Trapping of Decay Waves in Whistler Resonance Cones

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The decay of a right-hand polarized (whistler) wave is considered in the presence of a spatial gradient of the pump amplitude. It is shown that the low-frequency ion wave can be trapped in the vicinity of the gradient and sufficient conditions are derived for the threshold electric field and ion-wave frequency. The results of the calculation are in good agreement with experimental results.

Recent results have shown that the so-called cone phenomenon[1-4] which was thought to exist only in the near field can also exist in the far-field radiation pattern of a localized source.[5] The electric field in the regions of space defined by the resonance cone can be 10 to 20 dB greater than the field in the surrounding regions. It has also been shown that under certain conditions parametrically generated waves only appear in regions of space close to the resonance cone.[6]

The purpose of this Letter is to show that unstable trapping of the decay waves is possible for the interaction in which a whistler decays to a backward whistler plus an ion-acoustic wave. We will consider for simplicity the special case in which the wave vectors and the direction of the spatial variation of the pump amplitude are parallel to the external magnetic field (z direction). As the wave vectors and the spatial variation of the pump amplitude near the resonance cone are both nearly perpendicular to the cone, these restrictions are not very severe and could be relaxed.

Using the general results derived by Giles,[7] it can be shown after some tedious but straightforward calculations that the dispersion relation for this decay is

$$[(\Omega+i\gamma_2)^2-\delta^2][(\Omega+i\gamma_3)^2-\omega_3^{\,2}]+\alpha\delta\,\omega_3\Lambda^2(z)=0\,, \tag{1}$$

where

$$\alpha=\frac{\omega_2\omega_3}{\mu_2\mu_3}\left[\mathscr{E}_0\sum\frac{e_s\omega_{ps}^{\,2}}{2m_s}\,\frac{\omega_1\omega_2k_3+\Omega_s(k_2\omega_3+k_3\omega_2)}{(\omega_3^{\,2}-a_s^{\,2}k_3^{\,2})\omega_1\omega_2(\omega_1+\Omega_s)(\omega_2+\Omega_s)}\right]^2,$$

$$\mu_2=2-\sum_{s=i,e}\frac{\Omega_s\omega_{ps}^{\,2}}{\omega_2(\omega_2+\Omega_s)^2}\,,\quad \mu_3=2\sum_{s=i,e}\frac{\omega_{ps}^{\,2}\omega_3^{\,2}}{(\omega_3^{\,2}-a_s^{\,2}k_3^{\,2})^2}\,,$$

$\delta=\omega_1-\omega_2$, and $\Lambda(z)=E_1(z)/\mathscr{E}_0$. Here k_1, k_2, and k_3 are respectively the wave numbers of the pump, the whistler sideband, and the ion-acoustic wave, while $\omega_1-i\gamma_1$, $\omega_2-i\gamma_2$, and $\omega_3-i\gamma_3$ are the corresponding solutions of the linear dispersion relations; m_s, e_s, ω_{ps}, Ω_s, and a_s are respectively the mass, charge, plasma frequency, gyrofrequency, and thermal velocity of particles of type s (with $\Omega_e<0$). In addition, the amplitude of the pump wave is assumed to be normalized to its maximum value $\mathscr{E}_0$. It is important to notice that since $\omega_3\simeq c_sk_3$, where c_s is the ion-acoustic velocity, the quantity α depends linearly on ω_3. Hence we put

$$\alpha=\beta\omega_3\,.$$

Since we expect the complex root of (1), associated with the ion-acoustic wave, to be close to the solution of the linear dispersion relation we substitute $\Omega=\omega_3-i\gamma_3+\delta\Omega$ in (1) and neglect quadratic and higher-order terms in $\delta\Omega$. The real and imaginary parts of Ω will then be given approximately by

$$\Omega_R=\omega_3[1-0.5\beta\delta R(\omega_3)\Lambda^2(z)] \tag{2}$$

and

$$\Omega_I=\gamma_3[\Lambda^2(z)/T^2-1]\,, \tag{3}$$

where

$$R(\omega_3)=(\omega_3^{\,2}-\omega_{31}^{\,2})[(\omega_3^{\,2}-\omega_{31}^{\,2})^2+4\gamma^2\omega_3^{\,2}]^{-1},$$

and

$$T^2 = \gamma_3 [(\omega_3^2 - \omega_{31}^2)^2 + 4\gamma^2\omega_3^2] (\beta\gamma\delta\omega_3^2)^{-1},$$

