

mize  $\chi^2$ :

$$c = c_0 A^{1/3};$$

$$Q_0 = Q_0(\beta).$$

Complete details are given in a paper submitted elsewhere.<sup>6</sup>

The results of our calculation for <sup>181</sup>Ta, <sup>165</sup>Ho, and <sup>169</sup>Tm are displayed in Tables I, II, and III, respectively. We indicate the values of the parameters for which these calculations were made. The values of the parameters are generally within the acceptable region of fit for the respective nuclei. If more than one transition is indicated, the energy is the weighted average (weighted according to calculated intensities of the *E*1 radiation). The corrections listed with the weighted averages represent the difference we would obtain for the weighted result calculated with and without making the corrections.

One observes from the tables that there is considerable variation in the total corrections from nuclide to nuclide. The total corrections for the *K* x-ray transitions vary by as much as 3.09 keV and for the *L* x-ray transitions the corrections vary as much as 3.73 keV. We also note that within a single nuclide, there is a large variation. In <sup>165</sup>Ho we encounter a variation as large as

3.25 keV in *K* x-ray transitions and 2.88 keV in the *L* x-ray transition.

In looking at the contributions of the individual parts to the total correction we find that the extended-space part varies by 2.25 keV for both *K* and *L* x-ray transitions. The variation in the nuclear-polarization correction varies by as much as 2.50 keV in <sup>165</sup>Ho.

It is, therefore, not possible to account for the effects of either correction by the addition of constant amounts of energy to the values of the calculated energies. Nor can the total correction be obtained by adding a constant number to the energy of the transitions calculated without the corrections.

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## Elastic *n-d* Scattering with the Reid Interaction

C. Stolk and J. A. Tjon

*Institute for Theoretical Physics, University of Utrecht, Utrecht, The Netherlands*

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For the case of the Reid soft-core potential the elastic neutron-deuteron scattering is studied. The three-nucleon equations are solved exactly for the pure *s*-wave parts of the two-nucleon *T* matrix while the higher partial-wave components of the force are treated perturbatively. Results are presented at 14.1 and 46.3 MeV. In particular, it is found that the pronounced minimum in the elastic cross section at higher energies, calculated with pure *s*-wave interactions, is filled in predominantly by the *d*-wave component of the deuteron.

Although there exist by now various three-body calculations on polarization phenomena, all of these studies have been carried out with separable forces.<sup>1</sup> On the other hand, solutions have been presented only for simple local *s*-wave potentials.<sup>2</sup> One of the remarkable results of the latter calculations is the considerable improvement of the elastic cross section in the forward direction compared to the results of *s*-wave separable interactions, whereas there is a signifi-

cant effect in this region of the presence of *p*-wave forces in the separable case. Therefore it is interesting to study the effects of inclusion of higher partial-wave components in the nuclear force in the case of local interactions.

In this Letter we report on a calculation of *n-d* elastic scattering using the full Reid soft-core interaction.<sup>3</sup> For this interaction exact numerical solution of the Faddeev equations is precluded by computational limitations in view of the

large number of coupled channels. We therefore have taken resort to a perturbational treatment of the non-*s*-wave components in the nuclear force. The same procedure has been successfully applied in the case of separable interactions.<sup>4,5</sup>

The general analysis of the Faddeev equations proceeds in the same way as in Ref. 2. The transition matrix for *n-d* elastic scattering, which is given in the notation of Ref. 2 by

$$M = \langle f | (s - H_0) | i \rangle_2^A + \frac{1}{2} \langle f | \sum_{klm} T_k^l (s + i\epsilon) | i \rangle_m^A, \quad (1)$$

$k \neq m, \quad l \neq 1,$

with  $T_k^l$  satisfying the Faddeev equations

$$T_k^l(z) = T_l(z) \delta_{kl} - \sum_{m \neq l} T_l(z) G_0(z) T_k^m(z), \quad (2)$$

can be treated perturbatively with respect to the non-*s*-wave parts of the two-nucleon interactions. Because of the occurrence of *d*-wave components in the deuteron wave function we may accordingly write for the initial and final states  $|i\rangle = |i^0\rangle + \lambda |i^1\rangle$  and  $|f\rangle = |f^0\rangle + \lambda |f^1\rangle$ , where  $\lambda |i^1\rangle$  and  $\lambda |f^1\rangle$  represent the *d*-wave parts of the deuteron. Furthermore, the two-nucleon *T* matrix

can be split into two parts,

$$T_l = T_l^{(0)} + \lambda T_l^{(1)}, \quad (3)$$

where  $T_l^{(0)}$  is identified as the  $^1S_0$  and *s-s* elements of the  $^3S_1$ - $^3D_1$  channel of the two-nucleon *T* matrix. For the second term in Eq. (3) we take the *s-d* and *d-d* matrix elements of the  $^3S_1$ - $^3D_1$  channel; the  $^1D_2$ ,  $^3D_2$ ,  $^1P_1$ ,  $^3P_0$ , and  $^3P_1$  channels; and the *p-p* matrix elements of the  $^3P_2$ - $^3F_2$  channel of the two-nucleon *T* matrix.

