Confirmation of the Predicted L Dependence in the Radial Form Factor for Nucleus-Nucleus Inelastic Scattering

P. J. Moffa, J. P. Vary,* and C. B. Dover
Brookhaven National Laboratory, Upton, New York 11973†

and

C. W. Towsley, R. G. Hanus, and K. Nagatani
Cyclotron Institute,‡ *Texas A & M University, College Station, Texas 77843*
(Received 12 August 1975)

The multipolarity (L) dependence of the inelastic form factor for heavy-ion–induced excitations predicted by the folding model is confirmed in the analysis of the scattering of ^{14}N from ^{12}C and ^{16}O at 155 MeV. Agreement between the target deformation lengths ($\beta_L R_t$) of these measurements and those from electromagnetic and proton experiments is found when the strongly L-dependent folding-model form factor is employed.

The main difference between the radial part of collective inelastic form factors obtained by deforming the optical potential and those calculated in a folding model by deforming the density directly is the intrinsic L dependence in the latter model.[1,2] This L dependence is enhanced for heavy ion projectiles.[2,3]

In the present note, we test the validity of the L dependence predicted by the folding model by extracting deformation lengths for different multipole transitions from heavy-ion inelastic data and comparing them to values obtained with proton and electromagnetic probes. To this end it is desirable to perform and analyze experiments that (a) involve nuclei whose densities are well known and where the collective strengths of the target excited states of different L are reliably known from electromagnetic and/or inelastic proton studies; and that (b) are performed at as high an energy as possible so as to minimize the need for corrections to the folding model.

A program of heavy-ion experiments already in progress at the Texas A & M University cyclotron provides results well suited for this test. The elastic and inelastic scattering of ^{14}N from ^{12}C and ^{16}O at a nominal lab energy of 155 MeV are selected and analyzed with the folding model to demonstrate the validity of its predictions. For comparison an analysis with Woods-Saxon optical potentials and form factors is also presented.

The microscopic folding model we employ has been described elsewhere.[4] Its chief features are that it (a) represents the leading term of a systematic hole-line expansion for the nucleus-nucleus interaction[5]; (b) accounts for the fact that the projectile and target nucleons are both off shell; and (c) treats the projectile and target on a symmetric footing. The resulting optical potential $V(\vec{r}, E)$ is expressed as the convolution of the target density ρ_t and projectile density ρ_p with the complex energy-dependent and density-dependent effective nucleon-nucleon interaction $g(\vec{R}, E)$ [assumed local]:

$$V(\vec{r}, E) = \int\int d^3r'\, d^3r''\, \rho_p(\vec{r}\,')\rho_t(\vec{r}\,'')g(\vec{r}\,' - \vec{r}\,'' - \vec{r}, E). \tag{1}$$

For the scattering Brueckner g matrix we have adopted a simple *Ansatz:*

$$g(\vec{R}, E) = -2\pi\hbar^2[\bar{f}(E)/M]\exp(-R^2/r_0^2)(\pi r_0^2)^{-3/2},$$

where M is the nucleon mass. We take r_0 to be 1.4 fm consistent with the range of the spin-isospin–independent component of the t matrix at medium energies.[6] In the zero-density limit $\bar{f}(E)$ is the experimental complex forward nucleon-nucleon scattering amplitude evaluated at the relative *velocity* between a nucleon at rest in the projectile and a target nucleon at rest. The many-body effects on $\bar{f}(E)$ (Fermi motion, off-shell effects, and the Pauli principle) are then treated in an average manner in a Fermi-gas model.[4] For simplicity we take a single total density (3% of saturation density) characteristic of the most important contributions to (1) for the peripheral reactions we consider here. This yields $\bar{f} = 1.82 + 0.83i$ fm for a 155-MeV ^{14}N projectile which is used for the calculations described below.

The densities for ^{12}C, ^{14}N, and ^{16}O were selected as follows. For ^{12}C we adopted the charge density of Friar and Negele[7] obtained by inverting elastic-electron-scattering data. This density was corrected for finite-proton-form-factor contributions and then scaled to the total mass. For ^{14}N, the depth of the state-dependent, single-particle potentials of Malaguti and Hodgson[8] were adjusted slightly to reproduce accurately the experimental binding energies in ^{14}N. The resulting wave functions were squared and summed to yield the density.[4] A correction for spurious c.m. motion was applied according to the harmonic-oscillator prescription.[9] For ^{16}O, we selected density-dependent Hartree-Fock results of Negele[9] and applied the c.m. correction. Thus, with the ingredients of V determined in advance there are *no free parameters* in the model.

