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Fokker-Planck Velocity Diffusion Coefficient in Plasma Turbulence

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A direct method of measuring the Fokker-Planck velocity diffusion coefficient in plasma turbulence is developed. The method is consistent with, but more general than, the quasilinear approximation. The velocity variance $\langle \Delta v^2 \rangle$ is measured by integrating the space-time potential correlation function along unperturbed particle orbits. Correlation times and velocity diffusion coefficients are measured as a function of velocity in ion-beam–plasma turbulence and the experimental results are compared with the Fokker-Planck theory requirements.

The Fokker-Planck diffusion equation is often applied in plasma physics to describe the velocity diffusion due to a statistical Coulomb field between charged particles in a stable plasma.[1,2] This scheme appears to be more general, however, since it also includes quasilinear theory.[3,4] The Fokker-Planck (FP) diffusion coefficient can be simply related to measured experimental quantities. Furthermore, this measurement is self-contained and does not require the existence of a dispersion relation, as previously assumed in plasma turbulence.[5-7] In this Letter, we will first present the basic ideas underlying the diffusion-coefficient measurement. This approach will then be applied to an experiment on ion-beam-plasma turbulence. The experimental values of the diffusion coefficient will be calculated and discussed.

When the E-field fluctuations are independent of the particle trajectories on the time scale of interest, it has been shown[3] that the particle diffusion can be described by a FP equation

$$\frac{\partial f}{\partial t}=\frac{\partial}{\partial \vec{v}}\cdot\left(\overleftrightarrow{D}\cdot\frac{\partial f}{\partial \vec{v}}\right), \qquad (1)$$

where

$$\overleftrightarrow{D}(v)=\frac{\langle\Delta\vec{v}\Delta\vec{v}\rangle}{2\Delta t},$$

is the diffusion tensor. It is expressed in terms of the velocity variance at time Δt, about the initial velocity $\vec{v}$, due to the fluctuating field. Equation (1) holds when the velocity variance is proportional to Δt and small enough for the expansion procedure[2] leading to (1) to be valid.

In stationary, homogeneous turbulence, and in the absence of a magnetic field, the velocity variance can be simply related to the electric field correlation function by integrating the equation of motion along unperturbed orbits. When the correlation time (tensor) for a particle of velocity $\vec{v}$ is defined by

$$\overleftrightarrow{\tau}_c=(\langle E^2\rangle)^{-1}\int_0^\infty d\tau\,\langle\vec{E}(0,0)\vec{E}(\vec{r}=\vec{v}\tau,\tau)\rangle, \qquad (2)$$

then if $\Delta t \gg \tau_c$,

$$\overleftrightarrow{D}(\vec{v})=(q^2/m^2)\langle E^2\rangle\overleftrightarrow{\tau}_c, \qquad (3)$$

Thus by integrating along the unperturbed orbits of the electric field correlation function as measured in the laboratory (Eulerian) frame, we can generate the FP diffusion coefficient $\overleftrightarrow{D}(\vec{v})$.

Equation (3) can also be related to the electric field spectral density $|E(\omega,\vec{k})^2|$. We find

$$\overleftrightarrow{D}(\vec{v})=\frac{q^2}{2^{n+1}\pi^n m^2}\times\int d\omega\, d^nk\,\frac{\overleftrightarrow{kk}}{k^2}\,|\vec{E}(\omega,\vec{k})^2|\,\delta(\vec{k}\cdot\vec{v}-\omega), \qquad (4)$$

where n is the number of space dimensions.

This is similar to the usual quasilinear result since the diffusion is due to spectral components in resonance with the particles. The FP treatment has the advantage of being more general, however, since no dispersion relation $\omega(\vec{k})$ need be assumed. The FP diffusion model can be applied to spectral densities which are not necessarily concentrated along a plasma dispersion relation. In particular a broad spectral density,[8] which is sometimes taken as a characteristic of strong turbulence, can be treated using the FP model.

