the magnitude of the sublattice magnetization is mainly determined by temperature and not by field.

We consider the ordering treated in this paper to be characteristic for systems which consist of weakly coupled low-dimensional systems, resulting in a large spin reduction at low temperatures. Experiments are now in progress to study the long-range order and the field dependence of the sublattice magnetization for other ALC systems with a variety of $|J'/J|$ values.

Note added.—Recently Ishikawa and Oguchi[14] have calculated the zero-point spin-reduction for Heisenberg antiferromagnets, in which low-dimensional interactions dominate. Extrapolation to the ideal 1D system with $S=\frac{1}{2}$ gives a spin reduction $\Delta S=\frac{1}{2}$, which is the physically correct answer for the 1D Heisenberg antiferromagnet. Interchain interactions $J'/J \approx 10^{-3}$ result in a spin reduction of 60% in zero magnetic field.

*Now at Philips Research Laboratories, Eindhoven, The Netherlands.

[1]L. J. de Jongh and A. R. Miedema, Adv. Phys. 23, 1 (1974).

[2]J. C. Bonner, H. Kobayashi, I. Isujikawa, Y. Nakamura, and S. A. Friedberg, J. Chem. Phys. 63, 19 (1975).

[3]Y. Imry, P. Pincus, and D. Scalapino, Phys. Rev. B 12, 1978 (1975).

[4]M. J. Hennessy, C. D. McElwee, and P. M. Richards, Phys. Rev. B 7, 930 (1973).

[5]T. Nagamiya, K. Yosida, and R. Kubo, Adv. Phys. 4, 1 (1955).

[6]L. Hulthén, Ark. Mat. Astron. Fys. A26, 1 (1938).

[7]F. Keffer, in *Handbuch der Physik*, edited by H. P. J. Wijn (Springer, New York, 1966), Vol. 18, Part 2.

[8]M. L. Néel, Ann. Phys. (Paris) 5, 232 (1936).

[9]L. S. J. M. Henkens, K. M. Diederix, T. O. Klaassen, and N. J. Poulis, Physica (Utrecht) 81B+C, 259 (1976).

[10]M. W. van Tol and N. J. Poulis, Physica (Utrecht) 69, 341 (1973).

[11]R. F. Wielinga, thesis, University of Leiden, 1968 (unpublished).

[12]J. F. Cooke and H. A. Gersch, Phys. Rev. 153, A641 (1967).

[13]L. J. Azevedo, E. O. Mclean, W. G. Clark, and P. F. Seligmann, in *Proceedings of the Fourteenth International Conference on Low Temperature Physics, Otaniemi, Finland, 1975*, edited by M. Krusius and M. Vuorio (North-Holland, Amsterdam, 1975).

[14]T. Ishikawa and T. Oguchi, Prog. Theor. Phys. 54, 1282 (1975).

COMMENTS

Relativistic Particle Dynamics for an N-Body Interacting System

M. K. Liou
Department of Physics and Institute for Nuclear Theory, Brooklyn College of the City University of New York, Brooklyn, New York 11210

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A general method is proposed to construct a relativistic particle dynamics for an N-body interacting system and new separable three-body interactions of order c^{-2} are presented. Implications of the existence of nonunique solutions for practical application of the theory are discussed.

Recently, Foldy and Krajcik have constructed a representation of the inhomogeneous Lorentz group (IHLG) corresponding to an N-body interacting system, valid to order c^{-2}, which satisfies the condition of separability.[1,2] They used the work of Zhivopistsev, Perolomov, and Shirokov (ZPS)[3] as a basis to obtain an interesting result which shows that the Hamiltonian of the system should contain three-body terms as well as the two-body terms of ZPS. The purposes of this Comment are threefold: (i) To propose a general method which, at least in principle, if solutions exist, allows construction of a representation of the IHLG valid to all orders in c^{-1} for an N-body system with separable interactions.[4,5] (ii) To present new three-body interactions of order c^{-2} which satisfy the separability condition. (I have found that the three-body term obtained by Foldy and Krajcik is not the only possibility. Their form is just one of the infinite number of three-body interaction terms that I have obtained.) (iii) To discuss the effects arising from the existence of nonunique representations of the IHLG on the practical applications. Specifically, I comment on calculations done previously using this approach to estimate the relativistic corrections to nuclear binding energies.

