

relevant for the collapsed object (neutron star?). I have argued that the coherent reaction (vi) does not degrade the escaping neutrinos, even though the scattering cross section for $\nu p \rightarrow \nu p$ and $\nu n \rightarrow \nu n$ in the core is large (because of spin- $\frac{1}{2}$ and isospin- $\frac{1}{2}$ effects, it is some 13–14 times larger than what one obtains by simply putting $A = 1$ in Freedman's formula for $S=I=0$; see Bernabéu¹³ and Tubbs and Schramm¹³. R.H. ν 's may scatter even more strongly. While the ν 's will still get out with little energy loss (though some may be trapped), if their interaction with the core is strong enough, they can carry away angular momentum. What effect this may have on the envelope and in particular on the rotation of the subsequent neutron star is a question to be investigated elsewhere.

¹D. Z. Freedman, Phys. Rev. D **5**, 1389 (1974).

²J. R. Wilson, Phys. Rev. Lett. **32**, 849 (1974).

³S. Weinberg, Phys. Rev. Lett. **19**, 1264 (1967); A. Salam, in *Elementary Particle Theory: Relativistic Groups and Analyticity* (Nobel Symposium No. 8), edit-

ed by N. Svartholm (Almqvist and Wiksell, Stockholm, 1968), p. 367.

⁴J. R. Wilson, in Proceedings of the International School of Physics "Enrico Fermi," Varenna, Italy, 1975 (to be published).

⁵M. L. Perl *et al.*, Phys. Rev. Lett. **35**, 1489 (1975).

⁶R. Cowsik and J. M. McClelland, Phys. Rev. Lett. **29**, 669 (1972).

⁷J. B. Adams, M. A. Ruderman, and C. H. Woo [Phys. Rev. **129**, 1383 (1963)] have shown that the plasma decay proceeds mostly through the vector current.

⁸See, e.g., G. Beaudet, V. Petrosian, and E. Salpeter, Astrophys. J. **150**, 979 (1967).

⁹D. A. Dicus, Phys. Rev. D **6**, 941 (1972).

¹⁰K. O. Mikaelian, Phys. Lett. **57B**, 181 (1975), and Astrophys. J. **200**, 336 (1975).

¹¹R. B. Stothers, Phys. Rev. Lett. **24**, 538 (1970).

¹²I have not considered S , P , and T interactions which can flip the helicity of the incoming ν 's. This is discussed by B. Kayser, G. T. Garvey, E. Fischbach, and S. P. Rosen, Phys. Lett. **52B**, 385 (1974). Experiment [F. J. Sciulli, in *Particles and Fields, 1975*, edited by H. Lubatti and P. M. Mockett (Univ. of Washington, Seattle, Wash., 1975)] already rules out any substantial scalar or pseudoscalar terms in the neutral current.

¹³J. Bernabéu, CERN Report No. TH. 2073-CERN (to be published); D. L. Tubbs and D. N. Schramm, Astrophys. J. **201**, 467 (1975).

COMMENTS

Droplet Model for the Viscosity of Fluids near the Critical Point

David W. Oxtoby* and Horia Metiu†

The James Franck Institute, The University of Chicago, Chicago, Illinois 60637

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We generalize the droplet model introduced by Ackerson *et al.* to make it consistent with scaling laws for the free energy. We then use the model to predict a logarithmic divergence of the shear viscosity, in agreement with mode-coupling theories and experiment. We also study non-Newtonian (finite-shear-gradient) effects on the viscosity.

Recently Ackerson *et al.*¹ presented a simple droplet model for the Rayleigh light scattering spectrum of fluids near the critical point. They took a droplet size distribution of the form

$$N(l) = B \exp(-l^2/2\xi^2)/l^{5+\eta}, \quad (1)$$

where B varies slowly with temperature near the

critical point; ξ is the correlation length, and η is the exponent introduced by Fisher² to account for deviations from Ornstein-Zernike theory. It is defined by the asymptotic form of the pair distribution function:

$$G(r) \sim r^{-(d-2+\eta)} \text{ at } T = T_c. \quad (2)$$

The second assumption made by Ackerson *et al.* is that the density profile of a droplet of size l is given by

$$n(r) = A \exp(-r^2/l^2). \quad (3)$$

From these two assumptions [Eqs. (1) and (3)], Ackerson *et al.* were able to calculate both the total light scattering intensity and the Rayleigh linewidth, and found good agreement with experiment.¹

We show here that a simple generalization of Eqs. (1) and (3) is necessary to treat in a consistent fashion the free energy of the system and the higher-order cumulants of the density. With this generalization, the good agreement for total intensity and Rayleigh linewidth is retained, and in addition the behavior of the shear viscosity near the critical point may be predicted from the model.