Within this same microscopic approach, if we invoke a macroscopic collective wave function for the excited target states, we obtain[2,3] the *total* complex inelastic transition form factor $\mathfrak{F}_L{}^t$ of multipolarity L:

$$\mathfrak{F}_L{}^t(\vec{r}) = \beta_L R_t F_L{}^t(r) \sum_M Y_{LM}(\hat{r}) = \beta_L R_t \sum_M \iint d^3r'\, d^3r''\, Y_{LM}(\hat{r}'') \rho_p(\vec{r}')[\partial \rho_t(\vec{r}'')/\partial r''] g(\vec{r}' - \vec{r}'' - \vec{r}, \epsilon). \tag{2}$$

In Fig. 1, we display the real part of $V(r)$ and the real part of the *intrinsic* form factor $F_L{}^t(r)$ predicted for ^{14}N-induced excitations of ^{16}O at 154.3 MeV lab energy. Note especially the magnitude difference between the $L = 2$ and $L = 3$ intrinsic strengths from 7 to 8 fm, the important surface region. Here, the folding model predicts the $L = 2$ intrinsic strength on the average to be more than 60% stronger than the $L = 3$, corresponding to a factor of more than 2.5 in cross section. A slightly smaller difference is predicted for the ^{14}N + ^{12}C reaction.

In order to amplify these remarks we compare the folding model with the conventional phenomenological method by selecting a Woods-Saxon potential $V_{\rm WS}(r) = -(V_0 + iW_0)/\{1 + \exp[(r - R_0)/a]\}$ from the infinite class of potentials that agree in strength and slope with V at the strong-absorption radius,[10] 7.6 fm. The $V_{\rm WS}$ chosen also yields an intrinsic form factor $\partial v_{\rm WS}/\partial r$ in agreement with the folding-model 2^+ intrinsic form factor at this same radius. The resulting parameters are $V_0 = 33.2$ MeV, $W_0 = 15.1$ MeV, $R_0 = 5.92$ fm, and $a = 0.70$ fm. The real parts of $V_{\rm WS}$ and $\partial v_{\rm WS}/\partial r$ are shown in Fig. 1. Similarly, we find for ^{14}N-induced transitions of ^{12}C, $V_0 = 24.9$ MeV, $W_0 = 11.4$ MeV, $R_0 = 5.65$ fm, and $a = 0.70$ fm. Thus, we have performed no searches on the parameters of either the folding or phenomenological models for the potential and *intrinsic* inelastic form factors.

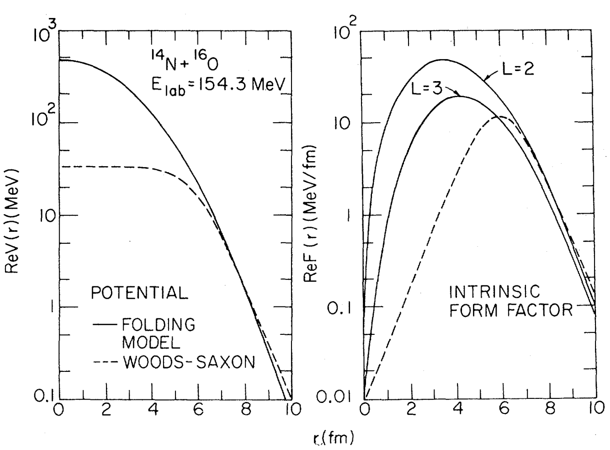

FIG. 1. Real parts of the potential, V, and intrinsic radial form factor, F, as a function of radius.

A beam of ^{14}N^{4+} ions at 155 MeV nominal energy from the Texas A & M University cyclotron was used to bombard a 430-μg/cm^2 natural-carbon target and a gas cell target containing enriched ^{16}O at a pressure of 100 Torr. Outgoing particles were detected in a solid-state counter telescope. A more detailed description of the experimental procedure will be given elsewhere.[11] The overall energy resolution was typically 400 keV, due mainly to kinematic broadening (the detector acceptance angle was between 0.2° and 0.5°), but was adequate to separate the peaks of interest except for the 3^- state in ^{16}O where the 0^+ state at 6.06 MeV was not resolved. Yields were extracted by shape filling, a procedure which added about 5% uncertainty to the inelastic cross sections. The error in the absolute normalization was estimated to be smaller than 15%. However, results from several independent runs showed that reproducibility was not better than 20%. This is presumably due to difficulties in adjusting the beam optics in the present system. The absolute angle was determined to ±0.3° by

