able $x$. The observed $x$ range corresponds to a missing mass $M$ of the recoil system $X$ of 13 GeV $\leq M \leq 20$ GeV. If the polarization is produced by the interference of two amplitudes $f_1$ and $f_2$, $P_\Lambda = 2\,\mathrm{Im} f_1 f_2^* / (|f_1|^2 + |f_2|^2)$, then it will have a weak $x$ dependence provided that $f_1$ and $f_2$ depend on $x$ in the same way, which cancels in the ratio.

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# Spin from Isospin in a Gauge Theory*

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We show that in an SU(2) quantum gauge field theory, with isospin symmetry broken spontaneously by a triplet of scalar mesons, isospinor degrees of freedom are converted into spin degrees of freedom in the field of a magnetic monopole.

Classical solutions to gauge theories[1] mix spatial and internal symmetry indices; for example an isotriplet vector field $A_a^{\,i}(\vec{r})$ is proportional to $\epsilon_{aij}\hat{r}^j$. We now know that such solutions play a role in the corresponding quantum theory; in a weak-coupling limit they approximate matrix elements of the quantum field between soliton-monopole states.[2] It is natural to ask the following: Does the conjunction of spin and internal symmetry persist in the quantum theory; and, when internal symmetry is spontaneously broken, do internal degrees of freedom reappear as spin degrees of freedom?

An affirmative answer to this question is indicated by our investigation[3] of the interaction of an isospinor, spin-$\frac{1}{2}$ fermion with a monopole, arising in an SU(2) Yang-Mills-Higgs model with spontaneously broken isospin symmetry.[4] We found in the quantum theory a tower of spinless states, describing dyons with magnetic charge $Q_m = \pm 1/e$ and electric charge $Q_e = \pm (N/2)e$ ($e$ is the gauge coupling constant and $N$ is an integer or zero). Moreover, the quantum Fermi field $\Psi$ effects a transition between them, $\langle Q_m, Q_e - e/2| \times \Psi | Q_m, Q_e\rangle \neq 0$. This matrix element is related to the static, zero-energy solution of a $c$-number Dirac equation in the monopole field. The solution $\psi_{\nu n}$ ($\nu$ refers to Dirac indices; $n$ to isospin indices) has vanishing lower components ($\nu = 3$, 4), while the upper components have the form

$$\psi_{\nu n} = f(r)(1/\sqrt{2})(s_\nu^{\,+} s_n^{\,-} - s_\nu^{\,-} s_n^{\,+}), \qquad \nu, n = 1, 2, \tag{1}$$

where $s^+$ ($s^-$) is a Pauli up (down) bispinor, and $f$ is a normalized wave function.

In the transition form factor (1), spin and isospin form an antisymmetric singlet. This explains mathematically the curious circumstance that a spinor field should possess nonvanishing matrix elements between spinless states. Evidently degrees of freedom of the spontaneously broken isospin symmetry survive as spin degrees of freedom, and couple to Dirac spin. For physical clarification, consider the nonvanishing crossed matrix element $\langle Q_m, Q_e - e/2; -Q_m, -Q_e| \times \Psi|0\rangle$, which describes the creation of two spinless dyons by a spin-$\frac{1}{2}$ field. Angular momentum is conserved, since as is well known, the total angular momentum of magnetic systems contains contributions from magnetic monopoles. In the above configuration, the electromagnetic angular momentum has the magnitude $(-Q_m)(Q_e - e/2) - (Q_m)(-Q_e) = Q_m e/2 = \frac{1}{2}$ and provides the requisite half-integer. Thus we suggest that in the soliton-monopole sector of the quantum theory total angular momentum $\vec{J}$ is the sum of conventional $\vec{M}$ (orbital plus spin) and $\vec{I}$ (isospin),

$$\vec{J} = \vec{M} + \vec{I}. \tag{2}$$

In this Letter we exhibit another, more dramatic, example of the conversion of isospin to spin, and derive Eq. (2). We consider a spinless isospinor field $U$, coupling to the usual SU(2) Yang-Mills-Higgs theory, with a classical solution $U = u(r)s$, where $s$ is an arbitrary, position-independent isospinor and $u$ vanishes as $r \to \infty$. The solution is degenerate; the degeneracy is a three-parameter isospin rotation of $s$. In the quantum theory, the multiplicity of classical solutions leads to a tower of monopole-soliton states labeled by a new degree of freedom, interpreted as spin. We confirm our interpretation by introducing collective coordinates to account for the classical degeneracy and by verifying that the canonically conjugate collective variable is indeed an angular momentum $\vec{J}$ which coincides with isospin $\vec{I}$.