with $\gamma = \gamma_2 - \gamma_3$ and $\omega_{31}^2 = \gamma^2 + \delta^2$. The above results will be valid provided that $|\delta\Omega| \ll |\omega_3 - i\gamma_3|$. This condition will be satisfied in the region that we shall investigate below ($\omega_3 \geq \omega_{31}$) provided that

$$1 + \delta^2/\gamma^2 \gg (\beta\delta/4\gamma^2)^2. \tag{4}$$

It has been pointed out by Arnush and Kennel[8] that in the neighborhood of a point $(k_3^{(0)}, z^{(0)})$ at which

$$\frac{\partial\Omega_R}{\partial k_3} = \frac{\partial\Omega_R}{\partial z} = \frac{\partial^2\Omega_R}{\partial k_3 \partial z} = 0, \quad \Omega_B^2 = \frac{\partial^2\Omega_R}{\partial k_3^2}\frac{\partial^2\Omega_R}{\partial z^2} > 0, \tag{5}$$

the equation of a ray is approximately $\ddot{z} + \Omega_B^2(z - z^{(0)}) = 0$. The conditions (5) together with (2) and (3) therefore imply that unstable trapping of the ion-acoustic sideband in the neighborhood of a maximum of $\Lambda^2(z)$ is possible provided that there is a value of ω_3 for which

$$\frac{d\,\omega_3 R(\omega_3)}{d\omega_3} = \frac{2}{\beta\delta}, \tag{6}$$

$$\omega_3 R(\omega_3)\frac{d^2\,\omega_3 R(\omega_3)}{d\omega_3^2} < 0, \tag{7}$$

and

$$T^2 < 1. \tag{8}$$

We now show that a sufficient condition for the existence of a solution satisfying (6)–(8) is that

$$\beta\delta/4\gamma^2 > 1 \tag{9}$$

provided also that $\gamma > 2\gamma_3$. We note first that the function $F(\omega_3) = \omega_3 R(\omega_3)$ has the following properties for $\omega_3 > \omega_{31}$: (i) $F > 0$, (ii) $F'' < 0$, and (iii) F' decreases monotonically from $1/2\gamma^2$ at $\omega_3 = \omega_{31}$ to zero at the maxima of F, which is located at $\omega = \omega_{32}$, where

$$\omega_{32} = \omega_{31}[\lambda + (\lambda^2 + 1)^{1/2}]$$

and

$$\lambda = \gamma^2/(\gamma^2 + \delta^2).$$

Hence if $F'(\omega_{31}) > 2/\beta\delta$, i.e., if (9) is satisfied, there must be some point in the interval $\omega_{31} < \omega_3 < \omega_{32}$ at which (6) and (7) are satisfied. Thus all that remains is to prove that $T^2 < 1$ is in this interval. This, in fact, follows immediately by noticing that $T^2 < 1$ in the interval $\omega_{31} < \omega_3 <_{33}$, where

$$\omega_{33} = \omega_{31}[\kappa + (\kappa^2 + 1)^{1/2}]$$

and

$$\kappa = (\gamma/\omega_{31})(\beta\delta/4\gamma\gamma_3 - 1)^{1/2},$$

and that $\omega_{32} < \omega_{33}$. We can obtain an approximate solution of (6) [that also satisfies (7)], which is valid near the threshold indicated by (9) by noting that when $\gamma^2 = 0.25\beta\delta(1-\epsilon)$, where $0 < \epsilon \ll 1$, then $\omega_3 = \omega_{31}[1 + \epsilon + O(\epsilon^2)]$. The corresponding real and imaginary parts will then be given by

$$\Omega_R = \omega_{31}\{1 - (1 - 4\gamma^2/\beta\delta)^2\}$$

and

$$\Omega_I = \beta\delta/4\gamma - \gamma_3.$$

Summarizing the above results, we may say that unstable trapping in the vicinity of the maximum of the pump wave amplitude will certainly occur when the conditions (4) and (9) are satisfied. Near threshold the unstable frequency $\Omega_R \sim (\gamma^2 + \delta^2)^{1/2}$. Hence, in view of (4), Ω_R must be of order δ and sufficiently above γ. The growth rate near threshold $\Omega_I \sim \gamma - \gamma_3$, while the threshold field will be given by