It is known that a two-body relativistic theory can be constructed by using a prescription proposed by Bakamjian and Thomas,[6] but an N-body relativistic theory constructed by the same prescription is not physically acceptable because the interactions introduced by this prescription do not satisfy Foldy's separability condition. Here, I shall call the two-body interactions obtained by the Bakamjian-Thomas (BT) prescription, the two-body BT interaction. In a recent article,[7] I have used the two-body BT interaction to derive the relativistic corrections to the potential to all orders in c^{-1} and have found that to order c^{-2} the correction terms reduce to Shirokov's result.[8] These results suggest that the sum over all pairs of the two-body BT interaction will reproduce, to order c^{-2} and lowest order in the potential, the results of ZPS. Since the sum over all pairs of the two-body BT interaction also provides all higher-order terms, it can be used as input to construct a most general relativistic theory valid to all orders in c^{-1} for an N-body system with separable interactions. This does not imply, however, that the expressions for the generators of the IHLG for the N-body interacting system can be obtained by simply adding the sum over all pairs of the two-body BT interactions to the expressions for the free generators. The reason is very simple: The expressions for the generators so obtained do not satisfy the Lie algebra of the IHLG. The basis of the method is, therefore, to introduce an unknown operator to the Hamiltonian and another unknown operator to the boost operator. By requiring the new expressions for the generators to satisfy the Lie algebra of the IHLG, one is able to determine these unknown operators, order by order, in a systematic fashion from the commutation relations.

Two-body BT interaction.—Consider a system which consists of two free particles, particle i and particle j, without interaction. Let m_i, $\vec{r}_i$, $\vec{p}_i$, and $\vec{s}_i$ be the mass, position operator, momentum operator, and spin operator for particle i, respectively. Then the nonrelativistic c.m. dynamical variables $(\vec{R}_{ij}^{NR}, \vec{P}_{ij}^{NR}, \vec{q}_{ij}^{NR}, \vec{r}_{ij}^{NR}, \vec{s}_i^{NR}, \vec{s}_j^{NR})$[9] for this two-particle system can be defined in terms of the individual particle dynamical variables $(\vec{p}_i, \vec{p}_j, \vec{r}_i, \vec{r}_j, \vec{s}_i, \vec{s}_j)$, and the relativistic c.m. dynamical variables $(\vec{P}_{ij}, \vec{R}_{ij}, \vec{q}_{ij}, \vec{r}_{ij}, \vec{s}_i, \vec{s}_j)$ can be obtained from the corresponding nonrelativistic c.m. variables through a unitary transformation.[7] If two symbols, $\vec{X}_{ij}^{NR}$ and $\vec{X}_{ij}$, are used to represent an arbitrary nonrelativistic and relativistic c.m. variable, we have $\vec{X}_{ij} = \exp(iu_{ij})\vec{X}_{ij}^{NR}\exp(-iu_{ij})$. The Hermitian operator u_{ij} was obtained in Ref. 7. The generators of the IHLG can be expressed in terms of the operator u_{ij} and the nonrelativistic c.m. dynamical variables. The Hamiltonian and the boost operator can be written as

$$H_{ij} = \exp(iu_{ij})E_{ij}^0\exp(-iu_{ij}),$$
$$\vec{K}_{ij} = \exp(iu_{ij})[-\tfrac{1}{2}(\vec{R}_{ij}^{NR}E_{ij}^0 + E_{ij}^0\vec{R}_{ij}^{NR}) - (\vec{P}_{ij}^{NR}\times\vec{S}_{ij}^{NR})/(M_{ij}^0 + E_{ij}^0)]\exp(-iu_{ij}), \tag{1}$$

