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Saturation of the Gentle Bump Instability in a Random Plasma

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It is shown that as a result of density fluctuations, the phase velocity domain of the gentle bump instability can be greatly broadened, leading to a different saturation of the instability. A part of the beam is accelerated and the turbulence power at saturation is lowered.

It has recently been pointed out that the growth rate of a high-frequency instability can be modified by a nonresonant interaction with low-frequency turbulence.[1,2] This effect is qualitatively similar[2] to the effect of density fluctuations on the spatial growth of an instability. The following investigation of the gentle bump instability in the presence of random inhomogeneities describes the saturation of this instability in the presence of nonresonant low-frequency turbulence. The saturation process presented here is different from the saturation of the gentle bump instability in the presence of a coherent ion fluctuation[3] which allows a part of the beam energy to be absorbed by the main plasma.

I consider a one-dimensional plasma with a density $n(x)=\bar{n}+\delta n(x)$ fluctuating around an average density $\bar{n}$. A fast beam (with mean velocity $u \gg v_{\text{th}}$, the thermal velocity), satisfying the O'Neil-Malmberg[4] criterion for the kinetic beam-plasma instability, is streaming from the point $x=0$ towards positive x. Provided that the typical scale for the density fluctuation is much greater than the typical wavelength of the instability, a given frequency ω is related to a local wave number $k(\omega,x)$ given by

$$\omega=\Omega(k(\omega,x),n(x)), \tag{1}$$

where $\Omega(k,n(x))=[\omega_p^2(x)+3k^2v_{\text{th}}^2]^{1/2}$ (Bohm-Gross relation) depends on x only through $n(x)$. With the relation $k(\omega,x)=\bar{k}(\omega)+\delta k(\omega,x)$, where $\bar{k}(\omega)$ is the average value of $k(\omega,x)$, one obtains

$$\delta k(\omega,x)=\delta n(x)k_D{}^2/[6\bar{n}\bar{k}(\omega)], \qquad (2)$$

provided that $\delta k(\omega,x)\ll\bar{k}(\omega)$. For simplicity, let us assume the domain of k of interest to be narrow enough to allow consideration of δk as independent of ω [and thus write it $\delta k(x)$]. Let $\varphi(\delta k)$ be the probability distribution of δk, Δ be the rms average of δk, and l_c be the correlation length of the fluctuation. With the assumption $\Delta\ll\bar{k}(\omega)$, one gets $\omega=\Omega(\bar{k}(\omega),\bar{n})$. The Landau relation,

$$\gamma(x,k)=\pi\Omega(k,n(x))\frac{\Omega^3(0,n(x))}{3k^3v_{\text{th}}{}^2n(x)}\frac{\partial f(x,v)}{\partial v}, \qquad (3)$$

defines the spatial dissipation function of the instability. $\sigma(x)$, a characteristic width of $\gamma(x,k)$, is defined in the following way:

$$\sigma(x)=\{\gamma_m{}^{-1}(x)\int\gamma(x,k)[k-k_1(x)]\,dk\}^{1/2},$$

where $\gamma_m(x)$ is the maximum of $\gamma(x,k)$ and $k_1(x)$ is defined by $\gamma(x,k_1(x))=0$. I now can give the basic assumptions of this Letter:

$$\Delta/l_c\ll k_c{}^2, \qquad (4)$$

$$\Delta\gg\sigma(0), \qquad (5)$$

$$\gamma_m(0)\,l_c\ll 1. \qquad (6)$$

Inequality (4), where k_c is a characteristic wave number of the problem, is the WKB condition ($dk/dx\ll k^2$). Inequality (5) implies that for a given frequency ω, the wave-number fluctuations are larger than the typical width of the k spectrum of the classical gentle-bump-instability growth rate, leading, as we will see, to a completely different average growth rate. According to inequality (6), the spatial growth of the instability must remain smaller than 1 for a distance of the order of the spatial fluctuation correlation length. It is a sufficient but non-necessary condition and the same final result can be found after some tedious calculations with the less stringent condition

$$\gamma_m(0)\,l_c\exp[\gamma_m(0)\,l_c][\sigma(0)/\Delta]^{3/2}\ll 1.$$

The nonlinear evolution of the spatial gentle bump instability is characterized by the quasilinear diffusion equation[5,6]