The model Lagrangian is

$$\begin{aligned}
&\mathcal{L} = -\tfrac{1}{4} F_a{}^{\mu\nu} F_{a\mu\nu} + \tfrac{1}{2}(D^\mu \Phi)_a (D_\mu \Phi)_a - (1/e^2) V_1(e^2\Phi^2) + (D^\mu U)^\dagger (D_\mu U) - (1/e^2) V_2(e^2 U^\dagger U, e^2\Phi^2);\\
&F_a{}^{\mu\nu} = \partial^\mu A_a{}^\nu - \partial^\nu A_a{}^\mu + e\epsilon_{abc} A_b{}^\mu A_c{}^\nu;\\
&(D^\mu \Phi)_a = \partial^\mu \Phi_a + e\epsilon_{abc} A_b{}^\mu \Phi_c;\\
&(D^\mu U) = \partial^\mu U - ie(\tau^a/2) A_a{}^\mu U;\\
&\Phi^2 = \Phi_a \Phi_a, \quad a = 1, 2, 3;\\
&V_1(\Phi^2) = (\lambda^2/2)[(\mu^2/\lambda^2) - \Phi^2]^2;\\
&V_2(U^\dagger U, \Phi^2) = m^2 U^\dagger U + g^2 (U^\dagger U)^2 - h^2 U^\dagger U[(\mu^2/\lambda^2) - \Phi^2].
\end{aligned} \tag{3}$$

For a range of values of $m^2$, $g^2$, and $h^2$,[5] the solution in the vacuum sector is $\varphi^2 = \mu^2/\lambda^2 e^2$, $U = 0$; while in the soliton-monopole sector one may decrease the energy by a nonvanishing position-dependent $U$. Thus we expect that the following static, degenerate solution exists:

$$\begin{aligned}
&A_a{}^0 = 0, \quad A_a{}^i = \epsilon_{aij} \hat{r}^j A(r),\\
&\Phi_a = \hat{r}^a \varphi(r), \quad U = u(r) \exp[-i\vec{\alpha} \cdot \vec{\tau}/2] s.
\end{aligned} \tag{4}$$

At $r = 0$, $\varphi$ and $A$ vanish, while $u$ is a constant. For large $r$, $\varphi$ and $U$ approach exponentially their vacuum values, while $A$ tends to $-1/er$. The arbitrary triplet $\vec{\alpha}$ parametrizes the degeneracy. The solutions are of order $e^{-1}$, and hence large for weak coupling.

Quantization is performed in the unitary gauge[6] where $\Phi_a$ points in the third isospin direction. Classical solutions in this gauge, obtained from (4) by an isorotation, are

$$\vec{A}_a = \hat{\eta}_a [A(r) + 1/er], \quad a = 1, 2; \tag{5a}$$

$$\vec{A}_3 = \vec{A}_D(\vec{r}); \tag{5b}$$

$$U = u(r) R \exp[-i\vec{\alpha} \cdot \vec{\tau}/2] s; \tag{5c}$$

where $R$ is an isorotation matrix effecting the desired transformation. $\vec{A}_D$ is a Dirac vector potential for a monopole of strength $1/e$: $\nabla \times \vec{A}_D = (1/e)\hat{r}/r^2$, $\nabla \cdot \vec{A}_D = 0$; and $\hat{\eta}_{1,2}$ is a doublet of orthonormal unit vectors, orthogonal to $\hat{r}$. The quantities $R$, $\vec{A}_D$, and $\hat{\eta}_a$ are to a large extent arbitrary, reflecting the remaining electromagnetic gauge freedom; we shall not need their explicit forms.

In the quantum theory we must allow for a multiplicity of monopole-soliton states so that the matrix elements of $U$, which are proportional to (5c), reflect the classical degeneracy parametrized by $\vec{\alpha}$. The spinorial form of (5c) strongly suggests this degree of freedom to be spin; we confirm this by calculating the angular momentum operator $\vec{J}$, and by showing that it generates rotations of $\vec{\alpha}$.