where $M_{ij}^0 = [m_i^2 + (\vec{q}_{ij}^{NR})^2]^{1/2} + [m_j^2 + (\vec{q}_{ij}^{NR})^2]^{1/2}$, $E_{ij}^0 = [(M_{ij}^0)^2 + (\vec{P}_{ij}^{NR})^2]^{1/2}$, and $\vec{S}_{ij}^{NR} = \vec{r}_{ij}^{NR}\times\vec{q}_{ij}^{NR} + \vec{s}_i^{NR} + \vec{s}_j^{NR}$. Now, we apply the BT prescription to introduce the internal interactions into this free two-body system. This can be done by modifying the mass operator of the free system M_{ij}^0 to include a potential V_{ij}^{NR}. That is, we write the mass operator for the interacting system as $M_{ij}^{NR} = M_{ij}^0 + V_{ij}^{NR}$. The potential introduced here is a rotationally invariant function of the nonrelativistic internal c.m. dynamical variables. This potential is also symmetric in its subscripts and is zero if subscripts are equal. Furthermore, it must vanish sufficiently rapidly at large $|\vec{r}_{ij}^{NR}|$ in order to satisfy the separability condition. In terms of M_{ij}^{NR} and $E_{ij}^{NR} = [(M_{ij}^{NR})^2 + (\vec{P}_{ij}^{NR})^2]^{1/2}$, the generators for the interacting system, H_{ij}' and $\vec{K}_{ij}'$, can be obtained from Eq. (1) by replacing M_{ij}^0 and E_{ij}^0 by M_{ij}^{NR} and E_{ij}^{NR}, respectively. (I assume that $\vec{P}_{ij}' = \vec{P}_{ij}$ and $\vec{J}_{ij}' = \vec{J}_{ij}$.) The two-body BT interactions are then defined as

$$V_{ij} = H_{ij}' - H_{ij}, \quad \vec{W}_{ij} = \vec{K}_{ij}' - \vec{K}_{ij}. \tag{2}$$

Further, V_{ij} and $\vec{W}_{ij}$ can be expanded in powers of c^{-1}. We have $V_{ij} = V_{ij}^{(0)} + V_{ij}^{(2)} + \ldots$, and $\vec{W}_{ij} = \vec{W}_{ij}^{(2)} + \vec{W}_{ij}^{(4)} + \ldots$. It is not difficult to show that Shirokov's result can be obtained from $V_{ij}^{(2)}$.[7,8]

N-body case.—The heart of the present construction is to write the expressions for the generators of the IHLG for the N-body interacting system in the form

$$H_T' = H_T + \tfrac{1}{2}\sum_{i,j} V_{ij} + V, \quad \vec{K}_T' = \vec{K}_T + \tfrac{1}{2}\sum_{i,j} \vec{W}_{ij} + \vec{W}, \tag{3}$$

with $\vec{P}_T' = \sum_i \vec{P}_i = \vec{P}_T$ and $\vec{J}_T' = \sum_i \vec{J}_i = \vec{J}_T$. In Eq. (3), $H_T = \sum_i E_i$ is the free Hamiltonian, $\vec{K}_T = \sum_i \vec{K}_i$ is the free boost, V_{ij} and $\vec{W}_{ij}$ are the two-body BT interactions defined by Eq. (2), and V and $\vec{W}$ are two unknown operators to be determined from the commutation relations. Here, $\vec{J}_i$, E_i, and $\vec{K}_i$ are defined in Eq. (5) of Ref. 7.

