stabilities may also be fed by the free energy available from the relative drift of electrons with respect to ions in the sheath region.[2] Davidson and Gladd[9] have examined the anomalous resistivity and heating associated with the lower-hybrid-drift instability in the regime where $v_E \lesssim v_{\mathrm{Th}i}$, where $v_{\mathrm{Th}i}$ is the ion thermal speed and v_E is the azimuthal component of the $\vec{E}\times\vec{B}$ drift velocity. These waves propagate perpendicular to the magnetic field (i.e., azimuthally). Finally, the observed anisotropy in the C V temperature may also be a source of free energy. Davidson and Ogden[10] have studied transverse electromagnetic perturbations, propagating parallel to a magnetic field, which can become unstable in the presence of an ion-temperature anisotropy, $T_{i\perp}/T_{i\parallel}>1$. Numerical results[10] for this electromagnetic ion-cyclotron instability show that after ~10 maximum growth times (~1 μsec) the ratio $T_{i\perp}/T_{i\parallel}$ seems to saturate at ~2.

The data and analysis presented here show that collisionless mechanisms play a major role in the ion heating and thermalization in theta-pinches with densities two orders of magnitude higher than in earlier work on turbulent heating of hydrogen and argon ions.[11] One might add that impurity ion temperatures substantially larger than deuterium ion temperatures have also been observed in a dense plasma focus.[12]

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Plasma Heating by the Dissipative Trapped-Electron Drift Instability*

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It is shown that energy loss in tokamaks due to cross field diffusion caused by the dissipative trapped-electron drift instability is compensated in part by the heating associated with this mode. Thus the energy loss rate presently predicted for tokamaks in the parameter range of this mode may be substantially overestimated.

It is generally accepted that instabilities driven by the diamagnetic drifts of trapped electrons and ions in tokamaks[1] can, in principle, strongly affect particle diffusion and heat conductivity in the relatively low-collision-frequency limit appropriate to controlled-fusion reactors. Indeed a great many of the present tokamak scaling predictions are based on a diffusion coefficient $D_\perp \cong \gamma/k^2$, where γ and k are the growth rate and wave number of trapped-particle drift instabilities.[2] However, diffusion is only one aspect of transport, and an instability which produces significant diffusion may be expected to contribute also to plasma heating and to the electrical conductivity. If the heating due to the mode is neglected, the energy loss predicted due to diffusion induced by these instabilities can be a substantial overestimate. The relationship between these various aspects of turbulent transport has been calculated[3,4] for the modes which dominate theta-pinch behavior, and there is considerable experimental evidence for these transport processes.[5]

In the present work we use the standard quasilinear theory[6] to calculate the anomalous diffusion and heating due to the dissipative trapped-electron mode, writing the distribution functions and electric fields as $f=\langle f\rangle+\delta f$, where δf is the small-scale fluctuation in f due to the instability and $\langle f\rangle$ is the distribution function averaged over the small-scale oscillations, $\langle f\rangle=\int_0^{2\pi}d\xi f/2\pi$, where ξ is the toroidal angle. Taking moments of the quasilinear Vlasov equation, using $\delta n=\int\delta f\,d^3V$ and $\delta(n\vec{V})=\int\delta f\vec{V}\,d^3V$, gives the following fluid equation for each species, neglecting the classical heat-transfer terms (Ohmic heating, etc.):

$$\partial(nM\vec{V})/\partial t+\nabla\cdot nM\vec{V}\vec{V}-nq(\vec{E}+\vec{V}\times\vec{B}/c)+\nabla nT=q\langle\delta n\,\delta\vec{E}\rangle, \quad (1)$$

$$\tfrac{3}{2}\partial(nT)/\partial t+\tfrac{3}{2}\nabla\cdot\vec{V}nT+nT\nabla\cdot\vec{V}=q\langle\delta\vec{E}\cdot[\delta(n\vec{V})-\vec{V}\delta n]\rangle, \quad (2)$$

where n, $\vec{V}$, T, $\vec{B}$, and $\vec{E}$ are the macroscopic fluid variables, and δn, $\delta(n\vec{V})$, and $\delta\vec{E}$ are the microscopic perturbations due to the waves. For the trapped-electron mode, with neglect of radial and poloidal structure, the perturbed potential is

$$\delta\varphi=\textstyle\sum_l\delta\varphi_l\exp[-il\xi+im\theta-i\omega t],$$

