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# Leptonic-Decay Distributions for Heavy-Lepton Pairs Produced by Colliding Beams

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We present a general differential cross section, in the colliding-beam rest frame, for $\mu e$-pair production by leptonic decays of a heavy spin-$\frac{1}{2}$ lepton pair, themselves assumed to be produced via one-quantum annihilation. Our results hold for general $V,A,S,T,P$ couplings and depend neither on the $\mu e$-universality assumption nor on the absence or presence of neutral-current effects. We also present distributions relevant to the events reported by a Lawrence Berkeley Laboratory-Stanford Linear Accelerator Center group.

Recently, a Lawrence Berkeley Laboratory–Stanford Linear Accelerator Center (LBL-SLAC) group has announced[1] the existence of anomalous lepton events in electron-positron collisions: Events of the form $e^+ + e^- \to e^\pm + \mu^\mp +$ missing momentum have been observed at a rate far beyond that expected from any conventional processes leading to such a signature. One of the most exciting explanations[1] for these events is that they are due to pair production of some new heavy lepton, which subsequently decays into an electron or a muon and neutrinos. If a new heavy lepton indeed exists, general investigations similar to those performed for the decay properties of the muon[2] should be repeated. While there are certain theoretical anticipations on the weak-decay properties of heavy leptons, each such property should undergo a thorough examination; for example, lepton-number conservation, $\mu e$ universality, the vector and axial-vector structure of the interactions, the masslessness of neutrinos involved in the decay, and effects of weak neutral currents must all be scrutinized.

Assuming that a new heavy spin-$\frac{1}{2}$ lepton, $U$, exists,[3] we have studied general methods of analysis of the correlated purely leptonic decays of $U^+$-$U^-$ pairs produced in the electron-positron colliding beams. We assume that the production $e^+ + e^- \to U^+ + U^-$ proceeds via a one-photon intermediate state and that the decays $U^+ \to l^+ + \nu_l + \bar{\nu}_U$ and $U^- \to l^- + \bar{\nu}_l + \nu_U$ (where $l$ stands for $e$ or $\mu$) are governed by a general local weak Hamiltonian. We restrict[4] ourselves to the case of unpolarized electron-positron beams and neglect all masses except that of $U^\pm$.

We first present the differential cross section for the process[5]

$$
\begin{array}{l}
e^+ + e^- \to \text{“}\gamma\text{”} \to U^+ + U^- \\
\qquad\qquad\qquad\quad U^- \to e^- + \bar{\nu}_e + \nu_U \\
\qquad\qquad\qquad\quad U^+ \to \mu^+ + \nu_\mu + \bar{\nu}_U
\end{array}
$$

from which any desired distribution can be derived. In order to demonstrate how such distributions can be used, we give expressions for some quantities which permit analytic evaluations. We also give a few predictions which depend only on the local nature of the weak-interaction Hamiltonian. Finally we present distributions relevant to the events reported by the LBL-SLAC group and compare them with the data.

We shall use the most general local Hamiltonian governing the decay $U \to l + \bar{\nu}_l + \nu_U$ which can be written in the form[2]

$$H_l=(1/\sqrt{2})\sum_i\int d^3x\,\bar{\psi}_l(x)O_i\psi_U(x)\bar{\psi}_{\nu_l}(x)(g_{l,i}O_i+g_{l,i}'\gamma_5O_i)\psi_{\nu_U}(x)+\text{H.c.},\tag{2}$$

where $O_i=1,\gamma_5,\gamma_\mu,\gamma_5\gamma_\mu,\sigma_{\mu\nu}$. We define parameters[6,7] $\rho_l$, $\xi_l$, $\delta_l$ in the same way as those occurring in the electron-energy distribution for the polarized-muon decay.[8] We further introduce the following definitions: $M$ is the mass of $U^\pm$ and $\gamma=1/(1-\beta^2)^{1/2}=E/M$, where $E$ is the beam energy. $\vec{k}_{l\pm}$ stands for the momentum of $l^\pm$ evaluated in the rest frame of $U^\pm$. The direction of the positron beam is defined to be along the $z$ axis. $d\Omega_l=d\cos\theta d\varphi$, where $\theta$ and $\varphi$ give the direction of $U^+$ momentum. $B_l$ is the branching ratio for the decay $U\to l+\bar{\nu}_l+\nu_U$. Finally, $y_l=2k_l/M$.

