As the amplitudes of the electron-plasma waves grow they scatter the beam electrons causing the nonlinear instability to saturate and finally quench.

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[6]These high-frequency signals are detected by a small double probe which couples to the external circuit through a transformer. By checking the directivity and sensitivity we confirmed that the probe detects the electric field. The resonant decoupling which may occur at the plasma frequency is suppressed by choosing a small probe-wire size comparable to the Debye length.

[7]A simple geometric analysis of the dispersion relations indicates that all electron-plasma waves distributed in k space participate in excitation of the parametric instability only when k_a is along the beam and equals $2\omega_0/u$. Therefore, the observed ion-acoustic wave is the most unstable mode.

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Turbulent Lifetimes in Mirror Machines*

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A quasilinear model is developed describing the time evolution of a mirror-confined plasma which is unstable to the drift-cyclotron loss-cone mode. Good correlation is obtained with experiments with and without an external stream of cold plasma.

Linear stability analyses of mirror machines with loss-cone ion distributions indicate that current high-density machines are unstable to the drift-cyclotron loss-cone (DCLC) mode with broadband frequency and wave-number spectra.[1,2] With the assumption that the ions are lost in a transit time after scattering into the loss cone, a previous quasilinear treatment predicted the trapped-plasma lifetime to be a few axial bounce periods.[3] This picture has seemingly been contradicted by experiments, as narrow-band spectra and containment times of several hundred bounce periods have been observed.[4,5] Added to this, when a cold-plasma stream is injected, the containment time increases several fold.[6,7]

In this note we show that with refinement the quasilinear picture is indeed consistent with experiment and provides a unified description both with and without plasma stream. The essential modification from past analysis is to observe that mirror plasmas do not necessarily fill the entire containment region of phase space available to them, but can be peaked at pitch angles nearly perpendicular to the magnetic field. Confinement is then improved, both because the unconfined phase-space region is given by $v_\perp^2 < (2q\Phi/M_i + v_\parallel^2)/(R-1)$, and so is reduced if $v_\parallel^2 \lesssim 2q\Phi M_i^{-1}$ (Φ is the ambipolar potential and R the mirror ratio), and because untrapped particles of small $v_\parallel$ at the midplane have a longer transit time. The peaked distribution is maintained by the dominant velocity-space transport processes: electron drag which alters only the particle speed, and turbulent diffusion which affects primarily the perpendicular velocity $v_\perp$. In such cases the scattering time of ion-ion collisions, leading to pitch-angle broadening, is much longer than the first two transport processes, hence ion scattering does not alter the sharply peaked nature of the distribution. If the plasma initially has a broad pitch-angle distribution, turbulence will rapidly shed the larger-pitch-angle particles, so that the plasma evolves to a narrow pitch-angle distribution for which improved lifetimes are obtained.

The rate at which the plasma fills the phase-space hole can be described by a quasilinear model discussed below. The diffusion coefficient in the model, arising from fluctuations of the unstable plasma, grows to a level that causes the plasma to partially fill the uncontained phase-space region, driving the plasma to marginal stability in such a way that the transit-time loss of the uncontained plasma is balanced by the turbulent diffusion.

Many of the physical parameters observed in the 2XIIB mirror experiment correlate closely with the results of the quasilinear calculation. This consistency of theory with present experiment gives credibility to the predictions of the linear theory for larger machines. The principal conclusion is that at high β and for sufficiently large radial scale lengths, mirror machines will be stable to the DCLC mode; while at somewhat smaller scale lengths the loss rate due to instability can be made sufficiently small that buildup is feasible. In the following, we describe the theory as it applies to the 2XIIB regime and indicate some of its implications for larger machines.