$$v\frac{\partial f}{\partial x}(x,v)=\frac{\partial}{\partial v}D(x,v)\frac{\partial}{\partial v}f(x,v), \qquad (7)$$

where f is the total distribution function of the beam-plasma system and $D(x,v)$ is the diffusion coefficient of the instability. The WKB condition (4) allows the description of the turbulence in term of plasmons.[7] Then the diffusion coefficient becomes[8]

$$D(x,v)=(\pi/\mathscr{E}_0)(e/m)^2k'(v,x)N(x,k'(v,x)), \qquad (8)$$

where $N(x,k)$ is the plasmon occupation number and $k'(v,x)$ is defined by $vk'(v,x)=\Omega(k'(v,x),n(x))$. The quantity $k'(v,x)$ fluctuates and can be written $k'(v,x)=\bar{k}'(v)+\delta k'(v,x)$. The quantity $\bar{k}'(v)$ is given by $v\bar{k}'(v)=\Omega(\bar{k}'(v),\bar{n})$.

The problem here is to calculate the evolution of $\langle f(x,v)\rangle$. Averaging Eq. (8), one has to evaluate $\langle D(x,v)\partial f(x,v)/\partial v\rangle$; that is, to calculate

$$A(x,v)=\langle N(x,k'(v,x))(\partial/\partial v)f(x,v)\rangle. \qquad (9)$$

Let us follow the motion between $x=0$ and $x=x_1$ of a group of plasmons of frequency $\omega=\Omega(k'(v,x_1),n(x_1))$ interacting with the particles. Their density in phase space is given by $N(x,k(\omega,x))$. The spatial variation of N is given by

$$dN(x,k(\omega,x))/dx=\gamma(x,k(\omega,x))N(x,k(\omega,x)). \qquad (10)$$

Let us take the initial noise to be a white noise N_0. Then integration of Eq. (10) yields

$$N(x_1,k'(v,x_1))=N_0\exp\{\int_0^{x_1}\gamma(x,\bar{k}'(v)+\delta k'(v,x_1)-\delta k(x_1)+\delta k(x))\,dx\}. \qquad (11)$$

A simple calculation gives $\delta k'(v,x_1)/\delta k(x_1)=3[\bar{k}'(v)/k_D]^2$. As it was assumed that $u\gg v_{\text{th}}$, $\bar{k}'(v)$ is much less than k_D and $\delta k'$ is negligible with respect to δk. So one gets $N(x_1,k'(v,x_1))\simeq N(x_1,\bar{k}'(v))$. Note that $N(x,\bar{k}'(v))$ is a function of all the fluctuations between 0 and x, unlike $\partial f/\partial v$ which is only a function of the fluctuation at point x. Taking into account the relation (6), one can calculate $A(x,v)$ for $x\gg l_c$. Condition (6) implies that $N(x,\bar{k}'(v))\simeq M(x,v)$, where

$$M(x,v)=N_0\exp\{\int_0^{x-l}\gamma(x',\bar{k}'(v)-\delta k(x)+\delta k(x'))\,dx'\}. \qquad (12)$$

l is of the order of l_c, and is chosen such that one can neglect the correlation between the fluctuation at point x and the fluctuations between 0 and $x-l$. As $M(x,v)$ and $\partial f/\partial v$ are uncorrelated, one gets

$$A(x,v)\simeq\langle M(x,v)\partial f(x,v)/\partial v\rangle\simeq\langle M(x,v)\rangle\partial\langle f(x,v)\rangle/\partial v\simeq\langle N(x,\bar{k}'(v))\rangle\,\partial\langle f(x,v)\rangle/\partial v. \qquad (13)$$

Finally, one obtains the diffusion equation

$$v\frac{\partial}{\partial x}\langle f(x,v)\rangle = \frac{\partial}{\partial v}\langle D(x,v)\rangle\frac{\partial}{\partial v}\langle f(x,v)\rangle, \qquad (14)$$

where

$$\langle D(x,v)\rangle = (\pi/\mathcal{E}_0)(e/m)^2 k(v)\langle N(x,\bar{k}'(v))\rangle. \qquad (15)$$