The theoretical decay distribution of $l^\pm$ in $U^\pm$ decay can of course be copied from the textbooks[2] but that is not what we are after. Rather, for practical purposes it is important to give the differential cross section for $\mu^\pm e^\mp$ production in the colliding-beam rest frame, taking into account correlations between those two leptons dictated by the one-photon production process of the $U^+U^-$ pair. We present in our next formula the key differential distribution from which (barring electron-positron beam-polarization effects and neutrino-mass effects to which we shall return elsewhere) all relevant information can be extracted directly in the frame just mentioned[8]:

$$\begin{aligned}k_ek_\mu\frac{d\sigma}{d^3k_ed^3k_\mu d\Omega_l}=B_eB_\mu\frac{36\alpha^2\beta}{\pi^2E^2M^2}\{&G(y_e,\rho_e)G(y_\mu,\rho_\mu)(1+\cos^2\theta+\gamma^{-2}\sin^2\theta)\\&-\xi_e\xi_\mu G(y_e,\delta_e)G(y_\mu,\delta_\mu)[k_{e,z}(1+\cos^2\theta-\gamma^{-2}\sin^2\theta)-k_{e,y}k_{\mu,y}\beta^2\sin^2\theta\\&+k_{e,x}k_{\mu,x}(1+\gamma^{-2})\sin^2\theta-(k_{e,x}k_{\mu,z}+k_{e,z}k_{\mu,x})\gamma^{-1}\sin^2\theta]\},\end{aligned}\tag{3}$$

where $G(y,\rho)=1-y+\frac{2}{3}\rho(\frac{4}{3}y-1)$. Using Eq. (3), certain quantities can be evaluated analytically. We give some examples: Let $\vec{k}_l'$ be the laboratory momentum of $l$, $x_l=k_l'/E$, $\theta_l$ is equal to the angle between $\vec{k}_l'$ and the positron-beam direction, and $\cos\theta_{\text{coll}}=-\vec{k}_e'\cdot\vec{k}_\mu'/k_e'k_\mu'$. Then, (a)

$$\langle\cos^2\theta_l\rangle=[5(1+2\gamma^2)]^{-1}\{3+4\gamma^2-\beta^{-1}\ln[(1+\beta)/(1-\beta)]\}.\tag{4}$$

Note that for a given $\beta$, $\langle\cos^2\theta_l\rangle$ is uniquely fixed. It is independent of the detailed structure of the Hamiltonian. Furthermore, $\langle\cos^2\theta_l\rangle$ depends only weakly on $\beta$. Namely, $\langle\cos^2\theta_l\rangle\simeq\frac{1}{3}$ for $0\leq\beta\leq0.7$ and it increases from 0.34 to 0.40 between $\beta=0.7$ and 1. (b)

$$\langle\cos\theta_{\text{coll}}\rangle=\beta^{-4}\left([Z(\beta)]^2+\xi_e\xi_\mu[\beta^2(2\gamma^2+1)]^{-1}\{(1+\beta^2)\gamma^2[Z(\beta)-\beta^3]^2+\tfrac{1}{2}[Z(\beta)]^{-2}\}\right),\tag{5}$$

where $Z(\beta)=\beta-\frac{1}{2}\gamma^{-2}\ln[(1+\beta)/(1-\beta)]$. With the $\mu e$-universality assumption, $0\leq\xi_e\xi_\mu\leq1$, in which case the sign of $\langle\cos\theta_{\text{coll}}\rangle$ as well as its upper and lower bounds can be predicted. Also, we note that $\langle\cos\theta_{\text{coll}}\rangle\to0.16\xi_e\xi_\mu$ as $\beta\to0$. Since $\xi_e\xi_\mu<0$ is a sign of $\mu e$-universality violation, $\langle\cos\theta_{\text{coll}}\rangle$ near the production threshold should be measured. (c)

$$\frac{d\sigma}{dx_l}=B_eB_\mu\frac{4\pi\alpha^2}{E^2}\left(1+\frac{1}{2\gamma^2}\right)K_2(x_l,\rho_l),\tag{6}$$

where

$$K_n(x,\rho)=A_n(x)-A_{n+1}(x)+\tfrac{2}{3}\rho[\tfrac{4}{3}A_{n+1}(x)-A_n(x)]$$

and

$$A_n(x)=\frac{1}{n}\left[\theta\left(x-\frac{1-\beta}{2}\right)+\theta\left(\frac{1-\beta}{2}-x\right)\left(\frac{2x}{1-\beta}\right)^n-\left(\frac{2x}{1+\beta}\right)^n\right].$$

