So far, we only considered the "little group" (i.e., the subgroup of Lorentz transformations that leave the energy-momentum of the bound state invariant). We found invariance if the elements of the little group are associated with a gauge transformation [by mapping the two SO(3) groups into each other]. This leads to the conservation of the angular momentum (12).

What about the more general elements of the Lorentz group? They also affect the collective coordinates and thus also the fields Q^{cl} and A^{cl}. The gauge for the new Q^{cl} and A^{cl}, after a Lorentz boost, is essentially free, so that the formulation of the more general Lorentz transformations will be much more ambiguous, contrary to those of the little group for which we could keep Q^{cl} and A^{cl} fixed. One consequence of this complication is that although it is easy to tell what the spin of the particle is, by consideration of the little group, it will be hard to derive a relativistic wave equation such as the Dirac or Klein-Gordon equation for the composite particles.

Of course the theory is expected to be fully Lorentz invariant and unitary, at least in the perturbation expansion because shifts such as in our Eq. (9) are known not to affect these properties essentially, even after renormalization.

After this work was completed, Goldhaber[6] showed how to extend the relation between spin and statistics for particles with both electric and magnetic charge: When two dyons are obtained as bound states of magnetic poles and electric charges, then the wave equation for the two compound objects may violate the spin-statistics theorem, but it contains more Dirac strings than necessary. These Dirac strings can be transformed away by means of an ordinary gauge transformation, and then a new minus sign restores the spin-statistics relation for the two dyons. These arguments are expected to apply also to the bound states we discussed here.

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Connection of Spin and Statistics for Charge-Monopole Composites*

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An object composed of a spinless electrically charged particle and a spinless magnetically charged particle may bear net half-integer spin, but the wave function of two such clusters must be symmetric under their interchange. Nevertheless, a careful study of the relative motion of the clusters shows that this symmetry condition implies the usual connection between spin and statistics.

If magnetic monopoles exist, then classical physics already tell us that a system of pole g and electric charge q has an angular momentum of magnitude gq/c, directed from charge to pole. In quantum mechanics this spin adds to orbital and intrinsic angular momenta, so that, for $gq/\hbar c = (2n+1)/2$, an otherwise integer-spin system will have net half-integral total angular momentum.[1] This holds equally well in the SU(2) gauge field formulation of charge-pole interactions (in fact, this spin may be used to derive the gauge field[2]), as has recently been emphasized by Jackiw and Rebbi[3] and Hasenfratz and 't Hooft.[4] These papers lend drama to a question that could have been considered years ago: Perhaps an object whose half-integer spin comes from the charge-pole contribution obeys Fermi-Dirac statistics, so that a fermion may be made out of bosons !

To evaluate this proposal, first take note of an elementary theorem about the statistics of composite particles. Here "statistics" is used, following a common but—at least in the present context—treacherous convention, to mean "permutation symmetry of indistinguishable-particle wave functions." In the second part of the Letter we shall see how to relate these statistics to observable quantities.

Theorem.—If electric charges q can combine with magnetic monopoles g to form composite objects with half-integer values for the product $gq/\hbar c$, then there must exist entities with the "wrong" connection between spin and statistics. For large g-q spacing the composites themselves must have wrong statistics.

Proof.—First suppose that the wave function for a collection of spinless monopoles widely separated from a system of spinless charges is symmetric under interchange of any two charges or any two poles. If this were not so, the theorem would hold at once. The Hamiltonian for interacting charges and poles preserves the permutation symmetry. Therefore, if a state with two isolated clusters, each containing a charge and a pole, can be formed beginning with poles widely separated from charges, then that state must still exhibit symmetry under interchange of charges or of poles. *A fortiori,* this state must be symmetric under interchange of charge-pole clusters. Now, a cluster formed in such a way may well not be the ground state carrying a particular combination of charge and pole strengths. Therefore, it may decay in a cascade to the actual ground state, producing a final state consisting of this ground state and a complicated superposition of radiation states, carrying integer angular momentum. The final states for two different clusters again must be symmetric under interchange. Hence, the ground states must be symmetric unless the radiation states are antisymmetric. Thus, either the ground state or the radiation state has the wrong connection between spin and statistics (Q.E.D.).

Remarks.—(1) The crucial assumption in the proof is that isolated charge-pole clusters can be formed from initially widely separate groups of charges and groups of poles. This seems essential to the notion that the cluster is composed from initially free charges and poles.

(2) It might be that the actual ground state has integer spin.[5] In that case, the radiation field has half-integer spin, but again either one or the other must have anomalous statistics.

