Calculation of the Pinch Dynamics in Relativistic Electron Beam Diodes

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The pinch dynamics of relativistic electron beams in large aspect ratio diodes are calculated numerically, including both ion and electron currents. The effect of the ions on the pinch phenomenon is demonstrated. The roles of the time scales for ion emission and flow are discussed. Fair agreement with experiment is obtained.

The electron-beam approach to inertially confined fusion involves obtaining tightly pinched beams.[1-4] It is evident[5,6] that the force of the self-magnetic field is responsible for the collapse of the beam. However, Poukey, Freeman, and Yonas,[7] who first performed numerical diode simulations, had to invoke the anode-plasma model to explain the pinch phenomenon. Blaugrund and Cooperstein[8] showed that in a hollow-cathode diode a hollow beam collapses at a velocity of a few millimeters per nanosecond, excluding the possibility of thermal anode-plasma generation. On the other hand, it has been demonstrated[9] that the anodes in such diodes are good ion sources. It has therefore been suggested[8,10-15] that these ions should have a decisive effect on the diode dynamics. Such ion currents in diode gaps were demonstrated and analyzed.[13-16] Blaugrund, Cooperstein, and Goldstein[8,13-15] studied the behavior of hollow cathode diodes. They concluded that the probable ion sources are adsorbed gases released from the anode surface as it is heated by the beam. The released gases are ionized by both primary and secondary electrons. The dependence of the pinch dynamics on anode material is determined by the specific heat and albedo. The model for the diode dynamics is as follows[17]:

In the early phase of the pulse the electrons flow nearly "laminarly" in a cylindrical shell. Ion emission from the anode starts when a threshold of absorbed energy density is reached, enough to start the release of adsorbed gases. The main effect of the ions is to neutralize the electron space charge, allowing the magnetic force to dominate. A second effect is that the ion space charge near the cathode enhances electron emission and increases the current.[10,11] As a result of these effects the electron trajectories bend and cross, and part of the current is pulled inside the cylindrical current shell. There, the electrons are driven towards the anode by the diode field, heating a fresh inner ring. After some dwell time, ions are emitted from this new area and beam collapse proceeds. The dwell time introduces the observed[8] dependence of the collapse velicity on anode material: For higher-Z targets a lower threshold of absorbed energy density is required in order to reach the temperature at which adsorbed gas release is significant and a higher collapse velocity is expected. The pinch velocity is determined by the combined effects of the dwell time and the time scale of the motion of the ions in the gap.

We test this model by a two-dimensional simulation, using the finite-size particle technique.[18,19] The time-dependent electromagnetic equations are not actually solved. Instead, we make use of the fact that the flow is composed of slow ions and fast electrons, and the time scale of the evolution of the system is determined by the ions. We therefore assume that at each moment the charge distribution of the electrons corresponds to an equilibrium stationary solution for the given ion distribution at that moment. At each time step the electric fields are calculated from the charge distribution by solving Poisson's equation. The magnetic fields are calculated from the known particle velocities. The ion simulation particles are moved at time steps of about 0.1 nsec, and these represent the physical time steps. The electron simulation particles are then moved by a series of much shorter, nonphysical, time steps in which the relativistic equations of motion are solved self-consistently, until a stationary state is obtained. A new cycle then begins by moving the ions again.

The equation of motion for the electron simulation particles is

$$d(\gamma m_0\vec{V})/dt=(e/c)[\vec{E}+(\vec{V}\times\vec{B})/c], \qquad (1)$$

where m_0 is the electron mass, $\vec{V}$ the velocity, c the velocity of light, $\vec{E}$ the electric field, $\vec{B}$ the magnetic field, e the electron charge, and $\gamma=(1-V^2/c^2)^{-1/2}$. Equation (1) is solved for the transformed variables $R=\gamma\beta_r$, $Z=\gamma\beta_z$, $\beta_{r,z}=V_{r,z}/c$, taking constant values for $\vec{E}$ and $\Gamma = \gamma/B$:

$$R = (z - z\cos bt + r\sin bt)\Gamma + \rho\cos bt + \zeta\sin bt, \qquad (2)$$

$$Z = (-r + r\cos bt + z\sin bt)\Gamma - \rho\sin bt + \zeta\cos bt, \qquad (3)$$

where $b = Be/(m_0c)$, $r, z = eE_{r,z}/(m_0c)$, and ρ and ζ are values of R and Z at the beginning of the time step. If necessary, the time step is subdivided according to two criteria: The potential difference seen by a particle in a single step Δt should not exceed a given fraction of its total energy, and Δt should not exceed a given fraction of the cyclotron period. Each step is repeated using properly averaged values of the fields. For the emission law of both electrons and ions we assume space-charge-limited flow. We follow Poukey[10,11] in using Langmuir's law for bipolar flow. This amounts to approximating the flow by a steady state between the electrode and the adjacent grid line.