The experiments were performed in a multipole[9] double-plasma[10] device (argon pressure 2.5×10^{-4} Torr, $T_e=4.2$ eV, $T_i=0.1-0.2$ eV, $n_e\sim3\times10^8$ cm^{-3}). A plane ion beam is injected into a target plasma at about the ion acoustic speed ($n_b/n_e=0.45$, $v_b/c_s=0.95$). As reported previously,[11] the ion-beam–plasma instability develops into a turbulent spectrum with a one-dimensional plane structure. The frequency spectrum peaks at 170 kHz. Two positively biased spherical probes are used, each smaller than a Debye length ($\lambda_D\sim1$ mm). One probe is fixed at the origin $x=0$ (4.1 cm in front of the beam injection grid) while the other can be moved along the x axis parallel to the direction of the beam velocity. Using a 1-MHz correlator, the correlation function is traced out for a given time delay τ as the movable probe is swept slowly along the x axis. Displacing the trace vertically by an amount proportional to τ (for each value of τ) gives the space-time correlation function as shown in Fig. 1. This function has the form of a Gaussian wave packet, moving from left to right with a maximum at the origin $x=0$, $\tau=0$.

Figure 1 gives the correlation function for the electron current fluctuations $\tilde{J}$. Dividing by the saturation currents J_s collected by each probe, we can calibrate the vertical axis of Fig. 1 in terms of the normalized potential fluctuations

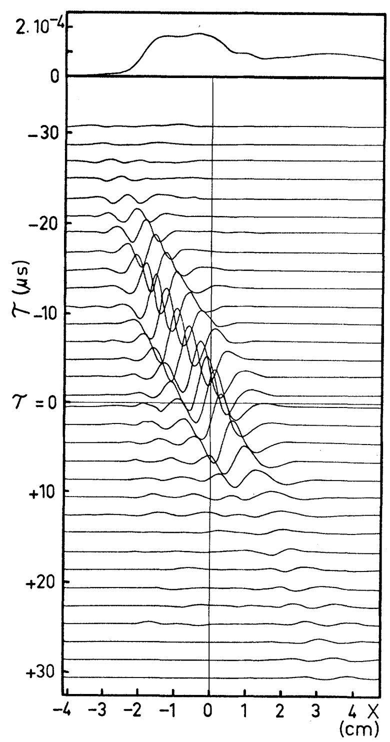

FIG. 1. Space-time correlation function of the potential fluctuations in one-dimensional ion-beam–plasma turbulence. Top: mean square relative potential fluctuation measured by the axially moving probe; vertical scale same as for correlation function. The origin ($x=0$) is the point at which the two probes lie at the same axial distance from the beam injection point.

$e\tilde{\varphi}/KT_e = \tilde{J}/J_s$. The top inset in Fig. 1 shows the mean square relative potential fluctuation as a function of x with the same normalized axis. The potential noise power is small near the beam source ($x = -41$ mm), grows rapidly near $x = -21$ mm with a growth rate $\langle k_i/k_r \rangle \simeq 0.25$, reaches a maximum of $\langle (e\tilde{\varphi}/KT_e)^2 \rangle = 1.7\times10^{-4}$ (rms $\sim 1.3 \times 10^{-2}$), and then saturates and decays very slowly.

To use Fig. 1, we must transform the measured normalized potential correlation function into the normalized electric field correlation function. Since the measured correlation is fairly harmonic with a mean wave number k_m [$=6\times10^2$ m^{-1} in Fig. 1], we have $\langle E(0,0)E(x,\tau)\rangle \simeq k_m^2 \langle \varphi(0,0)\varphi(x,\tau)\rangle$. Now to obtain the diffusion coefficient $D(v)$ from Fig. 1, we calculate the correlation time τ_c, Eq. (2), by integrating along the unperturbed orbits $x = v\tau$. In order to carry out the integration along a given orbit, a straight line of slope v is sketched outwards from the origin ($x=0$, $\tau=0$). The values of the correlation function are measured at the intersections of this line with the horizontal τ lines. Then τ_c is obtained by graphically integrating these values over τ and normalizing the integral by the value of the correlation at the origin. This procedure is carried out over different trajectories, corresponding to different velocities v. The diffusion coefficient is obtained by substituting these values of τ_c into (3), which can be written

$$D(v) = 10^{10}\tau_c(v) \text{ m}^2/\text{sec}^3,$$

where τ_c is in microseconds.