We consider the droplets to be fluctuations about the mean density, rather than fluctuations in the vacuum. Thus there will be equal numbers of droplets and "bubbles," corresponding respectively to regions with densities higher or lower than the mean density. The analog for binary mixtures is droplets rich in one or the other mixture component, while for magnetic systems droplets correspond to regions with a net excess of up or down spins. For simplicity we will deal with the critical isochore, $n = n_c$, although the theory may be applied easily to other regions in the phase plane.

We generalize Eqs. (1) and (3) by assuming instead that $N(l)$ is a homogeneous function³ of l and ξ :

$$N(l) = Bh(l/\xi)/l^y, \quad (4)$$

while $n(r) - n_c$ is a homogeneous function of l and r :

$$n(r) - n_c = \pm Af(r/l)/l^x. \quad (5)$$

We require that the functions h and f fall rapidly to zero when their argument is larger than 1. A and B are constants. The plus and minus signs in Eq. (5) refer, respectively, to droplets and bubbles, which should be present in equal proportions. A positive value for x allows small droplets to be more than a scaled version of the large ones. The model of Ackerson *et al.* corresponds to the assumptions that h and f are Gaussian and that $x=0$ while $y=5+\eta$.

We now assume that the droplets contribute independently to the total free energy of the sys-

tem, so we may write

$$F_{\text{tot}} = \int_0^\infty dl N(l) F_l, \quad (6)$$

where F_l is the free energy of a droplet of size l . For simplicity, we look first at the magnetic case, although the situation for fluids is completely analogous. F_l depends on the magnetic field H only through the coupling of the excess up (or down) spins to H . The number of excess spins corresponds to

$$\int d^d r [n(\vec{r}) - n_c] \propto l^{d-x}, \quad (7)$$

where d is the dimensionality of the system. Thus we have

$$F_l = F_l(Hl^{d-x}), \quad (8)$$

$$\begin{aligned} F_{\text{tot}} &= B \int_0^\infty dl l^{-y} h(l/\xi) F_l(Hl^{d-x}) \\ &= B \xi^{-y+1} F(H\xi^{d-x}), \end{aligned} \quad (9)$$

where

$$F(H\xi^{d-x}) \equiv \int_0^\infty dz z^{-y} h(z) F_l(H\xi^{d-x} z^{d-x}). \quad (10)$$

However, the known equation of state has the form^{3,4}

$$F_{\text{tot}} = C \epsilon^{2-\alpha} F(H/\epsilon^\Delta), \quad (11)$$

where $\epsilon \equiv (T - T_c)/T_c$, α is the exponent for the specific heat C_H , and Δ , the gap exponent, is given by $\beta + \gamma$, where β is the exponent describing the shape of the coexistence curve, and γ is the exponent for the isothermal compressibility. If we compare Eqs. (9) and (11) and use scaling laws we find

$$\begin{aligned} y &= d + 1, \\ x &= \frac{1}{2}(d - 2 + \eta). \end{aligned} \quad (12)$$

This method of choosing exponents by requiring the free energy to have the proper scaling form is similar to the approach used by Binder, Stauffer, and Müller-Krumbhaar,⁴ in studying a different kind of droplet model for the kinetic Ising lattice.

With Eq. (12), Eqs. (4) and (5) may now be used to study the static and dynamic properties of the system. The pair distribution function depends only on the combination

$$y + 2x = 2d - 1 + \eta,$$

a relationship which is satisfied by the choice made by Ackerson *et al.*,¹ $y = 5 + \eta$; $x = 0$ for $d = 3$. With the choices made in Eq. (12) the structure factor has the scaling form^{3,5}

$$I(k) \propto k^{-2+\eta} \hat{D}(k\xi). \quad (12a)$$

If h and f are chosen to be Gaussian, as Acker-
son *et al.* assume, one obtains one of the modi-
fied Ornstein-Zernike forms for $I(k)$ suggested
by Fisher and Burford^{5a}

$$I(k) \propto (k^2 + \xi^{-2})^{-1 + \eta/2}. \quad (12b)$$

Renormalization-group calculations^{5b} show, how-
ever, that $\hat{D}(x)$ must be more complicated than
the form implied by Eq. (12b). The results of
Ref. 1 for the Rayleigh linewidth are unchanged.
A difference between our work and theirs arises
though, if one looks at higher-order cumulants
of the density. For distances r_{ij} , all of order ξ ,
the relationship^{5c}

$$\langle \Delta n(\vec{r}_1) \dots \Delta n(\vec{r}_{2m}) \rangle_c \sim [\langle \Delta n(\vec{r}_1) \Delta n(\vec{r}_2) \rangle]^m \quad (13)$$

is satisfied only by the choice of x and y in Eq.
(12).

