within present experimental error to the correlation length ξ. Further investigation of the system is being carried out for different types and sizes of Brownian particles.

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Experiments on Orbital Dynamics in Superfluid ^{3}He-A*

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The response of the orbital degrees of freedom of superfluid ^{3}He-A to a magnetic field suddenly rotated through a small angle is a decaying exponential rather than an oscillatory response. Analysis of the field and temperature dependence of the time constant supports the Cross-Anderson orbital-viscosity theory as complemented by the Pethick-Smith relaxation-time theory but reveals a discrepancy in the susceptibility-anisotropy energy near T_c.

We report here the results of experiments on superfluid ^{3}He-A in which we study the dynamical response of the orbital degrees of freedom of the superfluid to sudden changes in an applied magnetic field (for general theoretical and experimental background see Leggett[1] and Wheatley[2]). There has been a great deal of concurrent theoretical interest in this subject.[3] The present work, all done at temperatures relatively near to T_c, is the first quantitative study of orbital dynamic effects and is based on the qualitative method and observations of Paulson, Kleinberg, and Wheatley.[4] We find that the response of the orbital system to a sudden field change is a damped exponential rather than an oscillatory response and that in the high-field limit the response is described well by the orbital-"viscosity" theory of Cross and Anderson[5] with the interpretation of relaxation times as given by Pethick and Smith.[6] The dynamical response as a function of magnetic field may be interpreted as measuring the "susceptibility-anisotropy" torque in terms of the experimentally known dipolar torque, and in this sense the first measurements of susceptibility anisotropy in ^{3}He-A are given here. As $T \to T_c$ an unexpected discrepancy between experiment and the simple theory develops.

In our experiments we measure the detected level of 15-MHz, continuous-wave, ultrasound which has passed through 3.81 mm of cerium-magnesium-nitrate- (CMN) cooled liquid ^{3}He in a region of our experimental cell which is shielded magnetically, using a Nb shield, and in which a field of less than 0.01 G has been trapped. Using solenoid and saddle-shaped geometry coils in this shield we can apply known magnetic fields both parallel and perpendicular to the direction

$\hat{q}$ of sound propagation. The perpendicular direction is also that of the axis between the CMN coolant and our shielded 10-mg-CMN thermometer and may be the general direction of superflow due to residual heat flow. Every effort was made in design to reduce the thermal resistance between the sound cell and the thermometer. Because of the known anisotropy of the attenuation of ultrasound[7] in ^{3}He-A and its interpretation,[8] our sound transmission measurements sense the orbital orientation of the superfluid.

We have performed three types of experiments. Most of our data were obtained using the first type in which a magnetic field $\vec{H}$ is suddenly (effective field transient time is ca. 30 μsec) *rotated* through a fixed angle (typically $15° \leftrightarrow 35°$) with respect to $\hat{q}$ and the time dependence of transmitted sound studied. We displayed both $15° \to 35°$ and $35° \to 15°$ changes on a storage oscilloscope and measured the time of their intersection, which we called $t_{1/2}$ as the time for half the change to take place. Such data can be accumulated rapidly. In a second type of experiment we put a steady field $\vec{H}$ at some angle such as 25° with respect to $\hat{q}$, superposed a small ac field, with angular frequency ω, perpendicular to $\hat{q}$, and then measured the phase ϕ of the resultant oscillating sound signal with respect to that of the ac magnetic field. Measurements of this type are more lengthy, and so they were used only as a check on the $t_{1/2}$ measurements. Plots of $\tan\phi$ versus ω were linear with a slope in good agreement with the corresponding $t_{1/2}/\ln 2$, thereby showing that the time response function to a step change has the form $\exp[-t\ln 2/t_{1/2}]$. In a third type of experiment, performed with $1 - T/T_c$ in the range 0.002 to 0.004, we applied a steady field of 5 G at 35° with respect to $\hat{q}$ with an incremental field perpendicular to $\hat{q}$ that made the initial field angle 55°. We then suddenly (time constant 8 μsec) turned off the incremental field and then looked, using signal averaging, for an oscillatory response in the transmitted sound corresponding to some response of $\hat{l}$ to the perpendicular ringing of the $\hat{d}$ vector.[2] These experiments gave a negative result.

