pleted, we learned that C. S. Lai also extended our standing-wave solutions to the case of partial reflection.[6]

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Collective Ion Acceleration in a Converging Wave Guide

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An ion-acceleration mechanism is proposed in which ions are trapped and accelerated by a negative-energy space-charge wave. Under certain conditions, an electron beam propagating down a slowly converging metal guide can be accelerated. A low-phase-velocity space-charge wave initiated at the large-radius end of the guide, because it is supported by the beam, will also accelerate. By trapping ions in the potential wells of a low-phase-velocity wave, ion acceleration down the converging drift tube can be achieved.

In recent years a number of ion-acceleration methods have been suggested.[1-9] In this paper we suggest an alternative to the auto-resonant accelerator proposed by Sloan and Drummond[7,8] for accelerating ions in the potential well of an accelerating negative-energy wave. In their paper the negative-energy space-charge mode of a relativistic electron beam is shown to have a phase velocity $v_{\mathrm{ph}} > c(1 - 1/\gamma_e)$ and is not considered for ion acceleration. In the case of a mildly relativistic beam the above inequality is not applicable and $v_{\mathrm{ph}} \ll c$ can be realized for space-charge waves. Instead of using a spatially decreasing magnetic field to accelerate the cyclotron wave, we utilize a converging guide to accelerate an electron beam which can in turn accelerate the phase velocity of both the space-charge and cyclotron mode. The range of phase velocities allowed by the converging-guide accelerator appears to be smaller than the range allowed for by the auto-resonant accelerator; however, we find there can be significant wave acceleration nevertheless. By initiating a low-phase-velocity negative-energy wave in the presence of ions, energy can be supplied to the field at the expense of the energy in the electron beam. Hence, ions can be trapped in both an accelerating and a growing longitudinal electric field.

The final velocity of the ions is equal to the final phase velocity of the wave which in turn is approximately the final velocity of the relativistic electron beam. Preliminary estimates indicate that the conversion efficiency of electron-beam to ion-beam energy should be larger than 0.5% and may be as high as 50%, depending on the details of the nonlinear saturation mechanism.

The electron-beam parameters as well as the maximum accelerating electric field appear to be substantially the same as those for the auto-resonant accelerator.

The propagation of an unneutralized relativistic electron beam in a uniform cylindrical guide immersed in an external magnetic field has been previously investigated.[10,11] We propose injection into a slowly converging guide, in the presence of a high uniform magnetic field, B_0, as shown in Fig. 1. The beam will propagate provided certain injection and stability conditions are satisfied.

The injection criterion,[3] which assures that the beam will not form a virtual cathode upon entering the guide, is

$$\gamma_{\rm inj} - 1 > (\omega_b r_b/2c)^2(1+2\ln r_g/r_b), \tag{1}$$

where $\omega_b = (4\pi|e|^2 n_e/m_e)^{1/2}$, n_e is the beam density in the laboratory frame, m_e is the electron rest mass, r_b is the beam radius, r_g is the guide radius, and $(\gamma_{\rm inj}-1)m_ec^2$ is the injection energy. The stability condition[12] is

$$\omega_b^{\,2} r_g/\Omega_0 c < \gamma_e, \tag{2}$$

where $\Omega_0 = |e|B_0/m_e c$ and $\gamma_e = (1 - v_e^{\,2}/c^2)^{-1/2}$.

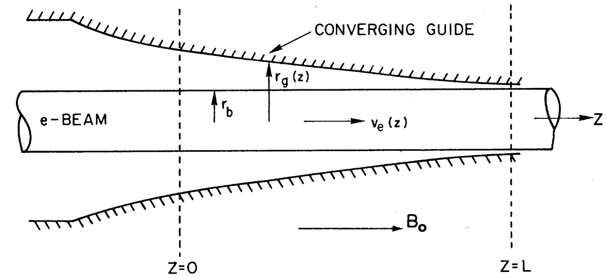

FIG. 1. Acceleration of an electron beam in an adiabatically converging wave guide.