The energy distribution[9] of the electron and muon will give information on $\rho$. This is in analogy with the electron-energy distribution in muon decay which gives the value of the Michel parameter. If $\mu e$ universality is valid, $d\sigma/dx_e=d\sigma/dx_\mu$ for all $x_e=x_\mu$, or equivalently, $\rho_e=\rho_\mu$. (d)

$$\begin{aligned}f_2(x_e,x_\mu)&\equiv\frac{1}{\sigma}\left(\frac{d\sigma}{dx_edx_\mu}-\frac{1}{\sigma}\frac{d\sigma}{dx_e}\frac{d\sigma}{dx_\mu}\right)\\&=-144\xi_e\xi_\mu\frac{1}{\beta^2}\left(\frac{2\gamma^2-1}{2\gamma^2+1}\right)[K_2(x_e,\delta_e)-2x_eK_1(x_e,\delta_e)][K_2(x_\mu,\delta_\mu)-2x_\mu K_1(x_\mu,\delta_\mu)].\end{aligned}\tag{7}$$

Note the factorization property of $f_2(x_e, x_\mu)$. We therefore have a prediction that $r(x_e, x_e', x_\mu) = f_2(x_e, x_\mu)/f_2(x_e', x_\mu)$ depends only on $x_e$, $x_e'$, and $\delta_e$. If $\mu e$ universality is valid, $r(x_e, x_e', x_\mu'') = r(x_e'', x_\mu, x_\mu')$ for all $x_e = x_\mu$, $x_e' = x_\mu'$, $x_e'' = x_\mu''$, or equivalently $\delta_e = \delta_\mu$.

The predictions given above are not directly comparable with the existing data. This is because the present experiment, as it is for most pioneering experiments, is not suitably designed for the detailed investigations under present consideration. The limitation in acceptance of the LBL-SLAC magnetic detector is such that, if the anomalous events are due to heavy-lepton production, only a small fraction of such events are actually detected.

We have evaluated $d\sigma/d\cos\theta_{\text{coll}}$ and $d\sigma/dk$ numerically including all cuts due to the acceptance of the apparatus. Before we present our results, we remark that if all events were accepted by the apparatus, the normalizations of these two quantities are completely fixed in terms of the branching ratios $B_e$ and $B_\mu$. In particular, the areas under $d\sigma/d\cos\theta_{\text{coll}}$ and $d\sigma/dk$ are easily seen to be independent of the parameters $\rho$, $\xi$, and $\delta$. In practice, however, the normalization does depend on the values of these parameters because of the acceptance of the apparatus. We have normalized the calculated distributions so that the areas under the distributions are the same as those for the experimental distributions. Our results, with some representative values for the parameters,[10] are shown in Figs. 1 and 2. Operationally, we have found that, for any value of mass $M$ between 1.6 and 2 GeV and for any value of $\rho$, the theoretical predictions are contained in the band defined by the two outermost curves in Figs. 1 and 2.

From the present data, it is impossible to draw any conclusions on the nature of the anomalous lepton events. As the statistics improve, however, if the anomalous lepton events are due to heavy-lepton production, we conclude that the peak present in the data for $d\sigma/d\cos\theta_{\text{coll}}$ at $\cos\theta_{\text{coll}} = 0.5$ should move to a larger value of $\cos\theta_{\text{coll}}$.

In conclusion, if heavy leptons exist, we can not stress enough the importance of general investigations on their properties. There is no doubt that such explorations will eventually enhance our understanding of weak interactions to a completely new level.

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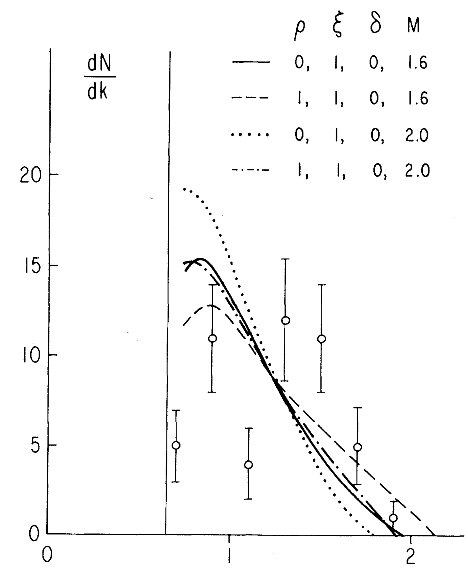


FIG. 1. The laboratory electron and muon energy distribution for $2E = 4.8$ GeV. Operationally, the theoretical predictions are contained in the bound defined by the two outermost curves for 1.6 GeV $\leq M \leq$ 2 GeV, and $0 < \rho < 1$.