We turn now to the most important new result
of the present paper: the determination of the
shear viscosity near T_c . Suppose the background
viscosity (in the absence of droplets) is $\bar{\eta}_b$. The
viscosity $\bar{\eta}$ of a dilute suspension of droplets has
been known for some time⁶; it is given by

$$\bar{\eta} = \bar{\eta}_b + C\bar{\eta}_b \varphi, \quad (14)$$

where C is a dimensionless constant of order uni-
ty and φ is the volume fraction of the droplets.
For our model

$$\begin{aligned} \varphi &= \int_0^\infty dl N(l) l^d \\ &\propto \int_0^\infty dl h(l/\xi)/l \\ &\propto \ln k_c \xi, \end{aligned} \quad (15)$$

where k_c is a cutoff preventing diverging contri-
butions from very *small* droplet sizes l . This
gives

$$(\bar{\eta} - \bar{\eta}_b)/\bar{\eta}_b \sim \ln k_c \xi. \quad (16)$$

This logarithmic divergence is also predicted by
the zeroth-order form of mode-coupling theory⁷
and is in good agreement with experiment.⁸ In
the extreme critical limit φ will become so large
that interactions between droplets can no longer
be neglected. In this limit our simple model will
certainly break down, and one must use other
methods, such as those based on the renormal-
ization group. We note that the most recent re-
normalization-group calculations⁹ suggest a very
weakly diverging shear viscosity, with an expo-
nent of about 0.04 in three dimensions; such a
weak power-law dependence will be extremely
difficult to differentiate from a logarithmic di-
vergence. In order to obtain the coefficient of

the logarithmic term in Eq. (16) (given by $\frac{8}{15} \pi^2$ in
the mode-coupling theory), one would need to
know the value of the constant B in Eq. (4); this
would require going beyond the postulate of homo-
geneous functional forms of Eqs. (4) and (5) to a
derivation of these equations and the constants
appearing in them.

Finally, we discuss the non-Newtonian shear
viscosity, that is, the effect on measurements of
 $\bar{\eta}$ arising from a finite shear gradient D . A di-
mensionless parameter enters into the droplet
suspension theory⁶:

$$k^{-1} \equiv \bar{\eta}_b D l / \sigma, \quad (17)$$

where l is the size and σ is the surface tension of
the droplet. When D is large enough so that k^{-1}
is of order unity, the droplet surface tension is
no longer strong enough to prevent the tearing
apart of the droplet by the shearing forces. Since
 $\sigma \sim k_B T / l^2$, we have

$$k^{-1} \sim \bar{\eta}_b D l^3 / k_B T. \quad (18)$$

Thus the only droplets that survive to contribute
to the increment in shear viscosity are those for
which

$$l \lesssim (k_B T / \bar{\eta}_b D)^{1/3}. \quad (19)$$

As a result, the upper limit to the integral in Eq.
(15) should be of order $(k_B T / \bar{\eta}_b D)^{1/3}$. For ξ
 $\ll (k_B T / \bar{\eta}_b D)^{1/3}$ the value of the integral will be
unaffected, but when

$$\xi \sim (k_B T / \bar{\eta}_b D)^{1/3},$$

the shear viscosity will level off to a constant val-
ue, and the logarithmic divergence will stop.
This is the same behavior predicted by one of us¹⁰
using mode-coupling theory; the same dimension-
less quantity $k_B T / \bar{\eta}_b D \xi^3$ appears in that theory
and determines the temperature at which the log-
arithmic divergence will level off to a constant.

The dynamic droplet model, simple as it is,
appears to give rather accurate predictions for
a wide variety of problems, and therefore de-
serves further study. In particular, we plan to
use the model to investigate the bulk viscosity
and sound attenuation near the critical point.

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¹B. J. Ackerson, C. M. Sorensen, R. C. Mockler, and W. J. O'Sullivan, *Phys. Rev. Lett.* **34**, 1371 (1975).

