Differential Gain and Bistability Using a Sodium-Filled Fabry-Perot Interferometer

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Differential gain and large hysteresis have been seen in the transmission of a Fabry-Perot interferometer containing Na vapor irradiated by light from a cw dye laser. Nonlinear dispersion, neglected in earlier work, dominates over nonlinear absorption in Na. The apparatus uses only optical inputs and outputs. Similar apparatus may be useful as an optical amplifier, memory element, clipper, and limiter.

We report the first demonstration of the use of a Fabry-Perot interferometer (FP) filled with a nonlinear medium as an optical device whose transmission is bistable.[1] Under suitable operating conditions such a system can provide differential optical gain.[2,3] Since all previous discussions[1,3] treated only nonlinear absorption, and the present apparatus utilizing Na vapor functions best when the nonlinearity is primarily dispersive, a model including nonlinear dispersion is presented. This experiment demonstrates the physical principles from which practical miniaturized optical memories, amplifiers, clippers, and limiters may be developed.

The transmission of an empty FP is described in terms of the input field E_I and transmitted field E_T, through the standard formula

$$E_T = Te^{i\Phi/2}(1-Re^{i\Phi})^{-1}E_I, \tag{1}$$

where R is the mirror reflectivity, $T=1-R$ is the mirror transmissivity, and Φ is the round-trip phase shift. In the empty FP case, $\Phi = 2\pi(2L/\lambda)$, where $L \gg \lambda$ is the mirror separation, and λ the light vacuum wavelength. If Φ is a multiple of 2π, then the FP transmission is 100%.

In our case, the FP is not empty, but has between the plates a nonlinear medium, so that Φ is a function of the fields inside the FP. These fields are directly related to E_T, through boundary conditions at the exit miror, and Φ is regarded as a function of E_T.

In the limit of small mirror transmission T, the FP transmission is insignificant, unless Φ is nearly a multiple of 2π. Let Φ differ from a multiple of 2π by $\Delta\Phi(E_T)$, which is small in absolute value. Then, using Eq. (1), approximately,

$$E_I = E_T - i(R/T)\Delta\Phi(E_T)E_T. \tag{2}$$

The phase increment $\Delta\Phi(E_T)$ is specified in terms of an additional contribution to a generalized refractive index, which owes its origin to nonlinear dispersive and absorptive polarization components of the medium, u and v, respectively.[4] The round-trip phase shift $\Phi = 4\pi nL/\lambda$, where L is the mirror separation, $n^2 = n_0^2 + 4\pi\chi$ specifies the refractive index n, and n_0 is a background refractive index. The nonlinear susceptibility $\chi = (u + iv)/E_c$, where E_c is the relevant cavity field. Let $\chi \ll 1$, so that

$$\Delta\Phi = \theta + 8\pi^2 L(u+iv)/\lambda n_0 E_c, \tag{3}$$

where the mistuning parameter $\theta = \Phi_0 - 2\pi M$, $\Phi_0 = 4\pi n_0 L/\lambda$, and $2\pi M$ is the nearest multiple of 2π. Since T and $|\Delta\Phi| \ll 1$, the forward and backward waves inside the cavity are of nearly equal and uniform intensity, so that we may set $E_c = T^{-1/2}E_T$. Thus,

$$E_I = E_T + \Gamma v - i\Gamma(u - \beta E_T), \tag{4}$$

where $\Gamma = 8\pi^2 RL/T^{1/2}\lambda n_0$ and $\beta = -R\theta/\Gamma T$ are subject to independent control through variation of T and θ. If $u = v = 0$, the transmission of the FP is $(1+\Gamma^2\beta^2)^{-1}$. A derivation of Eq. (4) through a treatment of boundary conditions (e.g., Ref. 3) shows that Eq. (4) applies even when u and v are unspecified functionals of E_c.