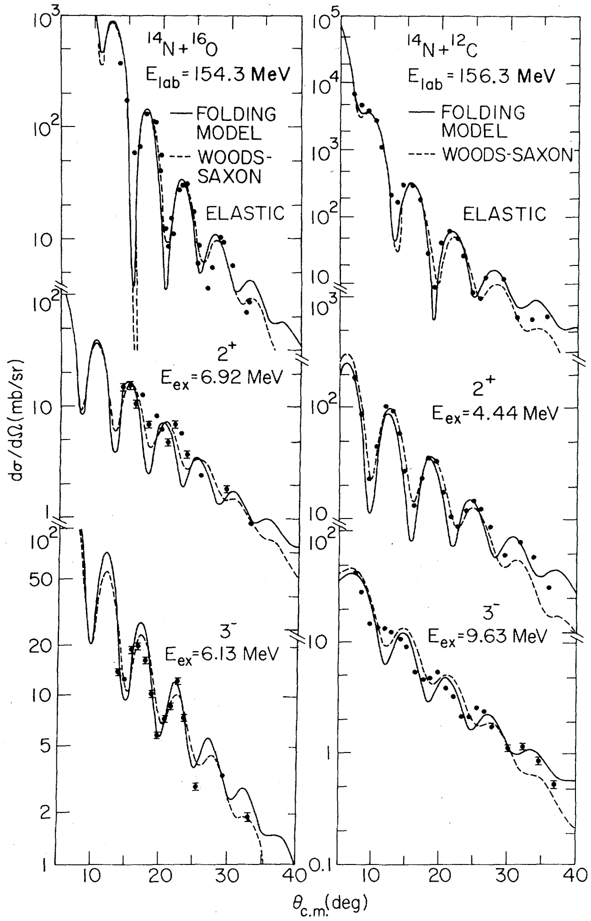

FIG. 2. Elastic and inelastic scattering cross sections for $^{14}N+^{16}O$ and $^{14}N+^{12}C$. Note that 0° is suppressed. Error bars indicate only the statistical errors. The data have a ±20% uncertainty in absolute values which is resolved in the usual manner by normalizing the measured to the calculated elastic cross sections. This does not affect either the *relative* elastic and inelastic cross sections or the relative **βR** values. There are 5% random uncertainties as discussed in the text. No corrections for finite angular acceptance or beam spot size have been applied.

a 0° measurement of the residual beam with the ion source off.

In Fig. 2 we compare the folding-model prediction with the data for the elastic scattering.[12] The agreement in the details of the elastic angular distributions provides a test of the folding model and its ingredients (densities and g matrix). Note that V_{WS} gives equally satisfactory agreement with the elastic data. Also shown in Fig. 2 are the inelastic cross sections evaluated in the distorted-wave Born approximation[13] and scaled to the data by the choice of $\beta_L R$ (meaning $\beta_L R_T$ or $\beta_L R_0$, depending on the model). The agreements of the calculated shapes are generally good, but, in detail, the more collective transitions (3^- in ^{16}O and 2^+ in ^{12}C) fit better. There are some ambiguous features in the fits to the other inelastic angular distributions, but the overall rate of fall-off is correctly reproduced. Shape difficulties for these states have also been encountered in phenomenological studies with α particles[14] and heavy ions.[15] There are idnications of improvement[3,14] in angular shape for these less collective cases when a microscopic target transition density is used in the folding model.

The central results of this effort are presented in Table I. For *all* transitions the deformation lengths extracted with the strongly L-dependent folding-model form factor agree well with those extracted from electromagnetic and inelastic proton studies. Additionally, the systematic difficulties found in the Woods-Saxon analyses shown in Table I are understood. In both experiments, the Woods-Saxon analyses predict a much smaller $\beta_3 R_0$ than the other measurements because of the lack of L dependence in the Woods-Saxon intrinsic form factor.

*Present address: Physics Department, Iowa State

TABLE I. Deformation lengths $\beta_L R$ in fermis extracted from this data with the L-dependent folding model and the phenomenlogical Woods-Saxon approach and compared with results of other experiments. Note that $R = 1.2A^{1/3}$ is assumed for electromagnetic $\beta_L R$.

Nucleus	State	This work: Folding	This work: Woods-Saxon	Electromagnetic and proton
^{16}O	2^+	0.90	0.67	0.86–0.88[a]
	3^-	2.1	1.2	2.0 –2.5[a]
^{12}C	2^+	1.6	1.3	1.5 –1.8[b]
	3^-	1.5	0.72	1.1 –1.4[b]

[a]See Ref. 16. [b]See Ref. 17.

University, Ames, Ia. 50010.

†Work supported in part by the U. S. Energy Research and Development Adminstration.

‡Work supported in part by the National Science Foundation.

[1]G. H. Rawitscher and R. A. Spicuzza, Phys. Lett. 37B, 221 (1971); V. R. W. Edwards and B. C. Sinha, Phys. Lett. 37B, 225 (1971).