(3) One might ask whence comes antisymmetric radiation? Since this is a monopole theory, pole-antipole states are an obvious possibility, but the same sequence of argument as in the proof above shows that these states could only produce such anomalous-statistics objects by emitting them in pairs. There is no natural mechanism, even with poles added, for emitting such radiation, and so one might as well concede that the composites themselves have the wrong statistics. In any case, when the clusters are still big, and could not yet have emitted anomalous radiation, they *must* have wrong statistics.

(4) What is the intuitive reason for the breakdown observed? Of course, in familiar theories the spin is carried by local fields, while in this case the charge-pole system is spread out; the angular momentum in the electromagnetic field is associated with both particles, and the fraction to be found outside a radius R about the cluster is $O(d/R)$, where d is the cluster size. Since this electromagnetic field is not a free radiation field, there is no meaning to assigning it statistics. Hence, the statistics are simply those of the constituents (the real degrees of freedom).

The "usual" connection between spin and statistics allowed for point particles in local field theory[6] is equivalent to the requirement that a pair of indistinguishable particles have even $L+S$, where L is the relative orbital angular momentum and S is the total spin of the pair. This may be reduced to the common statement about symmetry or antisymmetry of wave functions under interchange of particle coordinates, provided certain phase conventions are established. In particular, one depends on the choice of angular momentum eigenfunctions which acquire a phase $(-1)^L$ upon inversion of the relative position coordinate $\vec{r}$. Ordinary spherical harmonics exhibit this behavior. They are eigenfunctions of the orbital angular momentum operator $-i\hbar\vec{r}\times\nabla$, meaning that the realtive momentum operator is $-i\hbar\nabla$. However, magnetic effects imply the existence of velocity-dependent potentials which may (and in this case do) alter the phase produced by inversion of $\vec{r}$.

To identify the relevant velocity-dependent potentials, first consider the nonrelativistic interaction of one charge with one pole. Without loss of generality, the Hamiltonian for such a system may be written

$$H=[\vec{p}_q-q\vec{A}(\vec{r}_q-\vec{r}_g)/c]^2/2m_q + [\vec{p}_q+q\vec{A}(\vec{r}_q-\vec{r}_g)/c]^2/2m_g. \quad (1)$$

Here $\vec{A}(\vec{r})$ is the vector potential at the point $\vec{r}$ of a monopole located at the origin. The form given has the advantage that an infinitesimal displacement of q produces the same phase change as the opposite displacement of g, so that the effect of the interaction is confined to the wave function in the relative coordinate, and the usual separation of center-of-mass and relative motions may be accomplished. Any other choice could not change the physics, but would complicate the ensuing discussion. To see quickly the implications of Eq. (1) for the relative motion of two clusters, note that the velocity-dependent term is simply

$$-q(\vec{v}_q{}^0-\vec{v}_g{}^0)\cdot\vec{A}(\vec{r}_q-\vec{r}_g)/c,$$

with $\vec{v}^0\equiv\vec{p}/m$. For two charge-pole clusters, this gives a net velocity-dependent term

$$-q(\vec{v}_1{}^0-\vec{v}_2{}^0)\cdot[\vec{A}(\vec{r}_1-\vec{r}_2)-\vec{A}(\vec{r}_2-\vec{r}_1)]/c,$$

leading to the kinetic relative momentum[7]

$$\vec{p}=-i\hbar\nabla-q[\vec{A}(\vec{r})-\vec{A}(-\vec{r})]/c. \quad (2)$$

The combination $\vec{A}(\vec{r})-\vec{A}(-\vec{r})$ is well defined except at $r=0$, and its curl is the sum of monopole fields at $\vec{r}$ and $-\vec{r}$, which vanishes. This just reproduces a well-known classical result, that particles with the same ratio of magnetic to electric charge exert purely Coulombic forces on each other, and no long-range magnetic forces. One may check that if the clusters had finite size, it would lead only to dipole and higher multipole interactions between them (albeit violating parity and time-reversal symmetry) as long as the clusters could be enclosed in nonoverlapping spheres.

