longer-living $2S$ state. The narrowest observed components have a width of about 40 MHz, corresponding to a resolution of about 6 parts in 10^8, i.e., they exhibit more than an order of magnitude improvement over our earlier pulsed-laser saturation spectra.[9] A substantial improvement in the accuracy of the $1S$ Lamb shift is expected, if such a polarization spectrum is used as a reference for the $1S$-$2S$ two-photon spectrum.[9] A still further improvement in resolution should be possible if the laser linewidth is reduced by frequency stabilization. At low electric fields the natural linewidth of the quasi-forbidden $2S$-$4S$ transition is only about 1 MHz. A measurement of the H-D isotope shift to better than 0.1 MHz would confirm or improve the important ratio of electron mass to proton mass, and an absolute wavelength or frequency measurement to better than 6 MHz would yield a new improved value for the Rydberg constant. We are presently exploring these and other possibilities for new precision measurements.

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Analogy between the Laser and Second-Order Phase Transitions: Measurement of "Coexistence Curve" and "Susceptibility" for a Single-Mode Laser near Threshold*

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We report the first experimental determination of the laser "coexistence curve" and "susceptibility" below threshold. Both quantities are derived from an accurate measurement of the average output intensity as a function of the normalized net gain. Our data confirm the predictions of laser theory and give a quantitative support to the recently proposed analogy between the laser near threshold and second-order phase transitions.

It has been recently shown that the theoretical treatment of the threshold region of a laser bears a close analogy with the mean-field approach to a second-order phase transition, if one identifies the order parameter with the slowly varying amplitude of the laser electric field and the temperature with the unsaturated population inversion.[1,2] We report here the first measurement of the laser "coexistence curve" and "susceptibility" below threshold. Our data confirm the predicted mean-field-theory behavior of the laser transition in the threshold region.

The statistical properties of single-mode laser radiation near threshold have been extensively investigated in the past few years.[3] Various moments of the photon probability distribution and time-dependent correlations of both amplitude and phase fluctuations have been measured as functions of the average output intensity I of the laser. Our experiment concerns instead the dependence of I on the difference between the unsaturated population inversion σ and the threshold population inversion σ_t.

From the viewpoint of the proposed analogy, the quantity I does not always have the same meaning. Specifically, in the region above threshold where amplitude fluctuations are relatively small, I is the square of the laser order parameter, and therefore the behavior of $I^{1/2}$ versus $\sigma - \sigma_t$ gives the laser "coexistence curve."[4] Below threshold, where the order parameter is zero, I is the mean-square fluctuation of the field amplitude and is therefore proportional to the generalized "susceptibility" for the laser instability.

A point which distinguishes the laser case from second-order phase transitions is that the quantity σ is not directly measurable for a laser operating above threshold because saturation effects lock the population inversion to the threshold value σ_t for any $\sigma \geq \sigma_t$. In fact what we have actually measured is the average laser intensity as a function of the mode position under the Doppler line. For small displacements around the threshold position, the frequency of the laser mode is linearly related to the unsaturated gain which is proportional to σ.

To obtain meaningful results, we must not allow the cavity losses (which determine σ_t) to change while sweeping the mode position from below to above threshold. It should be noted that the feedback control schemes used in previous experiments on the laser near threshold stabilize the average laser intensity against variations of either gain or losses.[5] As a consequence, they cannot be employed here because they do not ensure the constancy of σ_t. We have therefore simply improved the passive stabilization of the laser cavity and devised a technique to measure the average intensity with good accuracy over the whole threshold region in a time interval short enough to make cavity drift effects negligible.

The experiment is performed with a He-Ne laser (Spectra Physics 119) working in a single longitudinal and transverse mode. The cavity losses and the discharge current are adjusted so that the threshold position is approximately 400 MHz away from the peak of the Doppler line. A piezoceramic mount on one of the cavity mirrors allows us to control the position of the laser mode. A frequency sweep equal to the difference in frequency between two adjacent longitudinal modes is obtained with a voltage variation of about 100 V. A cavity length of about 9.5 cm gives a calibration constant $A = 15.5$ MHz/V. The measurement of I as a function of $\sigma - \sigma_t$ is performed in a single experimental run by applying a voltage ramp to the piezoceramic which sweeps the mode frequency through the threshold region. The ramp duration is 35 sec and the slope is 33 mV/sec. The linearity of the voltage ramp is better than 0.1%. Since the total voltage variation is only 1.1 V, it can be assumed that the linearity of the piezoceramic response is better than 1%.