Our observations of bistability and differential gain were made with a 11-cm FP interferometer with Na vapor at 10^{-4} to 10^{-5} Torr pressure in the 2.5-cm region midway between the mirrors. The 50-mW single-mode dye laser[5] was tunable to any frequency within either of the D lines. Absolute frequencies were determined by Na vapor saturation spectroscopy.[6] At the FP, the optical beam had a diameter of about 1.65 mm and a maximum power of 13 mW. The FP structure had 90% reflecting one-hundredth-wave mirrors. The transmitted and incident intensities were displayed as vertical and horizontal deflections on an oscilloscope. The optical transmission of the properly tuned FP without Na vapor was typically about 45% since the mirrors were not perfectly flat and some loss due to deposited Na occurred. The vertical gain was adjusted so that the empty FP input-output relationship was a 45° line, as in Fig. 1(a).

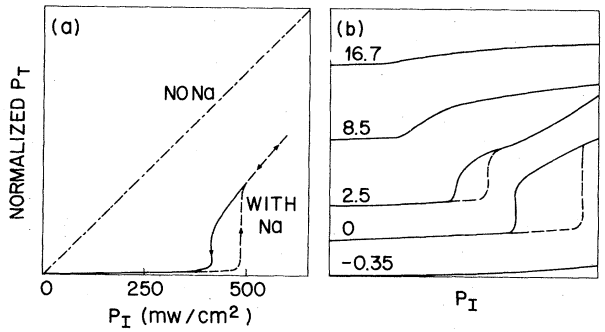

FIG. 1. (a) Optical bistability in Na vapor. The oscilloscope trace is dashed for increasing input intensities and solid for decreasing. The difference in switching intensities is 16% of their average [30% in Fig. 3(b)], i.e., only moderate laser stability is needed to make the device a stable memory element. The maximum input power was 12.7 mW in a 1.65-mm-diam Gaussian beam. The low-intensity sodium absorption coefficient was $\alpha L \approx 2.75$ with the laser frequency about 150 MHz above the Na $^2S_{1/2}, F=2$ to $^2P_{3/2}$ transitions. (b) Characteristic curve dependence on Fabry-Perot plate separation. The detuning is given in MHz; the free spectral range was about 1364 MHz.

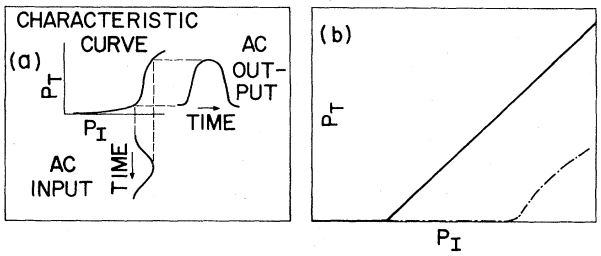

FIG. 2. (a) Output intensity versus input intensity in the differential gain mode. Experimentally, ac gains at 1 kHz exceeding 2 were observed (if there had been no FP losses, the differential gain would have been nearly 5). (b) Clipper and limiter action. All curves are data. The dashed curve refers to higher Na density. These observations were made at the D_1 crossover frequency. The output reflected from the front mirror was approximately complementary, i.e., nontransmitted light was reflected. The reflected output at larger intensities was not precisely constant, due to imperfectly flat mirrors, etc. By changing the laser frequency, we obtained reflected output-input curves which had a large region of nearly zero slope, thus demonstrating limiter action.

Our observations of differential gain and bistability are presented in Figs. 1 and 2. We tentatively ascribed our first observations of bistability to nonlinear absorption hysteresis effects already discussed.[1,3] In the absence of nonlinear dispersion, we set $u=0$ in Eq. (4); thus