For the Lagrangian (3), $\vec{J}$ is given by

$$\vec{J} = -\int d^3r\, \vec{r} \times [\Pi_a{}^i \nabla A_a{}^i + \Pi_a \nabla \Phi_a + (\Pi_U{}^\dagger \nabla U + \text{H.c.})] - \int d^3r\, \vec{\Pi}_a \times \vec{A}_a, \quad a = 1, 2, 3. \tag{6}$$

The canonical momenta are $\Pi_a{}^i = F_a{}^{0i}$, $\Pi_a = (D^0\Phi)_a$, and $\Pi_U = (D^0 U)$. In the unitary gauge, only $\Phi_3 \equiv \Phi$ and $\Pi_3 \equiv \Pi$ survive. We redefine the charged, massive vector fields by $\vec{A}_a \equiv \vec{W}_a$, $a = 1, 2$; and the massless photon field by $\vec{A}_3 \equiv \vec{A}$, $\vec{\Pi}_3 \equiv \vec{\Pi} = \vec{\Pi}_T + \vec{\Pi}_L$. The longitudinal part $\vec{\Pi}_L$ satisfies a constraint equation:

$$\nabla\cdot\vec{\Pi}_L = -e\epsilon_{ab}\vec{\Pi}_a\cdot\vec{W}_b - e[i\Pi_U{}^\dagger(\tau^3/2)U + \text{H.c.}] \equiv -ej^0, \quad a = 1, 2.$$

An integration by parts in (6) eliminates $\vec{\Pi}_L$ and $\vec{J}$ is expressed by unconstrained variables in the unitary gauge:

$$\vec{J} = -\int d^3r\, \vec{r}\times\{\Pi_a{}^i(\vec{\nabla}\delta_{ab} + e\epsilon_{ab}\vec{A})W_b{}^i + \Pi\nabla\Phi + [\Pi_U{}^\dagger(\nabla + \tfrac{1}{2}ie\tau^3\vec{A})U + \text{H.c.}] + (\vec{\Pi}_T - \nabla\nabla^{-2}ej^0)\times\vec{B}\} - \int d^3r\, \vec{\Pi}_a\times\vec{W}_a;$$
$$\vec{B} = \nabla\times\vec{A}, \quad \nabla\cdot\vec{A} = 0, \quad a = 1, 2. \tag{7}$$

To calculate $\vec{J}$ in the soliton-monopole sector, the quantum fields are shifted by the classical solutions, with all degeneracy parameters promoted to collective coordinates, i.e., time-dependent quantum operators.[7] We ignore the coordinates relevant to translations and charge rotations, and concentrate on the three-parameter degeneracy of (5c). To lowest order in the coupling constant, we need keep only the large, classical solutions. Thus we evaluate (7) with[8]

$$\begin{aligned} &\Phi = \varphi(r), \quad \Pi = 0; \\ &\vec{W}_a = \hat{\eta}_a[A(r) + 1/er], \quad \vec{\Pi}_a = 0, \quad a = 1, 2; \\ &\vec{A} = \vec{A}_D(\vec{r}), \quad \vec{\Pi}_T = 0; \\ &U = u(r)R\exp[-i\vec{\alpha}(t)\cdot\vec{\tau}/2]s \equiv RU(r,t); \\ &\Pi_U{}^\dagger = \dot{U}^\dagger(r,t)R^\dagger \equiv \Pi_U{}^\dagger(r,t)R^\dagger. \end{aligned} \tag{8}$$

$\Pi_U$ is nonvanishing since $U$ acquires time dependence from its collective coordinate $\vec{\alpha}(t)$. The other fields, not containing collective coordinates, are time independent and their conjugate momenta vanish. Now $\vec{J}$ becomes

$$\begin{aligned} \vec{J} &= -\int d^3r\, \vec{r}\times\Pi_U{}^\dagger(r,t)R^\dagger[\nabla + ie(\tau^3/2)\vec{A}_D(\vec{r})]RU(r,t) \\ &\quad - \int d^3r\, \vec{r}\times\{[\nabla\int d^3r'\,(4\pi)^{-1}|\vec{r}-\vec{r}'|^{-1} ie\Pi_U{}^\dagger(r',t)R^\dagger(\tau^3/2)RU(r',t)]\times\hat{r}/er^2\} + \text{H.c.} \\ &= -\int d^3r\, \vec{r}\times\Pi_U{}^\dagger(r,t)R^\dagger[\nabla + ie(\tau^3/2)\vec{A}_D(\vec{r})]RU(r,t) - \int d^3r\, \hat{r}\,\Pi_U{}^\dagger(r,t)R^\dagger(i\tau^3/2)RU(r,t) + \text{H.c.} \end{aligned} \tag{9a}$$

