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Normal Hadronic Decays of ψ and ψ'†

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A phenomenological model for decays of ψ and ψ' into normal hadronic modes is proposed. This model can explain the decay rate $\Gamma(\psi\to\pi\rho)$ as well as the puzzlingly large rate $\Gamma(\psi\to\varphi\pi^+\pi^-)$ in comparison to $\Gamma(\psi\to\omega\pi^+\pi^-)$. It also predicts many other relations which can be easily tested experimentally.

One year after the spectacular discovery of ψ and ψ', they remain yet a mystery. Their narrow widths may be qualitatively understood on the basis of the SU(4) quark-line rule[1] or a generalized nonet *Ansatz*.[2] However, the rule does not explain a relatively large experimental decay rate[3] of

$$\Gamma(\psi\to\varphi\pi^+\pi^-)/\Gamma(\psi\to\omega\pi^+\pi^-)=0.20\pm0.10 \tag{1}$$

since the quark-line rule would predict[4] a factor of $\frac{1}{100}$ or thereabouts times this rate. Second, there is still no trustworthy way to compute quantitatively various decay rates of ψ and ψ'. Third, following Abrams,[3] we observe the following interesting regularity for the ratios of decay rates of ψ and ψ':

$$\Gamma(\psi'\to2\pi^+2\pi^-\pi^0)/\Gamma(\psi\to2\pi^+2\pi^-\pi^0)=0.41, \tag{2a}$$

$$\Gamma(\psi'\to K^+K^-\pi^+\pi^-)/\Gamma(\psi\to K^+K^-\pi^+\pi^-)=0.58, \tag{2b}$$

$$\Gamma(\psi'\to p\bar{p})/\Gamma(\psi\to p\bar{p})=0.57, \tag{2c}$$

$$\Gamma(\psi'\to l\bar{l})/\Gamma(\psi\to l\bar{l})=0.46. \tag{2d}$$

The purpose of this note is to facilitate the understanding of these facts by the following phenomenological model. It also predicts many other relations which can be easily verified or nullified experimentally.

We assume that the decay interactions of ψ and ψ' into the normal (i.e., noncharmed) hadrons are described by the effective phenomenological Hamiltonians

$$\mathcal{H}_\psi(x)=\{g j_\mu^{(0)}(x)+f[j_\mu^{(3)}(x)+\tfrac{1}{3}\sqrt{3}\,j_\mu^{(8)}(x)]\}\psi_\mu(x), \tag{3a}$$

$$\mathcal{H}_{\psi'}(x)=\{g' j_\mu^{(0)}(x)+f'[j_\mu^{(3)}(x)+\tfrac{1}{3}\sqrt{3}\,j_\mu^{(8)}(x)]\}\psi_\mu'(x), \tag{3b}$$

where $\psi_\mu(x)$ and $\psi_\mu'(x)$ are vector field operators representing ψ and ψ' particles and $j_\mu^{(\alpha)}(x)$ ($\alpha=0,1,2,\dots,8$) are the usual nonet of the SU(3) vector currents. The terms proportional to f and f' in Eq. (3) result from the virtual electromagnetic process, so that these numerical values can be calculated from the known decay rates[3] of $\psi\to e\bar{e}$ and $\psi'\to e\bar{e}$ to be

$$f^2/4\pi=(4.65\pm0.58)\times10^{-6},\quad (f')^2/4\pi=(1.79\pm0.24)\times10^{-6}. \tag{4}$$

The first terms proportional to g and g' in Eq. (3) are SU(3) singlets and signify the new interactions of nonelectromagnetic origin. They may or may not represent effective interactions of more fundamental kinds just as the terms proportional to f and f' are effective interactions of electromagnetic origin.