$$E_I = E_T + \Gamma v,$$

where $v>0$, since there is no population inversion. If the total absorbed power (χE_c^2) is limited at high power, then it follows[3] that $v \propto \text{const}/E_T$ at high power, and the nonlinear absorber displays a region of negative resistance, $dv/dE_T<0$. In that region, therefore, the differential gain dE_T/dE_I exceeds unity, for values of $\Gamma > |dv/dE_T|^{-1}$. For larger Γ, $dE_T/dE_I<0$, so that a bistable region exists. Notice that these conditions can be met with some value of Γ (e.g., sufficiently high mirror reflectivity R) by a nonlinear absorber which displays a negative resistance region. In Na vapor, the saturation mechanism is optical pumping of atoms in the $F=1$ (or 2) ground state to the $F=2$ (or 1) ground states. For most laser frequencies, atoms tuned to resonance by the Doppler effect saturate at very low power, and recovery (T_1 process) occurs only after unpumped atoms enter the laser beam region. At higher powers, the resultant hole widens in proportion to the driving field (power broadening). Because the absorbed power (χE_c^2) is consequently approximately linear in driving cavity field, the negative resistance feature is quenched. It is only at very high powers, where the wings of the Doppler line are partially saturated, that negative resistance occurs.[3] Consequently, the absorptive model is inadequate for the explanation of these results.

Further experimental effects in disagreement with the absorptive model were discovered. The input-output curves behaved nonsymmetrically when the FP plate was changed from optimum bistability operation [Fig. 1(b)]. This asymmetry clearly indicates that nonlinear dispersion is important, because absorption effects alone involve only even functions of FP frequency mistuning.

Since the absorptive polarization easily saturates in Na vapor, we set $v=0$ in Eq. (4). The nonlinear dispersion does not easily saturate, however, since dispersion is due to moderately, but not exactly resonant atoms. A simple model, applicable to our case, is a nonlinear medium describable through an intensity-dependent refractive index,[7] so that $v=0$, and $u=f(|E_T|^2)E_T$, where f describes the nonlinear refractive index. Defining $P_I=|E_I|^2$, $P_T=|E_T|^2$ then Eq. (4) reduces to

$$P_I = P_T + \Gamma^2[f(P_T)-\beta]^2 P_T. \quad (5)$$

In Fig. 3(a), curves are presented for the case $f(P_T)=aP_T$, which represents the experimental

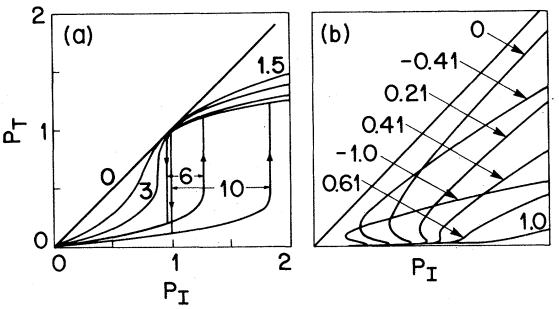

FIG. 3. (a) Purely dispersive model of gain and bistability [Eq. (5)], for the case $f(P_T)=aP_T$, corresponding to a linear dependence of refractive index on light intensity, so that $P_I=P_T+A(P_T-1)^2P_T$, where $A=R^2\theta^2 \times T^{-1}$ and P_I and P_T are measured in units of $P_0=a/\beta$. The inflection point, where maximum gain occurs in the nonbistable case, is at $P_T=2P_0/3$, with the maximum gain given by $(1-A/3)^{-1}$. The dividing case $A=3$ presents a point of infinite differential gain. Regions of negative slope are unstable; arrows indicate the hysteretic region in bistable cases. An analysis taking standing-wave effects into account essentially only replaces A above by twice its values. Numbers indicate the value of A. (b) Computer simulation of gain and bistability. The model was a homogenously broadened absorption line with center 1.5 full linewidths (at $\frac{1}{2}$ maximum) away from the laser frequency. Both absorptive and dispersive polarization components contribute, and standing-wave effects are included. The absorber is 1.2 Np thick at the laser frequency; $R=0.9$. Numbers refer to mirror displacements in units of $\lambda/4\pi F$, where finesse $F=\pi\sqrt{R}/(1-R)$. The model results illustrate differential gain, bistability, and clipping. The equations used are those of Ref. 3, generalized to take into account dispersive contributions and cavity mistuning.

conditions where a is a constant, corresponding to a linear dependence of refractive index on light intensity. In that case, we conclude that by adjusting Γ and β, one may achieve a region of (a) differential gain, wherein $dP_T/dP_I>1$ or (b) bistability, where Eq. (5) specifies a multivalued dependence of P_T on P_I. This conclusion is true for general $f(P_T)$, as long as $f(P_T)$ is not constant.