Following Coste *et al.*,[1] $\langle N(x,k)\rangle$ can be calculated. The spatial variation of the average number of plasmons having a frequency ω and a mean wave number k_0 is

$$\langle U(x,k_0)\rangle = N_0 \exp\left[\int_0^x \gamma_e(x',k_0)\,dx'\right], \qquad (16)$$

where $\gamma_e(x,k_0) = \langle\gamma(x,k_0+\delta k(x))\rangle$. As long as $\sigma(x) \ll \Delta$, $\gamma_e(x,k_0)$ has a simple expression

$$\gamma_e(x,k_0) = \sigma^2(x)\gamma_m(x)(d\varphi/d\delta k)[k_0 - \bar{k}'(v_1)], \qquad (17)$$

where v_1 is such that $\partial\langle f(x,v_1)\rangle/\partial v = 0$. The growth rate is centered around $\bar{k}'(v_1)+\Delta$, has a width Δ, and a maximum which is about $[\sigma(x)/\Delta]^2\gamma_m(x)$. The average number of plasmons having a given wave number k is simply related to $\langle U(x,k_0)\rangle$ by

$$\langle N(x,k)\rangle = \int \varphi(\delta k)\langle U(x,k-\delta k)\rangle\, d(\delta k). \qquad (18)$$

From Eqs. (17) and (18) we see that $\langle N(x,k)\rangle$ has a k spectrum width of the order of Δ as long as $\sigma(x) \ll \Delta$.

All the previous equations can now describe the saturation of the instability. I show in Fig. 1 the phase velocity domain of $\gamma(x,\bar{k}'(v))$, $\gamma_e(x,\bar{k}'(v))$, and $\langle D(x,v)\rangle$ (x is chosen such that it is large without being in the saturation domain of the instability). In the case without fluctuations[5,6], D is nonzero only in the positive part of γ, and a plateau appears in the distribution function between the main plasma and the bump which is destroyed. Unlike this case, here the whole beam is subjected to a diffusive effect, the magnitude of which is nearly constant over the beam width, as a consequence of condition (5). This is because, due to the wave-number fluctuation, the phase velocity of a given group of plasmons is sometimes in the positive-slope part of the beam-distribution function where it takes energy from the particles and slows them down, and sometimes in the negative-slope part of the beam-distribution function where it gives energy to the particles and accelerates them. Thus the first effect of turbulence will be to make the average beam-distribution function $\bar{f}_b$ evolve according to the equation

$$v\,\partial\bar{f}_b(x,v)/\partial x = D_0(x)\partial^2\bar{f}_b(x,v)/\partial v^2, \qquad (19)$$

where $D_0(x)$ is the diffusion coefficient. This corresponds to a heating of the beam. The width in velocity space of the beam, $\Delta u(x)$, will grow, and so will $\sigma(x)$, the k-spectrum width of the classical bump-on-tail growth rate γ, which is nearly proportional to $\Delta u(x)$. Somewhat further, at a certain point x_0, $\sigma(x_0) \simeq \Delta$. Then the average effect on $\gamma(x,k)$ is no longer important and $\gamma_e(x,k) \simeq \gamma_0(x,k)$, where $\gamma_0(x,k)$ is given by Eq. (3) calculated with $n(x) = \bar{n}$ and $f(x,v) = \langle f(x,v)\rangle$. The beam width is now $\Delta u(x_0) = [\Delta/\sigma(0)]/\Delta u(0)$. From this point, the later evolution of the instability becomes the classical quasilinear plateau formation. Comparison of the maximum growth rates at $x=0$ and $x=x_0$ shows that during the beam broadening the growth rate of the instability does not appreciably change.

In Fig. 2 I show the evolution of the total distribution function. Because of the beam broaden-

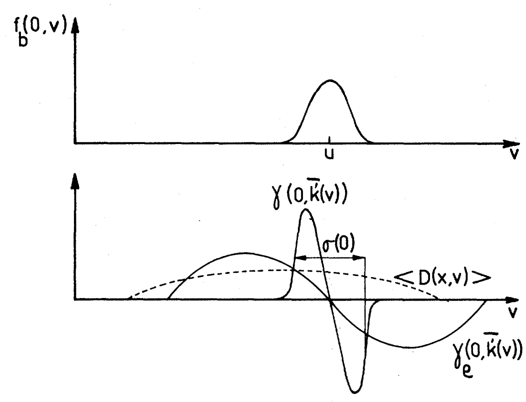

FIG. 1. Beam distribution function at $x=0$ and $\gamma(0,\bar{k}'(v))$, $\gamma_e(0,\bar{k}'(v))$, $\langle D(x,v)\rangle$ versus v. x is a distance between 0 and x_0.