What we imagine[9] occurs, following a change of $\vec{H}$ and the subsequent decay of the $\hat{d}$ ringing with $\hat{l}$ apparently in place, is that the susceptibility anisotropy of the ordered spin system ($\hat{d}$ vector) leads to a torque on the spins jointly proportional to H^2 (this is why we keep $|H|$ constant in the $t_{1/2}$ measurements) and to the small angle φ between $\hat{H}$ and $\hat{d}$. This opens out a small angle θ between $\hat{d}$ and the superfluid orbital vector $\hat{l}$, leading to a coherent dipolar torque on both the orbital state and the ordered spin system proportional to this angle. The orbital system then moves toward its new equilibrium state under the action of the dipolar torque balanced by an essentially equal but opposite viscous torque equal to the product of the Cross-Anderson viscous coefficient μ and the rate of angular displacement of the orbital state: More quantitatively, the dipolar and susceptibility-anisotropy torques result from free energies $\Delta F_D = -\frac{1}{2}\lambda_D(\hat{l}\cdot\hat{d})^2$ and $\Delta F_H = \frac{1}{2}\lambda_H(\hat{d}\cdot\vec{H})^2$, where $\lambda_D = \chi_N \Omega_{\parallel}{}^2/\gamma^2$ is known from empirical A-phase parallel-ringing angular frequencies[2,10] $\Omega_{\parallel}$ and $\lambda_H = \chi_N[1 - Y(T)]/[1 + \frac{1}{4}Z_0 Y(T)]$.[1] Here χ_N is the normal liquid susceptibility,[2] γ is the gyromagnetic ratio, Z_0 is a Landau parameter,[2] and $Y(T)$ is the Yosida function. Analyzing the problem as outlined above, i.e., assuming that $\lambda_D\theta = \lambda_H H^2\varphi$ and $-\mu d(\theta+\varphi)/dt = \lambda_D\theta$, we find that an exponential response is expected with a time $t_{1/2}$ given by

$$t_{1/2} = [\mu\ln 2/\lambda_D](1 + \lambda_D/\lambda_H H^2). \qquad (1)$$

For constant temperature and pressure our measurements are accurately characterized by an equation of the form

$$t_{1/2} = t_0(1 + H_0{}^2/H^2), \qquad (2)$$

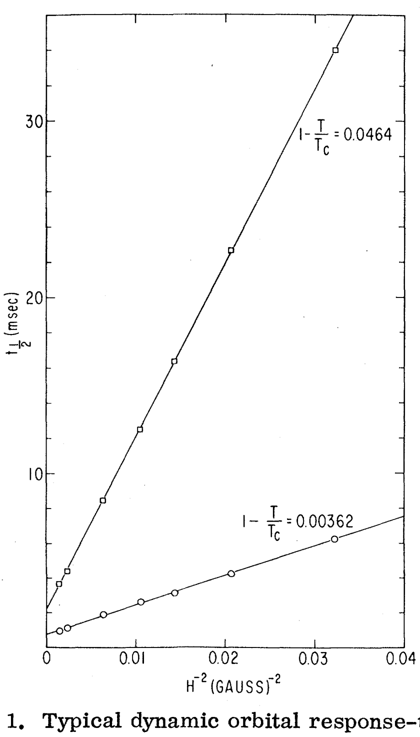

FIG. 1. Typical dynamic orbital response-time data at 24.02 bar for $1 - T/T_c = 0.0464$ (□) and 0.00362 (○) from which the quantities t_0 and $H_0{}^2$ were derived [Eq. (2)]. H is a magnetic field which is suddenly rotated.