As we expect the plasma to hover about marginal stability, we first estimate from linear theory the density and temperature of unconfined plasma required for marginal stability. We use a model distribution

$$F(v_\perp)=\frac{1}{v_H^{\,2}-v_h^{\,2}}\left[\exp\left(-\frac{v_\perp^{\,2}}{v_H^{\,2}}\right)-\exp\left(-\frac{v_\perp^{\,2}}{v_h^{\,2}}\right)\right]+\frac{\Delta}{v_W^{\,2}}\exp\left(-\frac{v_\perp^{\,2}}{v_W^{\,2}}\right), \tag{1}$$

where v_H and v_W are the hot- and warm-plasma thermal velocities, $v_h \sim (2q\Phi/M_i)^{1/2}$ is the parameter fixing the size of the hole in the hot-ion distribution, and Φ is the ambipolar potential. We seek the minimum warm- to hot-plasma density ratio Δ required for stability. The dispersion relation for a $k_\parallel = 0$ mode with $\beta < 1$ is[2]

$$1+\frac{\omega_{pe}^{\,2}}{\omega_{ce}^{\,2}}+\frac{\omega_{pe}^{\,4}}{\omega_{ce}^{\,2}k_\perp^{\,2}c^2}-\frac{\omega_{pi}^{\,2}}{\omega\omega_{ci}k_\perp R_p}+\frac{\omega_{pi}^{\,2}}{k_\perp^{\,2}}F(v_\perp=0)$$
$$+\frac{\omega_{pi}^{\,2}}{k_\perp^{\,2}}\sum\frac{\omega}{\omega-n\omega_{ci}}\int_0^\infty dv_\perp v_\perp J_n^{\,2}\left(\frac{k_\perp v_\perp}{\omega_{ci}}\right)\left(2\frac{\partial F}{\partial v_\perp^{\,2}}+\frac{\omega_H^{\,*}F}{\omega v_H^{\,2}}\right)=0, \tag{2}$$

where $\omega_H^{\,*}=k_\perp v_H^{\,2}/2R_p\omega_{ci}$, R_p is the local radial scale length, and other symbols have their familiar meanings. This equation has been analyzed in two ways that yield similar results; in a straight-line orbit approximation[8] and by a detailed numerical study of Eq. (2) by one of us (L.D.P.). In describing the results, we introduce the parameter $\epsilon = v_H/\omega_{ci}R_p$ and define $\beta=\omega_{pi}^{\,2}v_H^{\,2}/\omega_{ci}^{\,2}c^2$. For $\epsilon \leqslant 2\beta^{1/2}(m_e/m_i+\omega_{ci}^{\,2}/\omega_{pi}^{\,2})^{1/2}$ and $\beta > (m_e/m_i+\omega_{ci}^{\,2}/\omega_{pi}^{\,2})^{1/3}$, Eq. (2) has previously been shown to be stable to DCLC.[9] For larger ϵ, the marginal stability divides into three regions:

Region	Range	$\omega/(k_\perp v_h)$
I	$2\beta^{1/2}\left(\frac{m_e}{m_i}+\frac{\omega_{ci}^{\,2}}{\omega_{pi}^{\,2}}\right)^{1/2} < \epsilon \lesssim \beta^2\frac{v_h}{v_H}$	$\epsilon\frac{v_H}{v_h}$
II	$\beta^2 v_h/v_H \lesssim \epsilon \lesssim v_h/v_H$	$(\epsilon v_H/v_h)^{1/2}$
III	$v_h/v_H \lesssim \epsilon$	1

where $\omega/k_\perp$ is the phase velocity at marginal stability. The warm component is most effective when $v_W=\omega/k_\perp$, in which case the minimum density ratios for each region are given by

$$\Delta=\alpha\omega^3/k_\perp^{\,3}v_h v_H^{\,2}$$
$$\approx\begin{cases}2v_H/v_h(\epsilon/\beta)^3 \text{ in I}\\ 2(v_h/v_H)^{1/2}\epsilon^{3/2} \text{ in II}\\ \tfrac{1}{2}(v_h/v_H)^2 \text{ in III,}\end{cases} \tag{3}$$

where α is a constant of order unity. Note that the reduced Δ's at small ϵ are realized because the phase velocity of the unstable wave decreases as ϵ^{-1} increases, so that a smaller volume of phase space must be filled. If we use the precise dispersion relation given by Eq. (2), we find that the modes become resonant at harmonics, and a rough stability condition in III is $Q_n \equiv \int dv_\perp J_n^{\,2}(k_\perp \times v_\perp/\omega_{ci})\partial F/\partial v_\perp < 0$, generally with $n=1$ the most demanding.