Now suppose that we have approximately

$$(g')^2/g^2 \approx \tfrac{1}{2}, \quad f'/g' \approx f/g. \tag{5}$$

We then can readily understand the rough validity of Eqs. (2). In fact, the Hamiltonians predict

$$\Gamma(\psi' \to n)/\Gamma(\psi \to n) \approx \tfrac{1}{2} \tag{6}$$

for any particular final decay mode n which consists only of normal hadrons. Small numerical variations among Eqs. (2) would be the consequence of differences of phase volumes as well as a small deviation from the ideal relation $f'/g' = f/g$ as we may see from (4) and (5). The only other decay mode for which (6) can be tested at the moment is $\pi^0\rho^0$:

$$\Gamma(\psi' \to \pi^0\rho^0)/\Gamma(\psi \to \pi^0\rho^0) < 0.7.$$

Next, the coupling constant g can be estimated as follows. When we compute the inclusive decay rate $\Gamma(\psi \to \text{all normal hadrons})$ on the basis of Eq. (3a), we have to know numerical values of

$$\Delta_{\alpha\beta}(s) = \int d^4x\, e^{iqx} \langle 0 | j_\mu^{(\alpha)}(x) j_\mu^{(\beta)}(0) | 0 \rangle,$$
$$s = -q^2, \tag{7}$$

at the value $s = M^2$, where M is the mass of the ψ meson. Since M^2 is large, we may apply the idea of the asymptotic nonet symmetry[5] or the quark-parton model[6] to obtain

$$\Delta_{\alpha\beta} = \delta_{\alpha\beta} \times \text{const}$$

for all $\alpha, \beta = 0, 1, 2, \ldots, 8$. From this we compute

$$\Gamma(\psi \to \text{all normal hadrons})$$
$$= \tfrac{1}{16}\pi^{-1}(g^2 + \tfrac{4}{3}f^2)RM, \tag{8}$$

where R is the famous ratio of the total hadronic cross section to muon-pair cross section in the $e\bar{e}$ annihilation experiment. Using the experimental value[7] of $R = 2.5$ at this energy range, and assuming that all decay rates of ψ other than those into lepton pairs or normal hadronic channels are negligible, we now estimate

$$g^2/4\pi = (2.43 \pm 0.73) \times 10^{-5}. \tag{9}$$

From (4) and (9), we find

$$|g/f| = 2.28 \pm 0.50, \tag{10}$$

so that any interference term between g and f interactions affects decay rates considerably, depending upon the relative sign of f and g.

Applying this same formula (8) for ψ', and using (5), we estimate

$$\frac{\Gamma(\psi' \to \text{all normal hadrons})}{\Gamma(\psi' \to \text{all})} \simeq 15\%. \tag{11}$$

Together with known decay rates[3] of $\psi' \to l\bar{l}$ and $\psi' \to \psi + \text{any}$, this can account for 75% of all ψ' decays. The remaining unaccountable 25% is presumably due to new channels such as ψ'(3.7 GeV) $\to \psi_p$ (2.8 GeV) + pions or ψ_c (3.4 GeV) + photon, where ψ_p (2.8 GeV) and ψ_c (3.4 GeV) are newly discovered charmonium states.[8,9]

Next, let us consider $\psi \to \pi\rho$ decay, whose decay matrix element is proportional to $\langle \pi\rho | j_\mu^{(\alpha)}(0) | 0 \rangle$ for $\alpha = 0, 3, 8$. If we assume that the third quark q_3 does not contribute for this decay, or equivalently if we assume the nonet *Ansatz*,[2] then the matrix element can be calculated from the known decay rate of $\Gamma(\omega \to \pi^0\gamma) = 0.87$ MeV. However, we have to take into account the difference of the vector-vertex form factors involved for the two decay modes. Assuming the standard vector-dominance form of $F(q^2) = [1 + q^2/(m_\omega)^2]^{-1}$ for this form factor, we can compute the absolute decay rate for $\psi \to \pi^-\rho^+$ without introducing any arbitrary parameter to be

$$\Gamma(\psi \to \pi^-\rho^+) = \begin{cases} 0.98 \text{ keV for } g/f = +2.28, \\ 0.47 \text{ keV for } g/f = -2.28. \end{cases} \tag{12}$$

Experimentally, this value is 0.43 ± 0.1 keV. Considering the uncertainty of the vertex form factors, the agreement is very encouraging. I should remark that this form factor can in principle be measured experimentally from the reaction $e\bar{e} \to \pi^-\rho^+$.