Strong fulfillment of (2) leads automatically to the fulfillment of the equilibrium conditions on the beam.

In steady-state operation the combined field and particle energy flux through successive cross sections of the guide is conserved. Propagation of a beam through a converging guide accelerates the beam. Assuming conditions (1) and (2) are satisfied and that the convergence of the guide is slow, conservation of total energy flux yields

$$\gamma_e(z) = \gamma_e(0) + \tfrac{1}{4}\omega_b^{\,2}(0)(r_b^{\,2}/c^2)\{1+2\ln r_g(0)/r_b - [v_e(0)/v_e(z)][1+2\ln r_g(z)/r_b]\}, \tag{3}$$

where the arguments of the quantities refer to their values at the axial position, z. In obtaining (3) it has been assumed that the radius of the beam is held fixed by the guide magnetic field, B_0, and beam velocity is independent of radial position. From Eq. (3) it is easily seen that for a suitable choice of parameters it is possible to have $\gamma(L) > \gamma(0)$. However, $\gamma_e(L)$ cannot be greater than the injection value, $\gamma_{\rm inj}$. Given the initial beam current and energy we note that Eq. (3) has two physically acceptable solutions for $\gamma_e(z)$. Equation (3) also implies that a minimum γ exists, $\gamma_{\rm min} > 1$. In our analysis the $\gamma(z) \geq \gamma_{\rm min}$ branch of Eq. (3) is used.[13]

A physically simple explanation of the acceleration process is as follows. The injected beam carries electromagnetic energy flux in the space between it and the wall. As the walls contract the field energy flux reverts to the beam and accelerates it.

The well-known dispersion relation, for instance of the negative-energy longitudinal electric field on a relativistic electron beam, is

$$\omega_0 = k_{\parallel}(z)v_e(z) - \frac{\omega_b(z)}{\gamma_e^{\,3/2}(z)}\left[\frac{\omega_0^{\,2} - k_{\parallel}^{\,2}(z)c^2}{\omega_0^{\,2} - k^2(z)c^2}\right]^{1/2}, \tag{4}$$

where $k^2(z) = k_{\parallel}^{\,2}(z) + k_\perp^{\,2}(z)$, $k_{\parallel}(z)$ is the wave number of the axial electric field along the axis, and $k_\perp(z)$ is the perpendicular component. A pictorial display of the full dispersion relation is shown in Fig. 2 in the expanded and contracted guide regions. Since the system has no external time dependence but only a z dependence, ω_0 is a constant of the motion. Therefore, if a wave has positive phase velocity at $z=0$ and lies on the negative-energy dispersion curve, the phase ve-

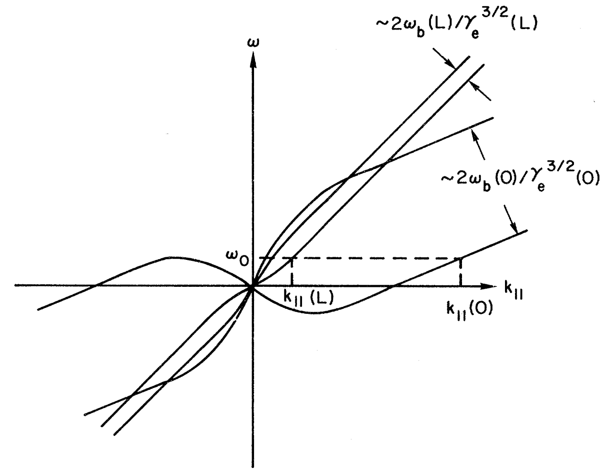

FIG. 2. Dispersion characteristics of the axial electric field in the expanded and contracted wave guide.

locity increases [$k_\parallel(z)$ decreases] as $\gamma_e(z)$ increases with z. One can show that the negative-energy cyclotron wave can also be accelerated by a converging guide. Equation (4) neglects the effect of a radial shear in γ. The inclusion of shear will modify the structure of the radial eigenmodes. The axial phase velocity of a single eigenmode however will still be independent of radius. The quantities $v_e(z)$, $\gamma_e(z)$ in Eq. (4) will then be replaced by some radial average. The resulting wave will still be accelerated by the converging guide.