²M. E. Fisher, *J. Math. Phys. (N.Y.)* **5**, 944 (1964).

³H. E. Stanley, *Introduction to Phase Transitions and Critical Phenomena* (Oxford Univ. Press, New York, 1971), Chap. 11.

⁴K. Binder, D. Stauffer, and H. Müller-Krumbhaar, *Phys. Rev. B* **12**, 5261 (1975).

^{5a}M. E. Fisher and R. J. Burford, *Phys. Rev.* **156**, 583 (1967).

^{5b}M. E. Fisher and A. Aharony, *Phys. Rev. B* **10**, 2818 (1974).

^{5c}L. P. Kadanoff, in *Critical Phenomena, Proceedings of the International School of Physics "Enrico Fermi," Course LI*, edited by M. S. Green (Academic, New

York, 1971), p. 118.

⁶A. Einstein, *Investigations on the Theory of Brownian Motion* (Dover, New York, 1956); for a survey of recent work, see S. J. Choi and W. R. Schowalter, *Phys. Fluids* **18**, 420 (1975).

⁷K. Kawasaki, in *Critical Phenomena, Proceedings of the International School of Physics "Enrico Fermi," Course LI*, edited by M. S. Green (Academic, New York, 1971), p. 342; D. W. Oxtoby and W. M. Gelbart, *J. Chem. Phys.* **61**, 2957 (1974).

⁸See, for example, H. L. Swinney and D. L. Henry, *Phys. Rev. A* **8**, 2586 (1973).

⁹E. D. Siggia, B. I. Halperin, and P. C. Hohenberg, *Phys. Rev. B* **13**, 2110 (1976).

¹⁰D. W. Oxtoby, *J. Chem. Phys.* **62**, 1463 (1975).

Comment on Selection-Rule Effects in Electron-Energy-Loss Spectroscopy of Ge and GaAs Surfaces

J. L. Freeouf

IBM Thomas J. Watson Research Center, Yorktown Heights, New York 10598

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Recent energy-loss and photoemission-partial-yield studies of (spin-orbit-split) d -core to surface-state transitions imply relative transition probabilities for the $d_{5/2}$ and $d_{3/2}$ components differing from their orbital degeneracies. This has previously been interpreted as implying s character for the unoccupied surface states of GaAs(110). I present an alternative explanation for this effect assuming only p character for these states and a small exchange interaction.

Recently electron-energy-loss-spectroscopy¹⁻³ (ELS) and photoelectron-yield-spectroscopy⁴⁻⁷ (PYS) techniques have been used to probe unoccupied surface states on semiconductor surfaces. The main content of most publications thus far has been to establish transition energies and from these to infer one-electron energies and a density of states for the unoccupied levels. However, recent publications have attempted to go beyond this and study the nature of these states. Thus far it has been established that in III-V semiconductors these states are localized about the cation^{2,4,6} and depend upon the crystal face,^{1,7} that a strong decay mode for these d -core to surface-state excitations is direct recombination,⁷ that these transitions are dependent upon the polarization of exciting photons,⁷ and that in some instances a metallic overlayer does not appear to perturb these states.^{4,6} An attempt to determine the orbital nature of these states (based on the relative transition probabilities for the $d_{5/2}$ and $d_{3/2}$ to surface-state excitations studied by ELS) recently appeared,¹ and it is this question that I wish to address. In particular, I shall offer an alternative explanation of the observed intensity

ratios which has very different implications as to the orbital character of the surface states. To properly define both the data and the interpretations I shall first discuss in greater detail the experimental techniques employed and the transitions studied.

Both ELS and PYS study the probability (as a function of excitation energy) for exciting a core $3d$ electron into an unoccupied surface or bulk state. In ELS the excitation energy is provided by an electron, whereas in PYS the transition is excited by a photon. In the photoexcitation case, dipole selection rules are valid. This has two effects: (1) There must be a one-electron dipole matrix element coupling the core d electron to the surface state, i.e., the method will probe only that portion of the surface state having p - or f -orbital character with respect to the origin of the d core level (as well as nonzero overlap with the d core level), and (2) since the initial state is d^{10} (1S), the final (ten-electron state) must be d^9 (surface state)¹ (1P). We shall neglect any partial- f -wave character of the surface state. ELS will satisfy these (dipole) selection rules at a sufficiently large primary electron energy, E_p ,