We can expand H_T, $\vec{K}_T$, V, and $\vec{W}$ in powers of c^{-1} and write them in the form $H_T = \sum_i m_i + H_T^{(0)} + H_T^{(2)} + \ldots$, $\vec{K}_T = \vec{K}_T^{(0)} + \vec{K}_T^{(2)} + \ldots$, $V = V^{(0)} + V^{(2)} + \ldots$, and $\vec{W} = \vec{W}^{(2)} + \vec{W}^{(4)} + \ldots$. Now, if we require the generators given by Eq. (3) to satisfy the Lie algebra of the IHLG, we obtain the following commutation relations:

$$[\vec{J}_T, V^{(0)}] = [\vec{P}_T, V^{(0)}] = [\vec{K}_T^{(0)}, V^{(0)}] = 0 \tag{4}$$

for zeroth order;

$$[(\vec{J}_T)_\alpha, (\vec{W}^{(2)})_\beta] = i\epsilon_{\alpha\beta\gamma}(\vec{W}^{(2)})_\gamma, \tag{5a}$$

$$[(\vec{P}_T)_\alpha, (\vec{W}^{(2)})_\beta] = i\delta_{\alpha\beta} V^{(0)}, \tag{5b}$$

$$[(\vec{K}_T^{(0)})_\alpha, (\vec{W}^{(2)})_\beta] + [(\vec{W}^{(2)})_\alpha, (\vec{K}_T^{(0)})_\beta] = 0, \tag{5c}$$

$$[\vec{J}_T, V^{(2)}] = [\vec{P}_T, V^{(2)}] = 0, \tag{6a}$$

$$[\vec{K}_T^{(0)}, V^{(2)}] = [V^{(0)}, \vec{B}^{(2)}] + [A^{(0)}, \vec{W}^{(2)}] - \sum_{i,j,k} [\vec{W}_{ij}^{(2)}, V_{ik}^{(0)}] \tag{6b}$$

for order c^{-2};

$$[(\vec{J}_T)_\alpha, (\vec{W}^{(4)})_\beta] = i\epsilon_{\alpha\beta\gamma}(\vec{W}^{(4)})_\gamma, \tag{7a}$$

$$[(\vec{P}_T)_\alpha, (\vec{W}^{(4)})_\beta] = i\delta_{\alpha\beta} V^{(2)}, \tag{7b}$$

$$[(\vec{K}_T^{(0)})_\alpha, (\vec{W}^{(4)})_\beta] + [(\vec{W}^{(4)})_\alpha, (\vec{K}_T^{(0)})_\beta] = [(\vec{W}^{(2)} - \vec{B}^{(2)})_\alpha, (\vec{W}^{(2)})_\beta] - [(\vec{W}^{(2)})_\alpha, (\vec{B}^{(2)})_\beta] - \sum_{i,j,k} [(\vec{W}_{ij}^{(2)})_\alpha, (\vec{W}_{ik}^{(2)})_\beta], \tag{7c}$$

$$[\vec{J}_T, V^{(4)}] = [\vec{P}_T, V^{(4)}] = 0, \tag{8a}$$

$$[\vec{K}_T^{(0)}, V^{(4)}] = [V^{(2)}, \vec{B}^{(2)}] + [V^{(0)}, \vec{B}^{(4)}] + [A^{(2)}, \vec{W}^{(2)}] + [A^{(0)}, \vec{W}^{(4)}] + \sum_{i,j} \{[\vec{W}_{ij}^{(2)}, V_{ij}^{(2)}] + [\vec{W}_{ij}^{(4)}, V_{ij}^{(0)}]\} - \sum_{i,j,k} \{[\vec{W}_{ij}^{(2)}, V_{ik}^{(2)}] + [\vec{W}_{ij}^{(4)}, V_{ik}^{(0)}]\} \tag{8b}$$

for order c^{-4}; etc. Here, $\vec{B}^{(2m)} = \vec{K}_T^{(2m)} + \tfrac{1}{2}\sum_{ij} \vec{W}_{ij}^{(2m)} + \vec{W}^{(2m)}$, $A^{(2m)} = H_T^{(2m)} + \tfrac{1}{2}\sum_{ij} V_{ij}^{(2m)}$ $(m = 0, 1, 2, \ldots)$, and $\alpha, \beta, \gamma = 1, 2, 3$.