The laser intensity is detected by a low-noise photomultiplier tube (ITT FW 130). In order to maintain linearity over a range of at least two decades in intensity and to improve the discrimination against detector noise and electrical disturbances, the measurement apparatus operates on digital signals. The beam intensity is always kept sufficiently low to ensure that the probability of producing more than one photoelectron in the resolving time of the detector is very small. Therefore the output of the detector consists of a train of nonoverlapping photoelectron pulses which are suitably amplified and standardized in shape. The intensity transient is recorded in the memory of a multichannel analyzer (Laben Correlatron) operating as a multiscaler. Since the sampling frequency is 5 Hz, the average intensity is measured at 175 different points. The value of 0.2 sec chosen for the duration of the sampling interval is long enough to average the intensity over many correlation times for intensity fluctuations. In fact, the longest correlation time for the laser under study is 40 μsec slightly above threshold.[6] To check that the effect of cavity drifts is negligible over the total duration of the ramp, three successive sweeps are recorded in the memory of the Laben Correlatron and compared one to another. In the given experimental conditions, the drift of the laser intensity is less than 1% over 1 min.

Figure 1 shows the measured behavior of the laser intensity as a function of the normalized net gain $\epsilon = (\sigma - \sigma_t)/\sigma_t$. The threshold point ($\epsilon = 0$) is obtained by extrapolation of the linear part of the plot above threshold. The parameter ϵ is easily connected to the voltage V applied to the piezoceramic mount, since the atomic line shape is known.[7] For small displacements of the mode position around threshold, ϵ is linearly related to V. The proportionality constant is determined by

$$\frac{d\epsilon}{dV} = A\,\frac{8(\nu - \nu_t)\ln 2}{\Delta\nu_D^{\,2}},$$

where ν and ν_t are the frequency at the peak of the Doppler line and the frequency of the laser mode at threshold, respectively, and $\Delta\nu_D$ is the width of the Doppler line. The abscissa scale of Fig. 1 is calibrated both with the normalized net gain ϵ and with the values of the pump parameter a defined by Risken.[8] The pump parameter is proportional to ϵ through a constant k characteristic of the laser under investigation. The con-

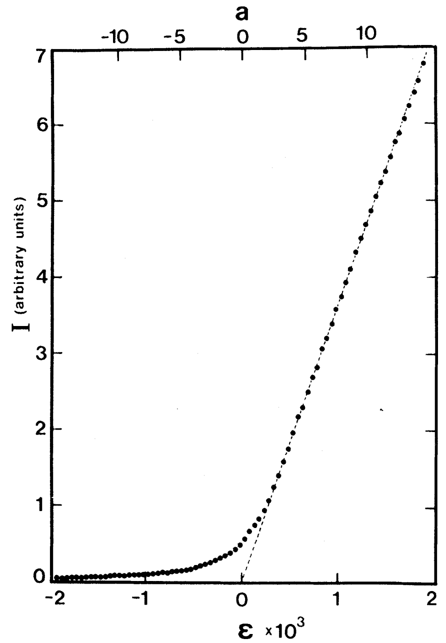

FIG. 1. The average laser intensity as a function of the normalized net gain ϵ (lower scale) and of the pump parameter a (upper scale). Errors are smaller than the dot size. The theoretical curve is not distinguishable from the line interpolating the experimental points.

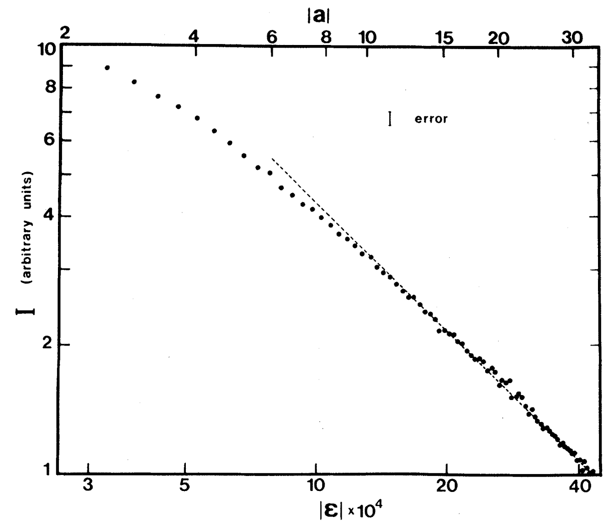

FIG. 2. Logarithmic plot of the average laser intensity below threshold as a function of ϵ and a.

stant k is determined by a best fit of the experimental points with the theoretical curve.[8,9] In as much as deviations from theory are smaller than the dot size in Fig. 1 for all points, the agreement is very good indeed. The best fit value is $k = 7.6\times10^3$.