The U-spin invariance of the interaction Eq. (3) predicts $\Gamma(\psi \to \bar{K}^+K^{+*})/\Gamma(\psi \to \pi^-\rho^+) = 1$, where this quantity is experimentally around 0.4. However, we should not take the discrepancy too seriously, since the same U-spin invariance appears to be rather badly satisfied in predicting $\sigma(e\bar{e} \to K^+ + \text{any})/\sigma(e\bar{e} \to \pi^+ + \text{any}) = 1$. Moreover, the problem is perhaps related to a similar experimental ratio[10] for $\Gamma(K^{0*} \to K^0\gamma)/\Gamma(\omega \to \pi^0\gamma)$ in comparison to the SU(3) prediction.

We can also compute the rate $\Gamma(\psi \to p\bar{p})$ if the electromagnetic form factors of the proton in this time-like region are known. This will be discussed elsewhere.

Finally, we come to the most difficult part of the explanation of the ratio in Eq. (1). As we shall see shortly, we can explain it if we implement the original nonet *Ansatz*[2] by a new supplementary rule peculiar to three-body (or more) decay problems. Within the framework of the nonet *Ansatz*, the decay matrix element for $\psi \to VPP$ of ψ into a nonet of a vector meson V and two pseudoscalar octet mesons P is a linear combination of the following four terms:

$$\mathrm{tr}(jVPP)+\mathrm{tr}(jPPV)+\mathrm{tr}(jPVP), \tag{13a}$$

$$\mathrm{tr}(jVPP)+\mathrm{tr}(jPPV)-2\,\mathrm{tr}(jPVP), \tag{13b}$$

$$\mathrm{tr}(jV)\,\mathrm{tr}(PP), \tag{13c}$$

$$\mathrm{tr}(jP)\,\mathrm{tr}(VP), \tag{13d}$$

since any term involving tr$V$ or tr$j$ is forbidden by the rule, where j represents the spurion matrix of a vector nonet $j_\mu^{(\alpha)}(x)$ appearing in Eq. (3). If we insist in the validity of the stronger quark-line rule,[1] then (13c) and (13d) are also forbidden. However, we will not adopt this view here. We now see easily that the two terms (13b) and (13d) do not contribute to the decays $\psi \to \omega\pi^+\pi^-$ and $\psi \to \varphi\pi^+\pi^-$. Hence, we need discuss only (13a) and (13c). The troublesome term is precisely that of (13a) which allows $\psi \to \omega\pi^+\pi^-$ but forbids $\psi \to \varphi\pi^+\pi^-$. However, I argue that the presence of (13a) is inconsistent with the spirit of both nonet and quark-line rules by the following reason. Because of an identity equation[11] in the SU(3) space, (13a) can be shown to be identically equal to

$$\mathrm{tr}(jP)\,\mathrm{tr}(VP)+\tfrac{1}{2}\,\mathrm{tr}(PP)[\mathrm{tr}(jV)-\mathrm{tr}j\;\mathrm{tr}V]+\mathrm{tr}V\,\mathrm{tr}(jPP)+\mathrm{tr}j\;\mathrm{tr}(VPP). \tag{14}$$

This implies that a sum (13a) representing connected quark-line diagrams is identical to a sum of disconnected quark-line terms in (14). Therefore, in order to be self-consistent, we demand that the nonet hypothesis (and quark-line rule) for three-body decay problems should be modified so as to forbid the appearance of the special combination (13a). Then, only (13c) is responsible for the decays $\psi \to \omega\pi^+\pi^-$ and $\psi \to \varphi\pi^+\pi^-$ and we find

$$\frac{\Gamma(\psi\to\varphi\pi^+\pi^-)}{\Gamma(\psi\to\omega\pi^+\pi^-)}=\left(\frac{\sqrt{3}\,g-\sqrt{2}\,f}{\sqrt{6}\,g+f}\right)^2=\begin{cases}0.15 \text{ for } g/f=+2.28,\\ 1.37 \text{ for } g/f=-2.28,\end{cases} \tag{15}$$

if we neglect the difference of phase volumes. Therefore, if we choose $g/f=+2.28$, then we can satisfactorily explain the experimental ratio of 0.20 ± 0.10.