[2]C. B. Dover, P. J. Moffa, and J. P. Vary, Phys. Lett. 56B, 4 (1975).

[3]P. J. Moffa, C. B. Dover, and J. P. Vary, to be published.

[4]J. P. Vary and C. B. Dover, Phys. Rev. Lett. 31, 1510 (1973); C. B. Dover and J. P. Vary, Brookhaven National Laboratory Reports No. BNL 19332 (unpublished) and No. BNL 19360 (unpublished); C. B. Dover and J. P. Vary, unpublished.

[5]J. Hüfner and C. Mahaux, Ann. Phys. (N.Y.) 73, 525 (1972).

[6]F. Petrovich, Ph.D. thesis, Michigan State University, 1970 (unpublished).

[7]J. L. Friar and J. W. Negele, Nucl. Phys. A240, 301 (1975).

[8]F. Malaguti and P. E. Hodgson, Nucl. Phys. A215, 243 (1973).

[9]J. W. Negele, Phys. Rev. C 1, 1260 (1970).

[10]J. S. Blair, Phys. Rev. 108, 827 (1957); N. Austern and J. S. Blair, Ann. Phys. (N.Y.) 33, 15 (1965).

[11]K. G. Nair, H. Voit, C. W. Towsley, M. Hamm, J. D. Bronson, and K. Nagatani, to be published.

[12]We are grateful to Dr. E. H. Auerbach for a copy of the elastic-scattering code ABACUS-HI.

[13]We would like to thank Dr. A. J. Baltz for providing us with a copy of the inelastic-scattering code.

[14]B. Tatischeff and I. Brissaud, Nucl. Phys. A155, 89 (1970); G. Tibell, A. A. Cowley, D. A. Goldberg, H. G. Pugh, N. S. Wall, and W. Reichart, University of Maryland Progress Report, 67, 1972 (unpublished).

[15]J. C. Hiebert and G. T. Garvey, Phys. Rev. 135, B346 (1964); P. K. Bindal, K. Nagatani, M. J. Schneider, and P. D. Bond, Phys. Rev. C 9, 2154 (1974).

[16]G. M. Crawley and G. T. Garvey, Phys. Rev. 160, 981 (1967); S. J. Skorka, J. Hertel, and T. W. Retz-Schmidt, Nucl. Data, Sect. A 2, 347 (1966), and references therein.

[17]G. R. Satchler, Nucl. Phys. A100, 497 (1967); M. P. Fricke and G. R. Satchler, Phys. Rev. 139, B567 (1965); R. M. Haybron, Nucl. Phys. 79, 33 (1966); P. H. Stelson and L. Grodzins, Nucl. Data, Sect. A 1, 21 (1965), and references therein.

Saturation of the Gentle Bump Instability in a Random Plasma

D. F. Escande
Laboratoire de Physique des Milieux Ionisés, Groupe de Recherche du Centre National de la Recherche Scientifique, Ecole Polytechnique, 91120 Palaiseau, France
(Received 17 June 1975)

It is shown that as a result of density fluctuations, the phase velocity domain of the gentle bump instability can be greatly broadened, leading to a different saturation of the instability. A part of the beam is accelerated and the turbulence power at saturation is lowered.

It has recently been pointed out that the growth rate of a high-frequency instability can be modified by a nonresonant interaction with low-frequency turbulence.[1,2] This effect is qualitatively similar[2] to the effect of density fluctuations on the spatial growth of an instability. The following investigation of the gentle bump instability in the presence of random inhomogeneities describes the saturation of this instability in the presence of nonresonant low-frequency turbulence. The saturation process presented here is different from the saturation of the gentle bump instability in the presence of a coherent ion fluctuation[3] which allows a part of the beam energy to be absorbed by the main plasma.

I consider a one-dimensional plasma with a density $n(x)=\bar{n}+\delta n(x)$ fluctuating around an average density $\bar{n}$. A fast beam (with mean velocity $u \gg v_{\text{th}}$, the thermal velocity), satisfying the O'Neil-Malmberg[4] criterion for the kinetic beam-plasma instability, is streaming from the point $x=0$ towards positive x. Provided that the typical scale for the density fluctuation is much greater than the typical wavelength of the instability, a given frequency ω is related to a local wave number $k(\omega,x)$ given by

$$\omega=\Omega(k(\omega,x),n(x)), \tag{1}$$

where $\Omega(k,n(x))=[\omega_p^2(x)+3k^2v_{\text{th}}^2]^{1/2}$ (Bohm-Gross relation) depends on x only through $n(x)$. With