All current mirror machines lie in region III; 2XIIB and PR-6 and PR-7 have v_H/v_h equal to 5 and 3.5, respectively. The spectra at marginal stability in region III at 2XIIB parameters are precisely those observed; $\omega \sim \omega_H^* \lesssim \omega_{ci}$ and $k_\perp v_H/\omega_{ci} = 4$ to 5, with wave propagation in the ion diamagnetic direction.[10]

The system is maintained at marginal stability by fluctuations that we describe by qualilinear theory. Further, to complete the description of the time evolution of the distribution it is necessary to include electron drag, the neutral beam input, and the plasma stream. Also, because the distribution is assumed highly peaked in pitch angle and we are considering $k_\parallel = 0$ modes, we may integrate the distribution over $v_\parallel$ and use an equation for the evolution of $F(v_\perp, t)$,

$$\frac{\partial F}{\partial t} = \frac{1}{v_\perp}\frac{\partial}{\partial v_\perp} v_\perp\left[2v_\perp\frac{F}{\tau_d} + D\frac{\partial F}{\partial v_\perp}\right] - \left[\int_0^\infty dv_\perp v_\perp S_{\rm cx} + \nu_L(v_\perp)\right]F + n\left[S_i(v_\perp) + S_{\rm cx}(v_\perp)\right] + S_0(v_\perp). \tag{4}$$

In Eq. (4) τ_d is the electron drag time, $n\tau_d = 1.5\times10^{12}[T_e(\text{keV})]^{3/2}$, $D(v_\perp, t)$ is the velocity-space diffusion resulting from fluctuations, S_i and $S_{\rm cx}$ are respectively the sources of particles due to ionization and charge exchange of a neutral beam, S_0 is the source of low-energy (stream) particles, and $n = \int dv_\perp v_\perp F$. In the loss rate ν_L are lumped all the processes of the higher dimensionality of the physical problem. Thus, ν_L is approximately the inverse transit time for untrapped particles and may be used to model certain aspects of pitch-angle scattering for trapped particles. As a model we take $\nu_L = \tau_{\rm tr}^{-1}\theta(v_h^2 - v_\perp^2)$ where $v_h^2 = q\Phi/M_i = \alpha T_e/M_i$ (α is ~ 2.5), $\tau_{\rm tr} \sim L_p/v_h$ with L_p the plasma axial length, and θ is a step function.

From rather general quasilinear considerations, the diffusion coefficient due to fluctuating potentials $\tilde\varphi_k$ of spectral width $\Delta\omega_k$ is given by[11]

$$D(v_\perp) = \frac{\pi q^2}{M_i^2 v_\perp^2}\sum_n n^2\omega_{ci}^2\int\frac{d^2k}{(2\pi)^2}|\tilde\varphi_k|^2 J_n^2\left(\frac{k_\perp v_\perp}{\omega_{ci}}\right)\frac{\Delta\omega_k}{(\omega_k - n\omega_{ci})^2 + \Delta\omega_k^2}. \tag{5}$$

Since the fundamental is generally the hardest to stabilize, we expedite the quasilinear calculation by taking $\omega = \omega_{ci}$ and assume the growth rate of $\tilde\varphi_k$ is proportional to Q_1 defined earlier. Because unstable waves have $\omega/k_\perp \lesssim v_h$, it follows that for $v_\perp > v_h$ and noise at $\omega \approx \omega_{ci}$,

$$D(v_\perp) \sim D(v_h)(v_h/v_\perp)^3, \tag{6}$$

so that the ratio of scattering times for hot particles to that for hole particles is of order $(v_H/v_h)^5$.

We model the electron temperature by the equation

$$d(\tfrac{3}{2}nT_e)/dt = \tau_d^{-1}n_H T_H - \eta T_e J, \tag{7}$$

where $J = \nu_L n_W = \nu_L\int_0^{v_h} dv_\perp v_\perp F$ is the flux of lost particles, and $\eta T_e J$ is the electron energy drain due to end loss. Fokker-Planck studies yield $\eta = 5.5$. However, additional cooling effects, such as secondary wall emission, can increase η. Typical 2XII operating conditions have[5] $\eta = 8$, which is used in this calculation.