We should note that if the new supplementary interpretation of the nonet *Ansatz* is correct, then we predict

$$\sigma(e\bar{e}\to\varphi\pi^+\pi^-)/\sigma(e\bar{e}\to\omega\pi^+\pi^-)=2 \tag{16}$$

for a sufficiently high-energy $e\bar{e}$ process by exactly the same reasoning. The experimental test of (16) is crucial.

So far we have worked exclusively within the framework of the SU(3). As a matter of fact, the SU(4) symmetry is largely irrelevant for the present model. However, if we wish, we could generalize the new nonet *Ansatz* for the SU(4) scheme. Some modifications are found to be necessary in that case, and the details will be discussed elsewhere since it is beyond the scope of the present model.

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Kinematical Constraints on the Observation of Slow Monopoles at the Top of the Atmosphere

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Relativistic kinematics of monopole production and propagation in Earth's atmosphere and magnetic field have been examined in the light of the observation of a magnetic monopole by Price *et al.* It has been shown that the observed monopole cannot result from nuclear interactions in the atmosphere and that the extraterrestrial origin of this monopole is very unlikely.

Over the last decade, searches for magnetic monopoles produced in cosmic-ray interactions[1] have yielded only negative results. A recent attempt to look for them in p-p collisions at 1500 GeV[2] has set an upper limit on the production cross section of 2×10^{-36} cm^2 for a mass $m_M < 20$ GeV. Further, magnetic monopoles have not been observed by high-energy interaction experiments using nuclear emulsion chambers exposed at balloon and mountain altitudes. This sets an upper limit to the flux of monopoles of $\sim10^{-15}$/cm^2 sr sec. However, recently Price *et al.*[3,4] have presented possible evidence for a downward-moving magnetic monopole ($\beta=0.5^{+0.1}_{-0.05}$) with a magnetic charge $g=137e$ and $m_M>600m_p$, observed under 3 g cm^{-2} of the atmosphere at a zenith angle of 11°. The flux was estimated to be 10^{-13}/cm^2 sr sec. If this is typically the galactic flux, then it is about two orders of magnitude larger than that necessary to remove the galactic magnetic field.[5] In this Letter, we examine the kinematics of the production and propagation of a magnetic monopole in the atmosphere and its interaction with Earth's magnetic field, and put severe constraints on the possibility of observing a monopole of the kind described by Price *et al.*[3] We further investigate the difficulties relating to the galactic origin of the observed monopole. A preliminary version of this work was reported at the Fourteenth International Cosmic Ray Conference at Munich.[6]

We first examine the possibility that the observed monopole was produced in a nuclear collision in Earth's atmosphere. In order to conserve the magnetic charge, monopoles have to be created in pairs and hence we consider the reaction of the type $NN\to M\bar{M}X$, where X represents the particles accompanying the monopole (M) production. At the threshold for the production of a monopole, the kinetic energy of the incident nucleon is $[(2m_M)^2-(2m_N)^2]/2m_N$, where m_N is the mass of the nucleon. Thus for the production of monopoles of mass $>600m_p$, the threshold energy is $>6.76\times10^5$ GeV. The monopoles emerge in the laboratory system with an energy $E_M = m_M^2/m_p$; the corresponding Lorentz factor γ_M is >600, which is very much larger than the observed value of only 1.155.

However, the laboratory energy of a monopole would be reduced if it were produced in the backward direction in the center-of-mass system by primary particles of considerably higher energy than the threshold. The minimum energy of the monopole in the laboratory system in a p-p collision can then be

$$E_M=\frac{E_p^{*2}}{m_p}-\frac{(E_p^{*2}-m_p^2)^{1/2}(E_p^{*2}-m_M^2)^{1/2}}{m_p}, \quad (1)$$

where E_p^* is the c.m. energy of the proton. This expression for E_M decreases monotonically to $(m_p^2+m_M^2)/2m_p$ as $E_p^*\to\infty$, which is a factor of only 2 smaller than the value at threshold; for $m_M>600m_p$, $E_M>1.69\times10^5$ GeV ($\gamma_M>300$). Thus we have shown that the magnetic monopole observed by Price *et al.*[4] cannot have been produced in a nuclear interaction in the residual atmosphere above their detector.

One possible means by which a monopole creat-