By scanning the laser frequency across the D_2 line, we found that the asymmetry indicating nonlinear dispersion vanished at three frequencies: about 400 MHz below the transitions from the ground state $F=2$, at the crossover halfway between $F=1$ and $F=2$, and about 300 MHz above $F=1$. Calculations show that at these three frequencies the dispersion is unchanged by optical pumping: At crossover the backward wave unpumps what the forward wave pumps; at the other frequencies the contribution to dispersion from an $F=1$ atom is equal to the contribution from an $F=2$ atom. At the lowest of the above frequencies, bistable operation was seen using maximum power and high absorption; this is the first demonstration of the previously predicted[1,3] absorptive bistability. Except at the three frequencies just discussed, the dispersion is light-intensity dependent; there bistability was easily observed. Calculated phase shifts[8] resulting from optical pumping are comparable to the FP instrument width, in agreement with the dispersion model described above. The asymmetry with respect to plate detuning is understood in terms of the sign of β in Eq. (5).

Observed "turn-on" (transition from weakly to highly transmitting operation in the bistable mode) times were 1–10 μsec. These times are approximately the time required for the intracavity field to optically pump the Na atoms. The turn-on times were shortest when the input was abruptly changed to its highest value. Turn-off times were typically 20 to 50 μsec. A thermal atom crosses the 1.65-mm beam in 5 μsec; thus the effective saturated diameter is several times larger.

In general, Eq. (4) applies, which should be, to be accurate, modified to take standing-wave effects into account. We have done this: Figure 3(b) presents results wherein the nonlinear polarization is derived from Bloch's equations.[9] The model used is spatially stationary homogeneously broadened two-level atoms. The parameters were chosen so that the curves would resemble our Doppler-broadened Na vapor results.

By taking u and v in Eq. (4) to be dynamical quantities dependent on $E_T(t)$, dynamical and stability problems may be investigated. In all cases we investigated, the negative (positive) slope region of a steady-state solution is unstable (stable). Solutions illustrated in Fig. 3(b) are therefore expected to exhibit hysteresis.

These effects should be observable in other systems. The contribution of nonlinear dispersion may allow operation over a wider frequency range, allow lower loss operation, and extend the number[10] of nonlinear materials that can be used. The medium properties, and hence the transmission, may be controlled with a weak third laser beam (e.g., incident from the side of the FP) leading to a class of three-port devices. Successful miniaturization may lead to a practical active component for integrated optical circuits.

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Fokker-Planck Velocity Diffusion Coefficient in Plasma Turbulence

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A direct method of measuring the Fokker-Planck velocity diffusion coefficient in plasma turbulence is developed. The method is consistent with, but more general than, the quasilinear approximation. The velocity variance $\langle \Delta v^2 \rangle$ is measured by integrating the space-time potential correlation function along unperturbed particle orbits. Correlation times and velocity diffusion coefficients are measured as a function of velocity in ion-beam–plasma turbulence and the experimental results are compared with the Fokker-Planck theory requirements.

The Fokker-Planck diffusion equation is often applied in plasma physics to describe the velocity diffusion due to a statistical Coulomb field between charged particles in a stable plasma.[1,2] This scheme appears to be more general, however, since it also includes quasilinear theory.[3,4] The Fokker-Planck (FP) diffusion coefficient can be simply related to measured experimental quantities. Furthermore, this measurement is self-contained and does not require the existence of a dispersion relation, as previously assumed in plasma turbulence.[5-7] In this Letter, we will first present the basic ideas underlying the diffusion-coefficient measurement. This approach will then be applied to an experiment on ion-beam-plasma turbulence. The experimental values of the diffusion coefficient will be calculated and discussed.

When the E-field fluctuations are independent of the particle trajectories on the time scale of interest, it has been shown[3] that the particle diffusion can be described by a FP equation

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial \vec{v}} \cdot (\vec{\vec{D}} \cdot \frac{\partial f}{\partial \vec{v}}), \tag{1}$$