The discussion in the first part of the Letter showed that a two-cluster wave function has the symmetry

$$\Psi(\vec{r},\xi_1,\xi_2)=(-1)^{2(s+gq/\hbar c)}\Psi(-\vec{r},\xi_2,\xi_1), \quad (3)$$

where s is the spin of a cluster and ξ_i stands for all the internal coordinates of cluster i, including spin orientation. We are ready now to determine the implications of this strange-looking symmetry for allowed quantum numbers of a cluster-pair state. Since we are dealing with a curl-free vector potential, we may write

$$\Psi(\vec{r},\xi_1,\xi_2)=e^{i\alpha}\Phi(\vec{r},\xi_1,\xi_2) \quad (4)$$

with

$$\vec{p}\Phi=-i\hbar\nabla\Phi \text{ and } \nabla\alpha=q[\vec{A}(\vec{r})-\vec{A}(-\vec{r})]/\hbar c.$$

Under inversion there is a phase shift $\alpha(-\vec{r})-\alpha(\vec{r})=\Delta\alpha$, with

$$\Delta\alpha=q\int_C\vec{A}(\vec{r}')\cdot d\vec{r}'/\hbar c, \quad (5)$$

where C is a closed contour, reflection symmetric about the origin, giving

$$\Delta\alpha=2\pi gq/\hbar c \ (\text{mod}2\pi), \quad (6)$$

since the integral is precisely half the total magnetic flux from g, mod(hc/q).

Equations (3) and (6) together imply for the exchange symmetry of Φ,

$$\Phi(\vec{r},\xi_1,\xi_2)=(-1)^{2s}\Phi(-\vec{r},\xi_2,\xi_1), \quad (7)$$

for any value of $gq/\hbar c$. Since Φ obeys a usual Schrödinger equation, it is the symmetry of Φ which is related in a familiar way to physical phenomena like many-body ground states or allowed spin combinations in 90° cluster-cluster scattering. The use of a similar procedure to obtain the relative wave function of a cluster and an anticluster would lead to the normal charge conjugation properties. Therefore, all results usually found from fermion (boson) field anticommutation (commutation) are true for charge-pole clusters. In terms of physical observables, they obey the usual connection between spin and statistics.

The first part of this Letter shows (for $gq/\hbar c$ half-integer) that the static fields of charge and pole in a given cluster produce an anomalous relation between cluster spin and permutation symmetry of a two-cluster wave function. The second part shows that the long-range interactions of charges with poles in different clusters produce an anomalous relation between wave-function symmetry and quantum numbers corresponding to physical observables. The two anomalies combine in such a way that, indeed, fermions can be made of bosons.

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Configuration Mixing of Two-Quasiparticle States in ^{250}Cf†

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The interaction of a $K^\pi = 5^-$ two-quasiproton band with a $K^\pi = 5^-$ two-quasineutron band in ^{250}Cf has been observed using proton and neutron transfer reactions and radioactive-decay measurements. Configuration-mixing calculations using a two-body neutron-proton force give an interaction matrix element in good agreement with the value derived from our measurements.

In recent years the interaction of two-quasineutron states with two-quasiproton states has been observed[1-5] between high K states in the hafnium region. Massmann *et al.*[6] have developed a theoretical treatment of configuration mixing due to a two-body neutron-proton force which reproduces the experimental mixing quite well. In this Letter we report the observation of this configuration mixing in a different region of deformation and, in particular, between two $K^\pi = 5^-$ bands of ^{250}Cf.

From an initial investigation[7] of the electron capture decay of 8.6-h ^{250}Es, $K^\pi = 5^-$ bands based at 1396 and 1478 keV were identified. Although definite spin-parity assignments were possible in this study,[7] the configurations of these states could not be uniquely determined. A portion of the decay scheme showing primarily the high K states of ^{250}Cf is given in Fig. 1. From single-particle-level systematics the only probable two-quasiparticle configurations for these states are $\{\frac{1}{2}^+[620]n;\frac{9}{2}^-[734]n\}_{5^-}$, $\{\frac{1}{2}^+[631]n;\frac{9}{2}^-[734]n\}_{5^-}$, and $\{\frac{3}{2}^-[521]p;\frac{7}{2}^+[633]p\}_{5^-}$. Since $\frac{9}{2}^-[734]n$ is the ground-state configuration of ^{249}Cf and the $\frac{7}{2}^+[633]$ proton orbital is the ground state of ^{249}Bk, the neutron and proton transfer reactions into the ^{250}Cf final nucleus should aid in delineating the correct Nilsson orbital assignments. The reactions ^{249}Cf$(d,p)^{250}$Cf and ^{249}Bk$(\alpha,t)^{250}$Cf were therefore studied to deduce the configurations of these $K^\pi = 5^-$ bands.

The charged-particle transfer experiments were performed using 12-MeV deuteron and 28-MeV α-particle beams from the Argonne FN tandem Van de Graaff accelerator. Reaction products were analyzed with an Enge split-pole magnetic spectrograph, and the particles were recorded using nuclear emulsion plates. Targets