These equations are to be solved for $V^{(2m)}$ and $\vec{W}^{(2m+2)}$. Solutions for these operators are not unique. To be physically acceptable, they must also satisfy the separability condition. This requires that if the N-body system is divided in any way into two subsystems A and B, and if every particle belonging to A is infinitely separated from every particle belonging to B, then $V^{(2m)}$ and $\vec{W}^{(2m+2)}$ should assume the separate forms $V_A^{(2m)} + V_B^{(2m)}$ and $\vec{W}_A^{(2m+2)} + \vec{W}_B^{(2m+2)}$, respectively. (Here $V_A^{(2m)}$ and $\vec{W}_A^{(2m+2)}$ involve dynamical variables referring to particles belonging to sybsystem A only, and $V_B^{(2m)}$ and $\vec{W}_B^{(2m+2)}$ involve dynamical variables referring to particles belonging to subsystem B only.) This requirement is satisfied if $V^{(2m)}$ and $\vec{W}^{(2m+2)}$ satisfy the strong-limit conditions

$$\lim_{a\to\infty} \|\exp(-i\vec{a}\cdot\vec{P}_A) V^{(2m)} \exp(i\vec{a}\cdot\vec{P}_A)\Psi_v\| = 0, \tag{9a}$$

$$\lim_{a\to\infty} \|\exp(-i\vec{a}\cdot\vec{P}_A) \vec{W}^{(2m+2)} \exp(i\vec{a}\cdot\vec{P}_A)\Psi_w\| = 0, \tag{9b}$$

where $\vec{P}_A$ is the total momentum of the subsystem A, and Ψ_v and Ψ_w are two dense sets of vectors of the Hilbert space.

The solutions for $V^{(2m)}$ and $\vec{W}^{(2m+2)}$ should include only many-body interaction terms which vanish when the system considered has only two particles. This is because the two-body BT interactions are the most general form for any two-body system. Any two-body interaction terms introduced into both $V^{(2m)}$ and $\vec{W}^{(2m+2)}$ can always be incorporated into two-body BT interactions.

Solutions for $V^{(0)}$ should include all many-body terms (from three-body up to N-body terms) of zeroth order. These terms are arbitrary, rotationally invariant functions of the nonrelativistic internal c.m. dynamical variables. If included, they must be constructed in such a way that the strong limit, Eq. (9a), is satisfied. Some of them can be constructed from the two-body interactions $V_{ij}^{(0)}$. The solutions for $V^{(0)}$ are then used to construct $\vec{W}^{(2)}$ which satisfies Eqs. (5) and (9b). Again, solutions for $\vec{W}^{(2)}$ are arbitrary many-body terms.

The solutions for $V^{(2)}$ include both particular solutions and the arbitrary solutions obtained from $V^{(0)}$ and $\vec{W}^{(2)}$. Particular solutions are those solutions which satisfy Eqs. (6), with $V^{(0)}$ and $\vec{W}^{(2)}$ set equal to zero, and Eq. (9a). I have found that the particular solution obtained previously by Foldy and Krajcik is not the only possibility. Their solution is just one of the infinite number of particular solutions I have determined. A general expression for these particular solutions can be written as

$$V^{(2)} = (2i)^{-1} \sum_{i,j,k} \{\vec{Q}_{ijk} \cdot [\vec{W}_{ij}^{(2)}, V_{ik}^{(0)}] + [\vec{W}_{ij}^{(2)}, V_{ik}^{(0)}] \cdot \vec{Q}_{ijk}\}, \tag{10}$$

with

$$\vec{Q}_{ijk} = (X_{ijk}\vec{p}_i + Y_{ijk}\vec{p}_j + Z_{ijk}\vec{p}_k)/(X_{ijk}m_i + Y_{ijk}m_j + Z_{ijk}m_k).$$