$$\mathcal{E}_H^2 = \frac{8\gamma^2}{\omega_2\delta}\sum_{s'}\frac{\omega_{ps'}^2 c_s^2}{(c_s^2 - a_{s'}^2)^2}\left(2 - \sum_{s'}\frac{\Omega_{s'}\omega_{ps'}^2}{\omega_2(\omega_2 + \Omega_{s'})^2}\right)\left(\sum_{s'}\frac{e_s\omega_{ps'}^2}{2m_{s'}}\frac{\omega_1\omega_2 + \Omega_{s'}(k_2 c_s + \omega_2)}{(c_s^2 - a_{s'}^2)\omega_1\omega_2(\omega_1 + \Omega_{s'})(\omega_2 + \Omega_{s'})}\right)^{-2}. \tag{10}$$

We can now use the results of Ref. 6 to compare these sufficient conditions with the actual experimental values. The experimental apparatus has been described elsewhere[5] and briefly is a chamber 120 cm long and 60 cm in diameter in a uniform magnetic field of 128 G. An rf source produces an argon plasma of approximately 6×10^{10} electrons cm^{-3} at a neutral gas pressure of 5×10^{-4} Torr. With an electron temperature of 2.6 eV the electron collision frequency is 1 MHz. The waves were transmitted from a 1-cm-diam loop and received by an electric probe with an exposed surface 3 mm long and 1 mm in diameter which could be rotated about the transmitter. Figure 1 shows the electric potential of the pump and sideband measured at a distance of 30 cm from the transmitter with a pump frequency of 150 MHz ($0.42 f_c$) and power of 30 W.

The sideband was separated from the pump by

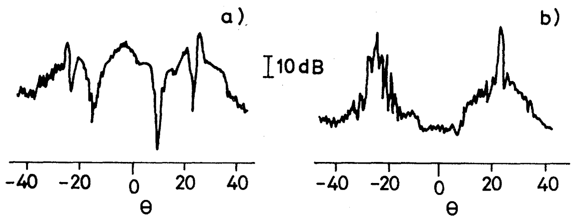

FIG. 1. Measured electric potentials (a) for the pump wave and (b) for the sideband wave, as a function of angle. The resonance cone angle is 28°.

0.15 MHz which was the frequency of the ion wave. The maximum of the sideband amplitude can be seen associated with the resonance cone ($\theta_c = 28°$).

Trapping only occurred when the ion-wave frequency was greater than the collision frequency, in agreement with Eq. (4). A similar condition has been derived for the parametric decay and trapping for a pump wave near the plasma frequency.[8]

The maximum field, measured near the transmitter, was of the order of 50 V/m, in good agreement with the value of 80 V/m predicted by Eq. (10). It should also be noted that Eq. (10) predicts minima for the threshold field for electron temperatures around 2 eV and densities around 5×10^{10} cm^{-3} if the pressure is kept constant at 3×10^{-4} Torr. This would explain the low powers needed for the parametric interaction in this machine and also in other rf-produced plasmas such as those generated by a Lisitano structure.

For present tokamaks, for example TFR, Eq. (10) predicts threshold fields of a few kilovolts per meter for a pump wave near the lower hybrid frequency. Although this is a sufficient rather than necessary field, the good agreement with the experiment described above suggests an error of not more than a factor of 2. It can also be seen from Eq. (10) that the threshold is linearly dependent upon the collision frequency. Consequently the presence of only a small number of neutrals would inhibit this phenomena.

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Generation of Trapped-Particle Modes with Well-Defined Initial Conditions by Injection of Electron Packets in a Plasma Wave

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This paper gives preliminary experimental results on synchronous injection of electron packets in the potential trough of an electrostatic plasma wave, and the trapped-particle modes which are generated. The initial conditions, wave amplitude, electron packet density, and phase of the packets with respect to the wave, may be varied independently and lead to detailed comparison with a recently developed theory.

This Letter reports preliminary experimental results of a direct approach to trapped-particle phenomena. These processes can occur naturally when electrostatic waves of finite amplitude are propagating in a plasma, if the correlation of the wave field is of the order of or lower than the characteristic time for field-particle interactions. A large number of papers have been devoted to the subject, from both theoretical[1] and experimental[2] points of view.

In experiments in which initial trapping is induced by high-amplitude, quasimonochromatic waves launched in the plasma, there is some uncertainty regarding the basic phenomena involved.