The rotation indicated by $R$ is evaluated by comparison with (4) and (5)—if we set $A(r) = -1/er$ in those equations, we recognize that (9a) is equal to

$$\begin{aligned} \vec{J} &= -\int d^3r\, \vec{r}\times\Pi_U{}^\dagger(r,t)[\nabla + ie(\vec{\tau}/2)\times\hat{r}/er]U(r,t) - \int d^3r\, \hat{r}\,\Pi_U{}^\dagger(r,t)i(\vec{\tau}\cdot\hat{r}/2)U(r,t) + \text{H.c.} \\ &= \int d^3r\, \Pi_U{}^\dagger(r,t)[-\vec{r}\times\nabla - i\vec{\tau}/2]U(r,t) + \text{H.c.} \end{aligned} \tag{9b}$$

The first term in the square brackets is the orbital angular momentum of $U$; for spherically symmetric fields, as in (8), it vanishes. The remainder is exactly the isospin generator $\vec{I}$. When terms that we have ignored are kept, $\vec{J}$ will of course also acquire conventional orbital and spin contributions. Thus we arrive at the promised derivation of (2).

For $\vec{\alpha}(t)$ pointing in a fixed direction $\hat{n}$, we have

$$U = u(r)R\exp[-i\alpha(t)\tau^n/2]s,$$
$$\dot{U} = -u(r)Ri\dot{\alpha}(t)(\tau^n/2)\exp[-i\alpha(t)\tau^n/2]s,$$
$$L = \int d^3r\, \mathcal{L} = \tfrac{1}{4}\dot{\alpha}^2(t)\int d^3r\, u^2(r) + \dots.$$

The momentum conjugate to $\alpha$ is $(\dot{\alpha}/2)\int d^3r\, u^2(r)$, which also is $I^n = J^n$. What appears to be an isorotation is in fact a spin rotation—in the quantum theory spin has been created from isospin![9]

To go beyond the details of our example, it is clear that spin will always emerge from isospin whenever there is an isospin-degenerate, classical solution in the field of a monopole. We have not carried out the construction of all the Lorentz generators in the monopole sector and verified all the commutation relations. This analysis, which we reserve for future investigation, is complicated since perturbative quantization of solitons always begins with a nonrelativistic approximation. However there should be no doubt that the spin which we have found is the correct relativistic quantity as we encountered it in $\int d^3r\,(x^i\theta^{0j} - x^j\theta^{0i})$. This is the proper Lorentz angular momentum and formally, as well as in the vacuum sector, satisfies, together with $\int d^3r \times x^i\theta^{00}$, the Lorentz algebra. Moreover Goldhaber has recently shown that our half-integer-spin dyons obey Fermi-Dirac statistics.[10]

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# Fermion-Boson Puzzle in a Gauge Theory*

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It is argued that magnetic monopoles in an SU(2) gauge theory may bind with an ordinary boson with isospin, to give bound states with spin. If the isospin of the free boson is integer or half-odd-integer, the total angular mometum of the bound state is integer or half-odd-integer, respectively. According to the spin-statistics theorem we can obtain fermions this way in a theory that started off with bosons only.

Recently it was shown by Coleman[1] that in a two-dimensional boson theory (the massive sine-Gordon theory) "solitons" occur that actually obey Fermi statistics. We present here a theory in four space-time dimensions where fermions can arise in a similar manner.

Let us consider the Lagrangian[2]

$$\mathcal{L} = -\tfrac{1}{4}G_{\mu\nu}{}^a G_{\mu\nu}{}^a - \tfrac{1}{2}(D_\mu Q^a)^2 - \tfrac{1}{8}\lambda(Q^2 - F^2)^2, \tag{1}$$

describing neutral massless photons, a massive charged vector boson, and a neutral Higgs boson. Here, $\lambda$ and $F$ are parameters, and

$$\begin{aligned} G_{\mu\nu}{}^a &= \partial_\mu A_\nu{}^a - \partial_\nu A_\mu{}^a + e\epsilon_{abc}A_\mu{}^b A_\nu{}^c, \\ D_\mu Q^a &= \partial_\mu Q^a + e\epsilon_{abc}A_\mu{}^b Q^c. \end{aligned} \tag{2}$$

This model has a magnetic monopole soliton, described by a classical solution of the form[3]

$$\begin{aligned} &(Q^a)^{\mathrm{cl}} = r_a Q(r), \quad (A_i{}^a)^{\mathrm{cl}} = \epsilon_{ial} r_l W(r), \\ &(A_0{}^a)^{\mathrm{cl}} = 0, \quad a = 1,2,3 \end{aligned} \tag{3}$$