which has the form of Eq. (1). An example of experimental data for the largest and smallest values of $1-T/T_c$ for which we were able to make measurements at 24.0 bar is shown in Fig. 1. At larger values of H^{-2} the quantity $t_{1/2}$ falls below the line shown, the effect being larger the closer the ^{3}He is to T_c. We believe that this effect can be explained by a heat-flow torque. Experimental results for the quantities t_0^2 and H_0^2 for pressures of 33.45, 29.34, and 24.02 bar are shown in Fig. 2. Over a significant range the quantity t_0^2 is adequately described by a linear dependence on $1-T/T_c$, although very close to T_c there appear to be significant deviations toward larger values of t_0^2. The linear dependence was anticipated by the Cross-Anderson theory[5] which predicts that

$$\mu = (\pi^2/64)\tau N(0)(kT_c)^2(\Delta_0/kT_c)^3, \tag{3}$$

where $N(0)$ is the density of states at the Fermi surface for spins of one sign, and Δ_0 is the amplitude of the anisotropic energy gap. The quantity τ has been predicted recently by Pethick and Smith[6] to be conceptually the same normal-state quasiparticle relaxation time as can be measured in the relaxation of the wall-pinned magnetization mode in the B phase.[11] For the Anderson-Brinkman-Morel (ABM) model for ^{3}He-A we have near T_c that[1] $\Delta_0/kT_c = \pi(\Delta C/C_N)^{1/2}(1-T/T_c)^{1/2}$, where $\Delta C/C_N$ is the empirical normalized specific heat jump at T_c. Setting $t_0 = \mu\ln 2/\lambda_D$ and noting that λ is proportional to $1-T/T_c$, near T_c, we find that $t_0^2 \propto 1-T/T_c$ as observed. Using empirical values[2] for $N(0)$ and $\Delta C/C_N$ and for λ_D,[10] we can evaluate τT_c^2 from the measured t_0 and find with an accuracy of about 10% that $\tau T_c^2 = 0.26$, 0.26, and 0.30 μsec mK^2 for pressures of 33.45, 29.34, and 24.02 bar. The value of τT_c^2 observed by Webb, Sager, and Wheatley[11] at 20.7 bar from wall-pinned mode relaxation in the B phase was 0.36 μsec mK^2, which can be said to support the interpretation of the present measurements considering the lower pressure and the uncertain accuracies of the various measurements.

To our surprise the quantity H_0^2 in Eq. (2) has a rather strong temperature dependence near T_c. If, as suggested by Eq. (1), we identify H_0^2/H^2 with the ratio $\lambda_D/\lambda_H H^2$ of the coefficients of the coherent dipolar to the susceptibility-anisotropy free energy then we can obtain empirical values of λ_H from the empirical values of λ_D and H_0^2 in $\lambda_H = \lambda_D/H_0^2$. Values of $\lambda_H/(1-T/T_c)$ so obtained are displayed in Fig. 3. Near T_c we can approxi-

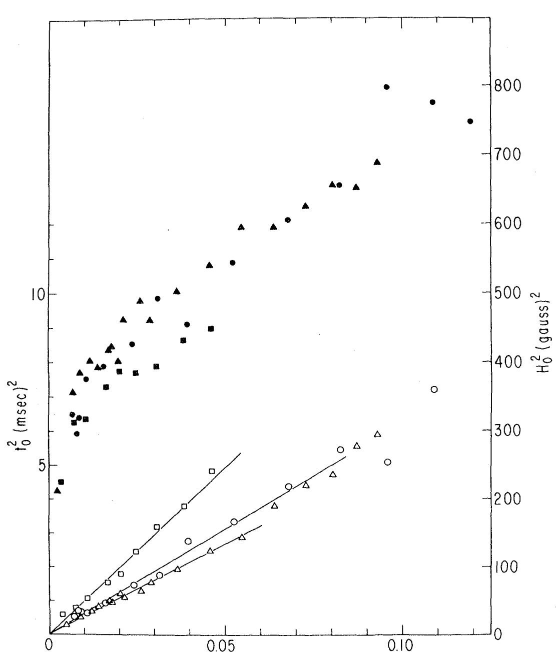

FIG. 2. Dependence on $1-T/T_c$ of t_0^2 (open symbols) and H_0^2 (closed symbols) for 33.45 bar (△,▲), 29.34 bar (○,●), and 24.02 bar (□,■).