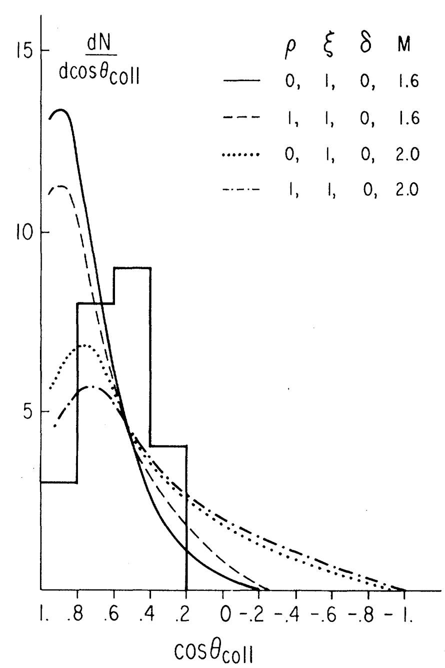


FIG. 2. The $\cos\theta_{\text{coll}}$ distribution.

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[1]M. L. Perl *et al.*, SLAC Report No. SLAC-PUB-1626, 1975 (unpublished), and LBL Report No. LBL-4228, 1975 (unpublished).

[2]See, for example, S. Källen, *Elementary Particle Physics* (Addison-Wesley, Reading, Mass., 1964), and references therein.

[3]Here we use the name suggested by Perl [M. L. Perl, SLAC Report No. SLAC-PUB-1592 (unpublished)] to stand for a heavy lepton in general.

[4]A general analysis relaxing these restrictions will be presented elsewhere.

[5]This process has been discussed previously by many authors: Y. S. Tsai, Phys. Rev. D 4, 2821 (1971); S. Kawasaki, T. Shirafuji, and S. Y. Tsai, Prog. Theor. Phys. 49, 1656 (1973); K. Fujikawa and N. Kawamoto, Institute for Nuclear Studies Report No. INS-Rep-239 (to be published); S. Y. Park and A. Yildiz, to be published; T. C. Yang, University of Maryland Technical Report No. 76-025 (unpublished). These authors have assumed $\mu e$ universality, and presence of only charged $V-A$ current (or $V-A$ and $V+A$ currents in case of Park and Yildiz) in the weak-interaction Hamiltonian for the $U$ decay.

[6]Ref. 2. Also, T. Kinoshita and A. Sirlin, Phys. Rev. 107, 593 (1957).

[7]$\rho_l = 3(b_l + 2c_l)/(a_l + 4b_l + 6c_l)$, $\xi_l = (3a_l' - 4b_l' - 14c_l')/(3a_l + 12b_l + 1)$, $\delta_l = 9(b_l' + 2c_l')/(-3a_l' + 4b_l' + 14c_l')$, where $a_l = |g_{l,S}|^2 + |g_{l,S}'|^2 + |g_{l,P}|^2 + 1$, $b_l = |g_{l,V}|^2 + |g_{l,V}'|^2 + |g_{l,A}|^2 + |g_{l,A}'|^2$, $c = |g_{l,T}|^2 + |g_{l,T}'|^2$, $a_l' = 2\,\mathrm{Re}(g_{l,S}^* \times g_{l,P}') + 2\,\mathrm{Re}(g_{l,S}' g_{l,P}^*)$, $b_l' = 2\,\mathrm{Re}(g_{l,V}^* g_{l,A}') + 2\,\mathrm{Re}(g_{l,V}' g_{l,A}^*)$, $c_l' = 2\,\mathrm{Re}(g_{l,T}^* g_{l,T}')$. As usual, $0 < \rho_l < 1$. Using Schwartz's inequality $|\xi_l| < 1$ and $|\xi_l \delta_l| < 1$. For $V-A$ interactions $\rho = \frac{3}{4}$, $\delta = \frac{9}{4}$, and $\xi = \frac{1}{3}$ and for $V+A$ interactions $\rho = 0$, $\delta = 0$, $\xi = 1$.

[8]By $\mu e$ universality we mean $g_{e,i} = g_{\mu,i}$, $g_{e,i}' = g_{\mu,i}'$ for all $i$. The process under consideration, Eq. (1), is particularly suited for the test of this hypothesis since $\rho_e = \rho_\mu$, $\delta_e = \delta_\mu$, and $\xi_e \xi_\mu \geq 0$ can all be checked with this *single* process.

[9]This reduces to the result of S. Y. Tsai when $\mu e$ universality and current-current interaction with $V-A$ charged current for the $U$ decay is assumed.

[10]We found that these results are not very sensitive to the particular choice of $\xi_e$, $\xi_\mu$, $\delta_e$, and $\delta_\mu$. We therefore choose these parameters arbitrarily to be the values shown in Figs. 1 and 2.