The solid curve in Fig. 2 shows the correlation time [and $D(v)$] as a function of v. These calculations correspond to the range of negative τ values in Fig. 1. The curve is sharply peaked at $v = 1.4$ km/sec, a velocity which is consistent with the phase velocity of the most unstable linear modes.[12] Around the peak, τ_c decreases to low and even (nonphysical) negative values. These minima are due to the inhomogeneity of the turbulence[13] since the trajectory $v = 1.4$ km/sec traverses the region of noise growth (see top of Fig. 1). Far from the peak, the diffusion coefficient extends into long wings in the regions of large positive and negative velocities. This behavior is not predicted by linear or quasilinear theory since no unstable normal plasma modes exist in this range. All of our measurements show a similar behavior. When τ_c is measured in the region of positive τ values (Fig. 1, downstream from the fixed probe), for

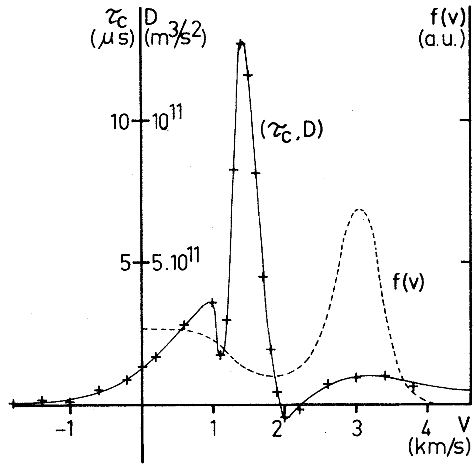

FIG. 2. Solid curve: Fokker-Planck diffusion coefficient (or correlation time) as a function of velocity, obtained from the negative-τ part of Fig. 1. The error bars due to the graphical integration of Fig. 1 are 2% at the maximum and 20% in the wings. The maximum uncertainties in the numerical constants ($\langle\varphi^2\rangle$, k_m^2) involved in τ_c and D are 30%. Dashed curve: mean ion velocity distribution function at $x = -0.8$ cm.

trajectories in the nearly homogeneous turbulence region, $\tau_c(v)$ still has a peak at 1.4 km/sec, but with a lower maximum [$(\tau_c)_{\max} = 6.6$ μsec] and a larger half-width (0.5 km/sec at half-maximum).

The dashed curve in Fig. 2 shows the ion velocity distribution at $x = -0.8$ cm as measured with an energy analyzer. The peak of the diffusion-coefficient curve lies in the plasma-ion distribution, in the range of v where the slope is negative, well separated from the beam distribution. The measured velocity-distribution curves at increasing distances have been compared to the FP spatial evolution predicted by (1), into which the measured values of $D(v)$ are inserted. The beam decay has been found consistent with (1). However, in the plasma distribution where a plateau should very rapidly form and stabilize under the peak of τ_c, a steady filling in of the plasma-ion distribution by the beam particles is observed. The ion-neutral collision mean free path is 20 cm, and does not play a significant role in the diffusion process which occurs over a few centimeters.

We now examine the hypotheses which have been used to establish the FP diffusion coefficient. The expansion procedure used requires that

$$\langle \Delta v^2 \rangle_{\Delta t} < (f^{-1}\partial^2 f/\partial v^2)^{-1}.$$

The lowest allowed value of $\langle \Delta v^2 \rangle_{\Delta t}$ is obtained for $\Delta t = \tau_c$. We must therefore satisfy

$$\langle \Delta v^2 \rangle_{\tau_c} \simeq \frac{2q^2}{m^2} \langle E^2 \rangle \tau_c^2 \ll \left(\frac{1}{f} \frac{\partial^2 f}{\partial v^2} \right)^{-1}.$$

When the measured value of $(f^{-1} \partial^2 f / \partial v^2)^{-1}$ is taken for velocities for which τ_c is maximum we find that τ_c must be less than 4.5 μsec. However, the measured peak value of τ_c is larger than this. Thus the use of the FP expansion is questionable for ion-beam–plasma turbulence for the class of velocities lying close to the peak of τ_c.

We can also estimate the effects of particle trapping on the orbits. Since the E-field spatial configuration moves at about the phase velocity corresponding to the peak of $\tau_c(v)$, particles in the velocity interval

$$\Delta v = 2(2e/m)^{1/2} (\langle \varphi^2 \rangle)^{1/4} = 1.14 \text{ km/sec}$$

can be trapped with a characteristic "ion trapping time"

$$\omega_b^{-1} = k_m^{-1} (m_i / e)^{1/2} (\langle \varphi^2 \rangle)^{-1/4} = 3.75 \ \mu\text{sec}.$$

We thus find that a large number of particles can bounce in potential wells. These strong perturbations in the particle orbits have not been taken into account when expanding the trajectories around unperturbed orbits $x = v\tau$.

The Fokker-Planck diffusion coefficient in a turbulent plasma has thus been experimentally measured using correlation techniques. The method can be applied to different types of weak turbulence and could be extended to other transport coefficients.

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