where θ is the poloidal angle. We take $m=lq\equiv lr\,B_\theta/RB_\xi$ with r the minor radius and R the major radius, thus assuming the mode is in a mode rational surface, corresponding to maximum growth. For the dissipative trapped-electron drift mode, the perturbed ion density is (see, e.g., Liu, Rosenbluth, and Tang[7])

$$\delta n_l^{\,i}=\frac{ne}{T_e}\frac{\omega^*}{\omega}\delta\varphi_l,\quad \omega^*=-\frac{m}{r}\frac{cT_e}{eB_\xi}\frac{1}{n}\frac{dn}{dr}\equiv k_\perp V_\perp^e,$$

with $V_\perp^e$ the electron diamagnetic drift velocity perpendicular to $\vec{B}$. The untrapped-electron density perturbation is

$$\delta n_l^{\,u}=(ne/T_e)[1-(2\epsilon)^{1/2}]\,\delta\varphi_l,$$

and the trapped-electron density perturbation is

$$\delta n_l^{\,T}=\left(1-i\frac{6}{\sqrt{\pi}}\frac{\eta\omega^*}{\nu^*}\right)\frac{ne(2\epsilon)^{1/2}}{T_e}\,\delta\varphi_l,$$

where $\epsilon=r/R$, $\eta=d\ln T_e/d\ln n$, and $\nu^*=\nu_{ei}/\epsilon$, where ν_{ei} is the 90° electron-ion collision frequency and we have assumed $\omega^*\ll\nu^*$. The frequency and growth of the dissipative trapped-electron mode are given by $\omega=\omega_0+i\gamma$, $\omega_0=\omega^*$, and $\gamma=6\eta\omega^{*2}(2\epsilon)^{1/2}/\nu^*\sqrt{\pi}$. Combining these results, the turbulent transport term in (1) is then

$$\langle\delta n^i\,\delta\vec{E}\rangle=\int_0^{2\pi}\frac{d\xi}{2\pi}\sum_l\delta n_l^{\,i}\exp[-il\xi+ilq\theta-i\omega t]\,\nabla\sum_{l'}\delta\varphi_{l'}\exp[-il'\xi-il'q\theta-i\omega t]=\sum_{l\neq0}k_\perp\frac{|\delta E_l|^2}{8\pi k_\perp^2\lambda_D^2e}\frac{2\gamma}{\omega^*},$$

where $\lambda_D^2=T_e/4\pi ne^2$ and $d|\delta E_l|^2/dt=2\gamma|\delta E_l|^2$. The perpendicular diffusion coefficient due to this mode has been estimated by many other authors.[2] In the context of the fluid approach it is given by solving Eq. (1) for $nV_\perp$ in quasiequilibrium, defining $D_\perp=(dn/dx)^{-1}(nV_\perp)$, and the energy in the waves by $\mathcal{E}=\sum|\delta E_l|^2/8\pi$; this procedure gives

$$\frac{q(nV_\perp)B}{c}=q\langle\delta n\,\delta E_\perp\rangle,\quad D_\perp=\frac{2\gamma}{k_\perp^2}\left(\frac{L_n}{\lambda_D}\right)^2\frac{\mathcal{E}}{nT_e}, \quad (3)$$

where $L_n\equiv n(dn/dx)^{-1}$. This is equivalent to the familiar estimate $D_\perp=\gamma/k_\perp^2$ if the wave energy is chosen equal to the expansion free energy[7] in a wavelength, $\mathcal{E}/k_\perp^2\lambda_D^2\simeq(nT_e/2)/(k_\perp L_n)^2$. Note that (3) is the diffusion coefficient calculated for ions. The same calculation for trapped electrons gives $D_\perp^T=D_\perp/(2\epsilon)^{1/2}$. For untrapped electrons, the diffusion coefficient is zero. Since the density of trapped electrons is $n_T=(2\epsilon)^{1/2}n_i$, the total diffusion rate for electrons and ions is equal. A more sophisticated calculation[7] including shear would reduce the localization distance of the mode and thus reduce the wave energy $\mathcal{E}$. In this paper $\mathcal{E}$ is treated as a parameter common to both heating and diffusion loss.