Ions enter the region $v_\perp < v_h$ by drag and diffusion from high energy, or by direct injection at $v_\perp = 0$ if $S_0 \neq 0$. They will be thermalized to the energy v_h^2 in a transit time provided $D(v_h) \gtrsim v_h^3/L_p$. The actual diffusion level and therefore particle lifetime will depend markedly on whether or not there is a source of particles S_0. The calculation above showed that marginal stability (or saturation of unstable growth) in region III, where the present experiment lies, requires a minimum Δ with $v_W \sim v_h$ having a loss rate ν_L. When $S_0 = 0$, these particles must come from the hot population, giving for it a loss rate $\sim \nu_L\Delta$. For the implied diffusion $D(v_H) \sim \nu_L\Delta v_H^2$ the steady state of Eq. (4) is one of higher temperature, so that hardening of the distribution results. With an adequate $S_0 \neq 0$, the hot particles need only supply the power loss of the warm component $\sim n_W v_h^2\nu_L$. A hot particle loss rate $\sim \nu_L\Delta v_h^2/v_H^2$ resulting from a diffusion $D(v_H) \sim \nu_L\Delta v_h^2$ supplies the necessary power. In this case, the diffusion rate is of order or less than the electron drag rate, so that hardening does not occur. From Eq. (5) the respective warm-particle diffusion rates are increased by $(v_H/v_h)^3$, so that the low-energy diffusion always exceeds v_h^3/L, the minimum diffusion rate needed to thermalize to v_h^2 in a transit time. While the marginally stable state is the same in both cases, the introduction of stream in 2XIIB is thus seen to allow plasma to achieve this state at a lower fluctuation level, extending its lifetime by roughly a factor v_H^2/v_h^2. The presence of the stream sufficiently decreases the fluctuation level and the electron temperature that the hot-ion lifetime becomes governed by the classical electron drag time.

Using $\Delta = \frac{1}{2}v_h^2/v_H^2$ as the minimum required for stability in region III, we obtain from Eq. (7) the maximum steady-state electron temperature in this region

$$T_e(\text{keV}) = 2.1\times 10^{-5} n^{1/4} T_H^{1/2}(\text{keV}). \qquad (8)$$

At $n=3\times 10^{13}$ cm^{-3}, this gives $T_e = 100$ eV at $T_H = 4$ keV and $T_e = 175$ eV at $T_H = 13$ keV, in good agreement with measured 2*XIIB* values.[12] Predicted electron temperatures in the presence of the Δ required for stability in region I and II may be obtained directly from Eq. (3) using the appropriate unstable phase velocities.

The one-dimensional diffusion Eq. (4) with the definition of D given by the described simplification of Eq. (5) and the prescription for T_e given by Eq. (7) has been solved numerically and has been described by one of us (H.L.B.) elsewhere.[13] At 13-keV ion energy the code is in very good agreement with the 2*XIIB* behavior with stream and neutral-beam input, predicting ion and electron temperatures, lifetime, and fluctuation level. The one free parameter in the code is the intensity of the zero-temperature source S_0, although the value used was in the range of the experimental equivalent.

The $T_e \propto T_i^{1/2}$ scaling in Eq. (8) correlates with preliminary experimental observations.[12] Because the hot-ion lifetime τ is dominated by electron drag, at fixed density $\tau \propto T_e^{3/2}$, there would result $\tau \propto T_i^{3/4}$. The experiment has definitely identified lifetime increasing with energy.[7,12] The exact scaling is complicated by sensitivity to stream and charge exchange, and quantitative comparison is under current experimental and theoretical investigation.

In summary, we have presented a model of the nonlinear saturation of the DCLC mode in mirror machines both with and without an external source of cold ions, the physical mechanism being the quasilinear partial filling of the ion loss cone. The plasma properties predicted by the model are consistent with observations in 2*XIIB*. The same theory when applied to machines of larger number of ion orbit radii predicts progressively smaller amounts of warm plasma required for stabilization of DCLC as the radius increases, with no such warm plasma required above a critical radius dependent upon the plasma β. In all intermediate regions requiring warm plasma for stabilization, the electron temperature is reduced below its classical value; thus the hot-ion lifetime is governed by energy drag on the electrons.

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