Here, $X_{ijk} \equiv X(m_i, m_j, m_k)$, $Y_{ijk} \equiv Y(m_i, m_j, m_k)$, and $Z_{ijk} \equiv Z(m_i, m_j, m_k)$ are arbitrary real constants (of the same order in c^{-1}) which may depend upon the masses m_i, m_j, and m_k. Since the condition $[V^{(2)}, \vec{P}_T] = 0$ requires that $\vec{Q}_{ijk} = \vec{Q}_{ikj}$, these constants are not completely independent; they must satisfy the following conditions: $X_{ijk} = X_{ikj}$, $Y_{ikj} = Z_{ijk}$, and $Z_{ikj} = Y_{ijk}$. This expression for $\vec{Q}_{ijk}$ makes it clear that the arbitrariness of the solution is characterized by one independent (continuous) parameter. The following operators are all special examples for $\vec{Q}_{ijk}$: $\vec{p}_i/m_i$, $(\vec{p}_j + \vec{p}_k)/(m_j + m_k)$, $\frac{1}{2}(\vec{p}_j/m_j + \vec{p}_k/m_k)$, $\frac{1}{2}[(\vec{p}_i + C_1\vec{p}_j)/(m_i + C_1 m_j) + (\vec{p}_i + C_1\vec{p}_k)/(m_i + C_1 m_k)]$, $[\vec{p}_i + C_2(\vec{p}_j + \vec{p}_k)]/[m_i + C_2(m_j + m_k)]$, and $(m_i^n\vec{p}_i + m_j^n\vec{p}_j + m_k^n\vec{p}_k)/(m_i^{n+1} + m_j^{n+1} + m_k^{n+1})$, where C_1, C_2, and n are arbitrary constants. The solution obtained previously by Foldy and Krajcik is the one given by Eq. (10) with a particular choice of $\vec{Q}_{ijk} = (\vec{p}_i + \vec{p}_j + \vec{p}_k)/(m_i + m_j + m_k)$. The expression for $V^{(2)}$ given by Eq. (10) is not, however, the most general form for particular solutions since we can always add to $V^{(2)}$ some three-body terms of order c^{-2} which commute with $\vec{J}_T$, $\vec{P}_T$, and $\vec{K}_T^{(0)}$. For instance, we can have solutions of the form

$$\overline{V}^{(2)} = V^{(2)} + \sum_{ijk} D_{ijk}^{(2)} V_{ij}^{(0)} V_{ik}^{(0)},$$

where $D_{ijk}^{(2)} = D_{ikj}^{(2)*}$ are constants of order c^{-2} constructed from m_i, m_j, and m_k. With an appropriate choice of $D_{ijk}^{(2)}$, for example, I have also found the following particular solutions:

$$\overline{V}^{(2)} = -i \sum_{i,j,k} (\vec{W}_{ij}^{(2)} \cdot \vec{Q}_{ijk} V_{ik}^{(0)} - V_{ik}^{(0)} \vec{Q}_{ijk} \cdot \vec{W}_{ij}^{(2)}), \tag{11}$$

where $\vec{Q}_{ijk}$ are the same operators used in Eq. (10). My result shows that even if those arbitrary terms obtained from $V^{(0)}$ and $\vec{W}^{(2)}$ are ignored, the expression for $V^{(2)}$ is still not unique.

The existence of many independent three-body interactions for $V^{(2)}$ will cause ambiguities in the practical applications. For example, the result obtained here valid to order c^{-2} can be applied to estimate the relativistic corrections to nuclear binding energies. Such calculations have been done, without including the three-body interaction terms, by several authors.[10-12] Ambiguities will certainly arise if one would like to improve these calculations by including these three-body terms in the calculations. Thus, these estimates of relativistic corrections are valid only if the contribution from all three-body terms is small compared with that from the two-body terms.

The solutions for $V^{(4)}$, $\vec{W}^{(4)}$, and all higher-order terms are extremely difficult to find and the complication increases as the order increases. Although one hopes to solve these problems, the existence of many independent solutions does raise a question of the practical value of these higher-order solutions, particularly if the solutions for these higher-order terms do not impose any additional constraint on the solutions for the lower-order terms.