The interpretation of our results is more involved than in usual experiments on second-order phase transitions because the laser order parameter is not directly measured in our case. Since the average intensity represents the mean square modulus of the electric field, the measurement of I generally contains information not only on the order parameter but also on the fluctuations of the field amplitude. Considering first the region above threshold, we know that relative fluctuations decrease rapidly as ϵ increases. Therefore except for the region closer to threshold, I can be considered to be just the square of the order parameter, and can be expected to follow the simple power law[4] $\epsilon^{2\beta}$, with $\beta=\frac{1}{2}$. Below threshold, where the order parameter is zero, I represents the mean square fluctuation of the field amplitude; hence it is the quantity analogous to the mean square fluctuation of the magnetization in a ferromagnet. The average intensity below threshold should therefore behave as the susceptibility at zero magnetic field of a ferromagnet, following the power law[4] $|\epsilon|^{-\gamma}$, with $\gamma=1$.

The plot in Fig. 1 shows clearly that above threshold, for $\epsilon \geqslant 0.5\times10^{-3}$, I is proportional to ϵ. As expected, the laser "coexistence curve" behaves consistently with a mean-field treatment. Deviations from linearity in the region $0 \leqslant \epsilon \leqslant 0.5\times10^{-3}$ are due to field fluctuations, as explained above.

We consider now the "susceptibility" behavior below threshold. Since the requirement of very good linearity of the photon-counting apparatus over the full range $-4\times10^{-3}<\epsilon<2\times10^{3}$ appreciably affects the statistical accuracy of the data well below threshold, we have repeated the measurement in the following way. At the start of the voltage ramp the attenuation of the laser beam is adjusted so as to get the required statistical accuracy for the region below threshold. When the slowly swept laser mode frequency is around the threshold value, a calibrated attenuator is inserted rapidly (less than half a second). In this way the intensity behavior above threshold is also reproduced with good linearity, and the threshold position can be determined by the same extrapolation procedure described above. The obtained results are presented in the logarithmic plot of Fig. 2. It is seen that the curve approaches a straight line with slope $\gamma=1$. The rounding effect shown by the points closer to threshold is due to the volume of the laser which sets an upper limit to the magnitude of field fluctuations. Of course, the asymptotic behavior of the "susceptibility" could be ascertained better by extending the measurements farther away from threshold. There are however some practical difficulties associated with the small signal-to-noise ratio and with the limited linearity of the piezoceramic response.

Since the laser is just an example from the broad class of physical systems in which an ordered state is created and maintained by an energy flux passing through the system, the analogy discussed in Ref. 1 can be generalized and applied to different instabilities in open systems,[2] such as some electronic devices[10] and fluid instabilities.[11] Measurements of the "coexistence curve" have recently been reported for the Rayleigh-Benard[12] and the Taylor[13] instabilities. No "susceptibility" measurement has however been performed, mainly because the region in which fluctuations are relatively large is not as easily accessible in fluid instabilities as it is in the laser case.

Finally it should be pointed out that all the previously reported experiments on the laser statistics near threshold have been interpreted by relying on the theoretical relation connecting I with the pump parameter a. The accurate check of this relation here provided puts the comparison between laser theory and experiments on a firmer basis.

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Nonlinear Optical Reflection and Transmission in Overdense Plasmas

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We show that our previous exact standing-wave solutions of the Maxwell cold-plasma equations may be generalized to describe partially reflected waves. Using these solutions, we find the transmission and reflection coefficients for intense circularly polarized light incident on a plasma slab.

Exact analytical solutions are available for two extreme cases of circularly polarized electromagnetic radiation in a cold plasma with fixed ions. The *traveling-wave* solutions of Akhiezer and Polovin,[1] which are "exact" for a homogeneous plasma, form the basis for an analysis by Max and Perkins[2] of waves in a slightly inhomogeneous medium where no reflection occurs. In the opposite extreme, the exact *standing-wave* solutions we reported recently allow an investigation of waves in an inhomogeneous medium where total reflection occurs.[3] Neither extreme is entirely realistic, as even when energy dissipation is negligible, a finite plasma does not reflect all incident radiation, and total transmission may occur only in special cases. Here we report a new