelengths. The wave frequency is close to one-half of the cyclotron frequency when the two beams have equal velocities.

In some prior experimental observations,[4,5] the mode at one-half of the electron cyclotron frequency was observed to grow in time, saturate in amplitude for a brief period, and then disappear. What in fact occurs is that a nonlinear interaction takes place between the growing wave and the beam electrons. This has the net effect of slightly increasing the temperature of the beams. The presence of finite beam temperature in turn leads