The heating rates are calculated in the same way as $D_\perp$. Using the continuity equation $-i\omega\delta n=-i\vec{k}\cdot\delta(n\vec{V})$ gives for the turbulent contribution to the heating equations of ions and electrons

$$\frac{3}{2}\frac{dT_j}{dt}\bigg|_T=\frac{q}{n}\langle\delta\vec{E}\cdot[\delta(n\vec{V})_j-\vec{V}_j\delta n_j]\rangle,$$

$$\frac{3}{2}\frac{dT_i}{dt}\bigg|_T=\frac{T_i}{nT_e}\frac{2\gamma\mathcal{E}}{k_\perp^2\lambda_D^2}=\frac{T_i}{L_n^2}D_\perp, \quad (4)$$

$$\frac{3}{2}\frac{dT_e}{dt}\bigg|_T=\frac{2\gamma\mathcal{E}}{nk_\perp^2\lambda_D^2}=\frac{T_e}{L_n^2}D_\perp. \quad (5)$$

The turbulent heating rate $dT_e/dt|_T$ for trapped and untrapped electrons is the same. Note that (4) and (5) represent a heating contribution that should be specifically included when modeling the effect of the dissipative trapped-electron instabilities.

The importance of this turbulent-heating term is easily seen by comparing it to the energy loss due to diffusion. Combining electrons and ions, the diffusive heat loss can be written approximately as

$$\frac{d}{dt}\frac{3}{2}n(T_e+T_i)\bigg|_D \simeq -\frac{\partial}{\partial x}n(T_e+T_i)\frac{D_\perp}{L_n}. \qquad (6)$$

The anomalous heating is given by

$$\frac{d}{dt}\frac{3}{2}n(T_e+T_i)\bigg|_T = +n(T_e+T_i)\frac{D_\perp}{L_n^2}.$$

Thus we find that the anomalous heating substantially balances the loss by diffusion and reduces the energy loss via this mode.

Cooling effects caused by plasma motion induced by the instability are automatically included by the assumption of an anomalous resistivity. For example, expansion cooling as the plasma diffuses outward is included in the fluid equations without regard to what the diffusion is being caused by.

The conclusion of this paper is that if one also includes heat loss caused by anomalous diffusion but one does not include the anomalous heating, then the energy-loss rate for this mode may be substantially overestimated.

The example given here was worked out in detail for the most uncomplicated case possible, to show the technique and to indicate the effect. Actually, the fact that the heat production nearly cancels the diffusion heat loss is quite a general result for drift waves, at least insofar as the usual[1] estimate is accepted for the heat conductivity. To see the generality of the result, note that, for example, the ion heating term is, using $\delta(nV)=(\omega/k)\delta n$,

$$\tfrac{3}{2}\,dnT_i/dt = q\langle \delta n_i \delta E(\omega/k - V_i)\rangle,$$

while the heat-conduction cooling term [noting that $V_i=(cT/qB)(d\ln n/dx)$] is also of order $q\langle\delta n_i\delta E(V_i)\rangle$. Thus the net ion heating, after subtracting out the heat-conduction loss, is

$$\tfrac{3}{2}\,dnT_i/dt = q\langle(\omega/k)\delta n_i\delta E\rangle. \qquad (7)$$

This vanishes to lowest order (in ω/ω_{ci} and $\omega/k_\parallel v_i$) since

$$\left\langle\frac{\omega}{k}\delta n_i\delta E\right\rangle = \int_{-\infty}^{\infty}\frac{ne}{T_e}\frac{\omega^*}{k}k(\delta\varphi_k)^2\,dk = 0, \qquad (8)$$

since $\omega^* = k_\perp V_\perp^e$. The result depends simply on the response of the ion density to the perturbed-wave electric field. For waves of low frequency ($\omega \ll \omega_{ci}$) and large phase velocity along the field $k_\parallel(T_i/m_i)^{1/2} \ll \omega$, $\delta n_i \sim 1/\omega$, and the results (7) and (8) are valid. The introduction of shear, curvature, or the retention of correction terms of order $(k_\parallel V_{Ti}/\omega)^2$, gives the residual heating (or cooling) due to the mode after conduction is subtracted out. This is clearly a small correction. For example, including shear in the calculation is equivalent to retaining correction terms of order $(k_\parallel V_{Ti}/\omega)^2$ in the calculation of δn_i and gives a small additional heating. With the use of the results of Ref. 7, the total heating term is

$$\frac{dnT_i}{dt} = \left(\frac{T_i}{nT_e}\right)\frac{2\gamma}{k_\perp^2\lambda_D^2}\,\mathcal{E}\left[1+\frac{L_n}{L_s}\left(1+4\frac{T_i}{T_e}+3\frac{T_i^2}{T_e^2}\right)\right]. \qquad (9)$$

The leading term is of the same order, and tends to cancel, the usual estimate of heat-conduction loss, Eq. (6); the residual heating effect due to retaining shear in the calculation is smaller by L_n/L_s.