We simulated the experiment of Blaugrund and Cooperstein.[8] The diode is shown in Fig. 1. We used simplified boundary conditions: The recess in the cathode is simulated by suppressing emission up to the recess radius. The taper is simulated by choosing an average gap of 4.7 mm. Along the line from radius 4 to 5 cm a logarithmic drop in potential is prescribed, as for two coaxial infinite cylinders. Shank emission is neglected. Each simulation particle represents 4×10^{10} physical particles. A step pulse of 750 kV is applied. We obtain an estimated limit for the threshold energy for ion emission by assuming that in the experiment the threshold on the anode area irradiated by the beam before pinching starts is reached simultaneously with the critical current. This yields a threshold of 35 J/cm^2 for aluminum anodes. We assume that the ions are singly ionized carbon.

The results confirm the main features of the model. As soon as ion flow becomes important the initially thick ring trace of the beam on the anode narrows, and beam collapse proceeds. In Fig. 2 distributions of the electrons are shown at various times. Beam collapse takes 8 nsec, in fair agreement with experiment. Also shown in Fig. 2 are typical electron trajectories. In Fig. 3 the current density distribution on the anode is shown in the steady-state, fully pinched flow. 42% of the beam current reaches the center 11 mm diam, as compared with the experimental value of 75%. The calculated total current in the final stage is 200 kA, in agreement with experiment. The ion current at that stage is 10 kA.

In order to study the relative contributions of

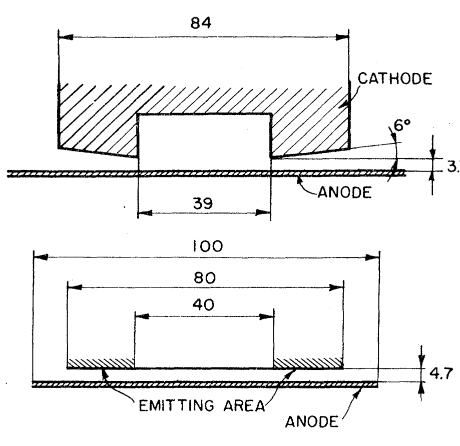

FIG. 1. Actual (top) and simulation (bottom) diode configuration.

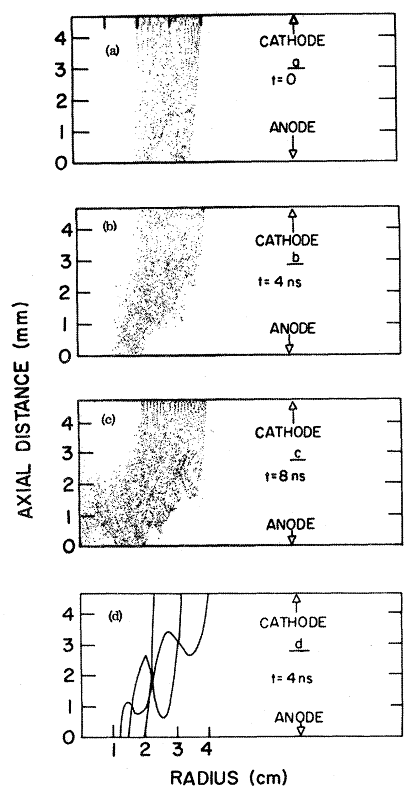

FIG. 2. Pinch dynamics: (a)–(c) Electron charge distribution at times 0 (start of ion emission), 4 nsec, and 8 nsec, respectively. Dots represent simulation particles. (d) Typical electron trajectories at time 4 nsec.

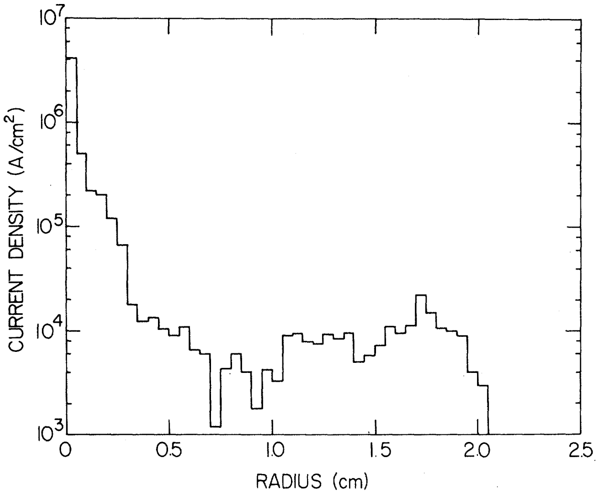

FIG. 3. Current density distribution on the anode in the fully pinched state.

the dwell time and the ion transit time we calculated the pinch dynamics in the same diode taking the threshold energy for ion emission to be zero. This should be a good approximation to the situation with high-atomic-number anodes such as gold where the threshold energy should be much smaller than that of aluminum. The calculated collapse time is about a factor of 2 shorter than that for aluminum. This ratio is in agreement with the experimental results obtained by Blaugrund, Cooperstein, and Goldstein[13] using somewhat different diode configurations.

It is concluded from our calculations that this model of ion and electron flow in the gap can explain the main phenomena of pinch dynamics in large-aspect-ratio diodes.

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