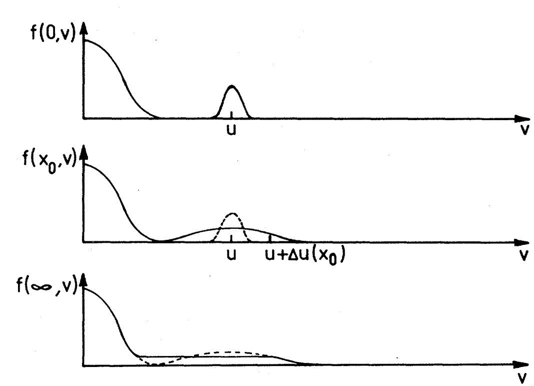

FIG. 2. Total distribution function at $x=0$, $x=x_0$, and at saturation of the instability.

ing, the saturation turbulence spectrum will be widened towards high phase velocities. As a part of the beam particles are accelerated during the beam broadening, the total saturation turbulence power will be lower here than in the classical case. The quantitative calculation of this power variation is generally difficult. An approximate value can be given when $\Delta u(0) \ll \Delta u(x_0) < \frac{1}{2}u$. The ratio of the total turbulence power at the equilibrium with fluctuations to that without fluctuations is then $(4-r)/3$, where $r = [1+\Delta u(x_0)/u]^3$. For example, in an experiment with parameters of the same order as in the experiment[9] of Roberson and Gentle, I calculate that a density fluctuation of 5% would lead roughly to a reduction of the turbulent spectrum power at saturation by a factor 3. The appearance of accelerated beam particles in some beam-plasma experiments has been explained[8] by an induced-wave scattering process. This Letter shows that a few percent of ionic random fluctuations yields the same result.

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Amplification of Extraordinary Waves through Relativistic Magnetized Plasma

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We observe amplification of extraordinary waves which is interpreted as negative absorption of the relativistic magnetized plasma confined in a mirror machine. Experimental results are explained well by the relativistic kinetic theory with the assumption of an isotropic peaked distribution, $f(p) \propto p^l \exp(-\epsilon/T)$.

The possibility of negative absorption in a relativistic magnetized plasma has been suggested by Twiss[1] and Bekefi, Hirshfield, and Brown.[2] An expression for the absorption coefficient α_ω in the vicinity of the nth harmonic of the electron-cyclotron frequency ω_0 for extraordinary waves was given in Ref. 2 where an isotropic-peaked distribution function, $f(p) \propto p^l \exp(-\epsilon/T)$, was assumed. ($p$ and ϵ are the momentum and relativistic energy of the electrons, respectively, l is a measure of the departure from a Maxwellian, and T is the average kinetic energy of electrons.) We observed the amplification of extraordinary-mode microwaves transmitted through a relativistic plasma during the heating period in a mirror machine[3] and compared it with the theory of Ref. 2. Amplification for ordinary waves was also observed but was rather weak.[2,4]

The experimental apparatus is shown schematically in Fig. 1. The relativistic plasma was generated in a mirror magnetic field by means of electron-cyclotron heating. The heating microwave power P_{mh} could be varied up to 5 kW and the pulse duration was about 60 msec with a repetition rate of $\frac{1}{2}$ sec^{-1}. The ambient gas pressure was about 2×10^{-4} Torr. The electron plasma frequency was determined by absolute intensity measurement of the x-ray bremsstrahlung from the plasma; it was less than 1.3 GHz with a heating power of 1 kW. The average energy of the hot electrons was about 30 keV as estimated from the analysis of x-ray spectra. The probe microwave power, amplitude modulated at 1 kHz, was 16 dBm (0 dBm = 1 mW) and was applied to the