Turning to the electrons, we simply note that from Poisson's equation

$$-ik\,\delta\varphi = 4\pi e(\delta n_i - \delta n_e),$$

so that for long-wavelength modes $k^2\lambda_D^2 \ll 1$, $\delta n_i \simeq \delta n_e$, and since $V_e = -V_i(T_e/T_i)$ the results (7)–(9) hold also for electrons, with $dT_e/dt = (T_e/T_i) \times dT_i/dt$.

Similar corrections, for example curvature corrections,[8] will clearly give corrections to $\langle\omega\delta n_i\delta E\rangle$ of the form shown in Eq. (9). A detailed survey of the many possible cases, as well as a consistent treatment of the conduction-loss term (which we have taken from Ref. 1) is beyond the scope of this Letter and will be the subject of a paper in preparation. We also note that the heating and loss of energy calculated here is perpendicular to the magnetic field. In the general spirit of toroidal magnetic confinement, we assume that the loss time will far exceed the particle collision time, so that the heating (and loss) will not much alter temperature isotropy. The instability will, since a component of $\vec{k}$ lies in the toroidal direction, also produce both an enhanced drag on the toroidal current and an enhancement of the toroidal current (through the enhanced radial diffusion which generates a bootstrap current). These effects also essentially cancel each other, as shown in a calculation published elsewhere.[9]

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Parametric Excitation of Nonneutral-Plasma Surface Ripples*

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Parametric excitation of the equilibrium surface ripples of a drifting, magnetically confined, nonneutral plasma is investigated by passing it through a spatially periodic magnetic field. Resonances are observed at $2\Omega_0$, Ω_0, and $\Omega_0/2$ (Ω_0 is the natural frequency for the ripple). The ripple amplitude of the equilibrium surface is measured as a function of the phase angle of the excitation, the number of pump periods, and the magnitude of the magnetic field perturbations.

This Letter reports on the parametric excitation of equilibrium surface ripples on a magnetically confined, drifting, nonneutral-plasma column. The apparatus used to create this nonneutral plasma is depicted schematically in Fig. 1. The electron gun injects a 30-μsec-pulse electron beam into the steady magnetic field (20–50 G) through a magnetic step that establishes the desired rigid-rotor nonneutral-plasma equilibrium mode. This method of creating a nonneutral plasma establishes an equilibrium in which the surface has a spatially periodic shape. The amplitude and phase of these equilibrium surface ripples are determined by the parameters of injection. The rigid-rotor $[n_0(r) = \text{const}]$ equilibrium and the equilibrium surface ripples of this nonneutral plasma, together with a description of the apparatus, the parameters of the experiment, and the diagnostic techniques used to study them are given elsewhere.[1,2] The surface envelope of a drifting, magnetically confined, nonneutral-plasma column in the frame moving with the axial ve-

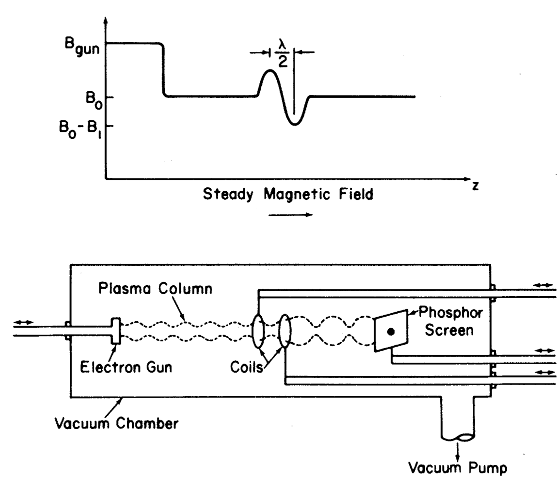

FIG. 1. Schematic of the experiment with two coils, each on a separate probe stick. This setup is used to obtain resonance response curves A versus ω. In other experiments, one of the probes has two coils and the other has four coils. This enables fixed-frequency pumping over one, two, and three pump periods. The magnetic field step is used to set up a plasma with a particular $\omega_p{}^2/\omega_{c0}{}^2$ and rotation mode.