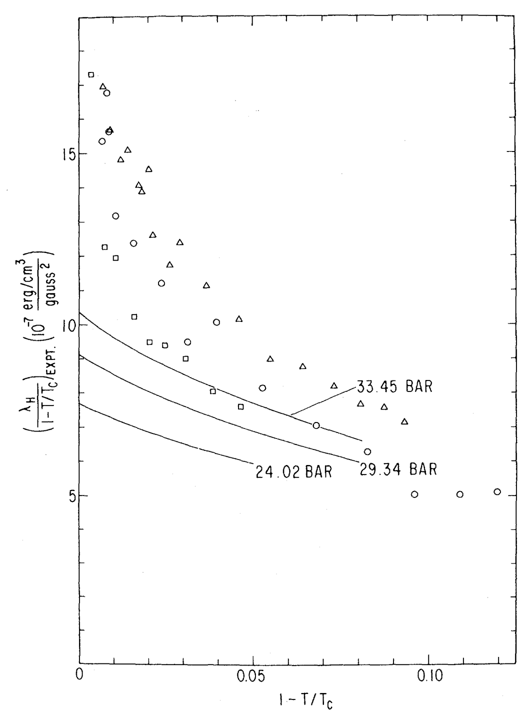

FIG. 3. Ratio of ^{3}He-A susceptibility-anisotropy free-energy parameter λ_H (see text) to $1-T/T_c$ as estimated theoretically near T_c (solid lines) and as derived from measurements using assumptions given in the text, for 33.45 bar (△), 29.34 bar (○), and 24.02 bar (□).

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$$\frac{\lambda_H}{1-T/T_c} \simeq \frac{2\chi_N \Delta C/1.43C_N}{1+\frac{1}{4}Z_0[1-(2\Delta C/1.43C_N)(1-T/T_c)]}$$

from theory. Farther from T_c we do not know accurately how to deal with the strong-coupling effects. The corresponding calculated values of $\lambda_H/(1-T/T_c)$ near T_c are shown as solid lines in Fig. 3, and suggest that although theory and experiment may be in reasonable agreement at lower temperatures, near T_c the apparent susceptibility-anisotropy torque is substantially larger than expected. This effect can be seen already in Fig. 1, where the ratio of the slopes of the two lines is significantly different from the ratios of the corresponding values of $(1-T/T_c)^{1/2}$. Near T_c, where the parallel-ringing period is increasing while the large-field orbital response time is decreasing, one might suspect that the transient ringing period of $\hat{d}$ might act to increase t_0, and hence to decrease H_0^2 and increase $\lambda_H/(1-T/T_c)$. However, calculation suggests that this effect is not large enough to account for the results. Furthermore, the effect can already be discerned in the low-field results shown in Fig. 1, where $t_{1/2}$ is already quite large but the ratio of $t_{1/2}$ at given H^{-2} for the two values of $1-T/T_c$ is much larger than the ratio of the corresponding $(1-T/T_c)^{1/2}$. It is, no doubt, this effect which led to our failure to observe a $(1-T/T_c)^{1/2}$ dependence for $t_{1/2}$ in our early work[4] where we used primarily only one set of initial and final fields.

The source of susceptibility anisotropy is the presumed zero susceptibility along $\hat{d}$ of superfluid pairs. The decrease of susceptibility in the B phase has a similar source. There still exists an unresolved discrepancy betweeen static and dynamic magnetism in the B phase[2] in which the static magnetism near T_c appears to decrease more rapidly than expected, in the same sense as the present discrepancy.

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