Now let us examine the effect of a small number of heavy ions trapped in the potential troughs of the wave. As the wave accelerates, the ions also accelerate as long as the change in the inertial potential (in the wave frame of reference) across a wavelength is small compared to the peak-to-peak potential perturbation of the wave. As the ions accelerate, they gain energy. Since the wave which traps them is a negative-energy wave, as the ions gain energy, the wave grows. Thus by properly tapering the wave guide radius as a function of z, it should be possible to insure that a large number of ions remain trapped throughout the entire wave guide.

The initiation of a negative-energy wave can be accomplished by placing a time-varying potential across two grids at the input of the system. The frequency of the applied potential as well as the spacing of the grids is determined by the dispersion characteristic of the space-charge waves at the input.

The ions can be either externally supplied[14] or locally generated by the beam itself (i.e., a two-stream-instability process). The self-electric-field of the beam radially draws in and contains the ions in the transverse direction.

The total time-averaged energy flux through any cross section of the guide is conserved. The negative-energy-wave flux and ion-energy flux at $z=L$ must sum to the initial value at $z=0$, which is approximately zero. In particular, at $z=L$, we have

$$P_w+P_i\approx 0, \tag{5}$$

where $P_w=P_{\mathrm{em}}+P_e$ is the wave energy flux,

$$P_{\mathrm{em}}=\tfrac{1}{2}c\int_0^{r_g(L)}\langle\vec{\mathrm{E}}\times\vec{\mathrm{B}}\rangle_t\cdot\hat{e}_z r\,dr$$

is the time-averaged electromagnetic energy flux,

$$P_e=\frac{v_e(L)}{4}\frac{\omega_b^2(L)}{\gamma_e(L)^3}\frac{\omega}{[\omega-v_e(L)k]^3}\int_0^{r_g(L)}\langle|E_z|^2\rangle_t r\,dr$$

is the time-averaged particle energy flux, and

$$P_i=2\pi\int_0^{r_b}n_i(L)v_i(L)[\gamma_i(L)-1]Mc^2 r\,dr$$

is the ion energy flux. In writing the expressions for the energy flux it has been assumed the fields are axially symmetric. Since $\omega<v_e k$ for a negative-energy wave, the particle energy flux P_e is negative. The amplitude of the axial electric field $|E_z|$ grows until an appreciable number of beam electrons are trapped. An estimate for the maximum electric field amplitude is given by the trapping condition

$$|E_z|_{\max}\approx\frac{\alpha k_\parallel}{|e|}\frac{(\gamma_{ew}-1)m_ec^2}{\gamma_{\mathrm{ph}}}, \tag{6}$$

where $\gamma_{\mathrm{ph}}=(1-v_{\mathrm{ph}}^2/c^2)^{-1/2}$, $v_{\mathrm{ph}}=\omega_0/k_\parallel$ is the phase velocity of the wave, and $(\gamma_{ew}-1)m_ec^2$ is the final energy of a beam electron as measured in the wave frame [$\gamma_{ew}=\gamma_e\gamma_{\mathrm{ph}}(1-v_ev_{\mathrm{ph}}/c^2)$]. The factor α in Eq. (6) takes into account the fact that the wave does not stop growing when the first electron is trapped. That is, if $\alpha=1$, Eq. (6) gives the amplitude of the electric field necessary to trap an electron at the top of the wave potential well. A recent analysis,[15] on the saturation of the relativistic two-stream instability, shows that the wave actually e-folds three or more times beyond the value given by Eq. (6) with $\alpha=1$. Therefore, α will certainly be at least 1 and may be as high as 10. (Numerical calculations that will yield accurate solutions for α are in progress.) Since the ions are trapped, their velocity is approximately equal to the phase velocity of the wave, $v_i\approx v_{\mathrm{ph}}$. Using the maximum electric field given by (6) in the energy-flux relation, Eq. (5), we find that the efficiency, η, is given by