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[1]L. L. Foldy and R. A. Krajcik, Phys. Rev. Lett. 32, 1025 (1974), and Phys. Rev. D 12, 1700 (1975).

[2]The separability condition was discussed by L. L. Foldy [Phys. Rev. 122, 275 (1961)] and by F. Coester [Helv. Phys. Acta 38, 7 (1975)]. See also Ref. 1 for more references.

[3]F. A. Zhivopistsev, A. M. Perolomov, and Yu. M. Shirokov, Zh. Eksp. Teor. Fiz. 36, 478 (1959) [Sov. Phys. JETP 9, 333 (1959)].

[4]The three-particle case was first discussed by Coester, Ref. 2.

[5]The N-particle case was also discussed by S. N. Sokoloff, Institute of High Energy Physics, Serpukhov, Report No. IHEP 74-135, 1974 (to be published). I am grateful to Dr. F. Coester and Professor L. L. Foldy for bringing Sokoloff's paper to my attention.

[6]B. Bakamjian and L. H. Thomas, Phys. Rev. 92, 1300 (1953).

[7]M. K. Liou, Phys. Rev. D 9, 1091 (1974).

[8]Yu. M. Shirokov, Zh. Eksp. Teor. Fiz. 36, 474 (1959) [Sov. Phys. JETP 9, 330 (1959)].

[9]See Eq. (1) of Ref. 7 for definitions.

[10]V. K. Gupta, B. S. Bhakar, and A. N. Mitra, Phys. Rev. Lett. 15, 974 (1967).

[11]B. S. Bhakar, Phys. Rev. C 2, 905 (1970).

[12]F. Coester, S. C. Pieper, and F. J. D. Serduke, Phys. Rev. C 11, 1 (1975).

Comment on Direct Lepton Production in Proton-Nucleon Collisions

F. E. Taylor
Physics Department, Northern Illinois University, De Kalb, Illinois 60115
(Received 24 February 1976)

A unified description is given of all available data on direct lepton production in proton-nucleon collisions. It is shown that the lepton cross sections exhibit a simple scaling behavior when expressed in terms of the radial variable, x_R. A striking similarity is noted between the behavior of the lepton and negative kaon production cross sections.

A considerable amount of data has been accumulated on the direct production of leptons in proton-nucleon collisions covering a wide range in $\sqrt{s}$, $p_\perp$, and θ^* (center-of-mass angle of direct lepton).[1,2] The unexpectedly large magnitude of this production makes it difficult to explain in terms of known mechanisms[2] and this process remains an intriguing enigma.

To gain some insight into the production of direct leptons, it is of value to examine the invariant cross section, or equivalently, the ratio of direct leptons to pion production over a wide range of kinematic variables and to look for a systematic behavior. In particular, it is of interest to analyze the direct lepton production data in terms of the radial scaling phenomenology, which has been shown to be a good approximation to inclusive hadron production in proton-proton collisions.[3] There is no *a priori* reason why this description of inclusive production of hadrons should work for direct lepton production, and a violent disagreement with the radial scaling picture could mean a radically different production process than that of hadron production. Conversely, an agreement with radial scaling would furnish a systematic view of the process and could lead to information about the quantum numbers involved in the direct lepton production process.

Taylor *et al.* have previously shown[3] that the invariant cross section for single-particle hadron production in p-p collisions can be described by the form:

$$E d\sigma/dp^3 \cong F(p_\perp, x_R), \qquad (1)$$

where $x_R \cong 2p^*/\sqrt{s}$, p^* is the three-momentum of the hadron in the proton-nucleon center-of-mass frame, and $\sqrt{s}$ is the total energy in the center-of-mass frame. To a good approximation all hadron production cross sections exhibit scaling when expressed in this form. It has also been found to be a fair approximation to assume factorization,[3,4]

$$F(p_\perp, x_R) \approx g(p_\perp) f(x_R),$$

where $g(p_\perp)$ is roughly the same function for all hadrons, and $f(x_R)$ is a function in which resides