$$\eta=P_i/P_b=\tfrac{1}{2}\alpha^2[J_1(k_\perp r_b)(\gamma_{ew}-1)ck_\parallel/\omega_b\gamma_{\mathrm{ph}}]^2v_{\mathrm{ph}}\{v_ek_\parallel\gamma_e^{3/2}[1+(k_\perp\gamma_{\mathrm{ph}}/k_\parallel)^2]^{3/2}\omega_b^{-1}-(k_\perp\gamma_{\mathrm{ph}}^2/k_\parallel)^2\}\times[v_e(\gamma_e-1)]^{-1}, \tag{7}$$

where all the quantities are evaluated at $z=L$, and $P_b=\pi r_b^2n_e(L)v_e(L)[\gamma_e(L)-1]m_ec^2$ is the energy flux of the electron beam. It has been assumed that at $z=L$ the beam fills the cylindrical guide and that the fields satisfy the linear dispersion relation as well as the appropriate boundary conditions, hence the appearance of the Bessel function J_1 in the expression for η.

To get a feeling for the ion beam currents and efficiencies of acceleration, which can be achieved with readily available electron beams, a rather modest example is considered. We choose an electron beam carrying a current of ~5 kA, $r_b = 0.5$ cm, and $\gamma_e(0) = 1.2$ [$\gamma_{\rm inj} \approx 2.2$, $\omega_b(0)r_b/c = 1.5$]. For the converging guide $r_g(0)/r_b = 1.5$ and $r_g(L)/r_b = 1.0$. The magnetic field needed to satisfy the equilibrium condition (2) and to assure a constant beam radius is ~5 kG. Using Eqs. (3) and (4) we find that $\gamma_e(L) \approx 1.8$, $\gamma_{\rm ph}(L) \approx 1.3$, and $r_b k_\parallel(L) \approx 0.13$ when $v_{\rm ph}(0) = \omega_0/k_\parallel(0) = 0.05c$ and $r_b k_\parallel(0) \approx 1.8$. From Eq. (5), for $\alpha = 10$ the conversion efficiency of electron beam to ion beam energy flux is 50%, resulting in an ion beam current of ~0.5 A. The final energy of the ions is $[\gamma_{\rm ph}(L) - 1]Mc^2 \approx 0.3$ GeV. Calculations show that the length of such an accelerator would be ~15 m. At $z = L$, the peak-to-peak electric field of the accelerating wave is ~1×10^5 V/cm. On the other hand, if $\alpha = 1$, the efficiency is about 0.5%, the length is 150 m, and the peak-to-peak accelerating voltage is ~10^4 V/cm. Thus, the details of the nonlinear saturation clearly play a crucial role in determining the viability of any negative-energy-wave accelerator. The effect of radial variation has not been considered in the above example. The numerical estimates given are meant to be a simple example and were not chosen by a parameter survey.

We have presented a concept that promises compact and efficient acceleration of large ion currents to high energies. As can be surmised from the example, only modest use is made of existing technology. In addition, the acceleration of multiply stripped heavy ions to multi-GeV energies is also possible, with minor changes in the efficiency and length of the system.

Partial charge and/or current neutralization of the beam will lower the initial phase velocity of the wave and supply a source of ions for acceleration. Inclusion of density and velocity gradients and the study of nonlinear stability of the accelerating wave is under way.

The acceleration of the beam in the converging guide can also be used to accelerate the phase velocity of the negative-energy cyclotron mode. The use of this to construct an ion accelerator is being considered.

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