With use of the above separations, Eq. (1) can be simplified in a straightforward way up to the first order in the parameter  $\lambda$ . In so doing, the *S*-matrix element *M* can be expressed in terms of the solutions to the Faddeev equations with only *s*-wave two-body *T*-matrix elements. The latter equations are solved in the same way as in Ref. 2 employing the Padé method. As a check on the computer code, we used Pieper's separable potential set *D* to recalculate his numerical results at 10.04- and 22.7-MeV neutron lab energy.<sup>4</sup> The three-nucleon scattering amplitudes were in agreement with his results. Stability checks indicate that the numerical inaccuracy in our results is a few percent.

Subsequently, using the Reid soft-core potential, calculations were carried out up to 50-MeV neutron lab energy. Some results at 14.1 and 46.3 MeV are shown in Figs. 1-5. The differen-

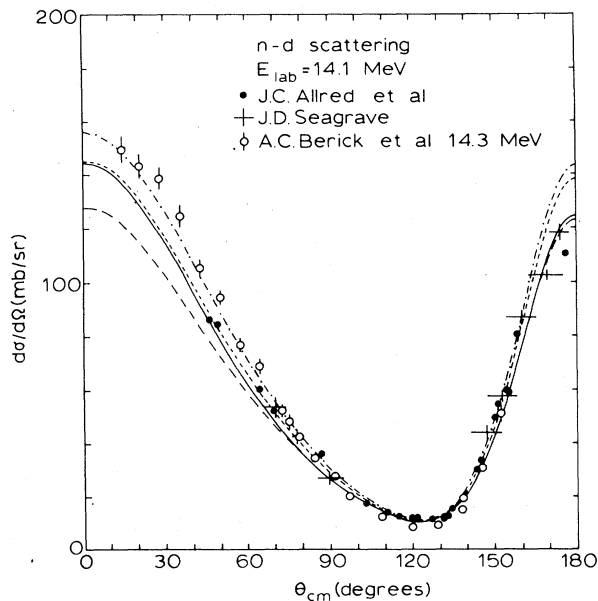


FIG. 1. Differential cross section at 14.1 MeV for various choices of two-nucleon interactions: ---, pure *s* waves in  $^1S_0$  and  $^3S_1$  and pure *s*-wave deuteron wave function normalized to 1. - · -,  $^1S_0$ ,  $^3S_1$ ,  $^1P_1$ ,  $^3P_0$ ,  $^3P_1$ ,  $^3P_2$ , and pure *s*-wave deuteron. —,  $^1S_0$ ,  $^3S_1$ - $^3D_1$ , and deuteron including *d*-wave component. ———, full Reid soft-core potential.

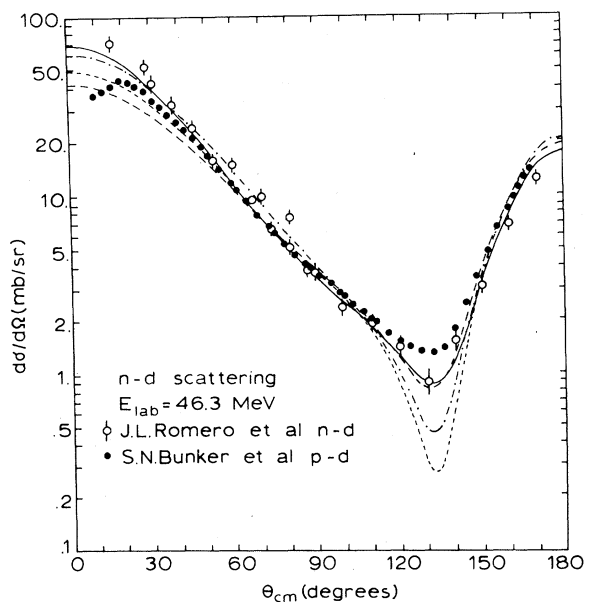


FIG. 2. Differential cross section at 46.3 MeV. The curves are the same as in Fig. 1.

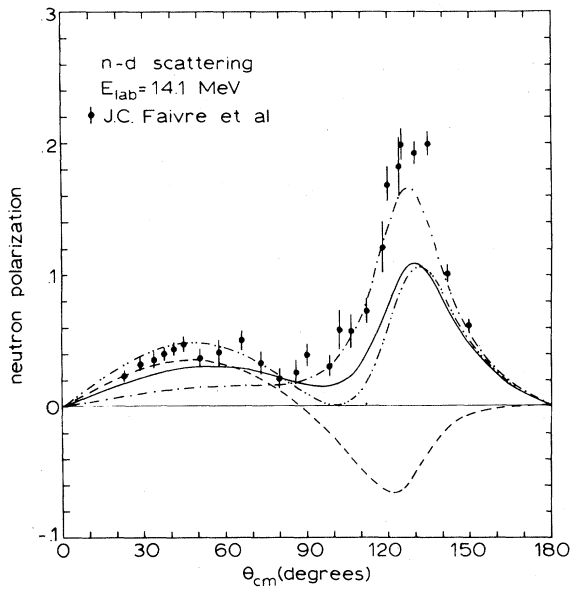


FIG. 3. Neutron polarization at 14.1 MeV: - · - ·,  $^1S_0$ ,  $^3S_1$ - $^3D_1$ ,  $^1P_1$ ,  $^3P_0$ ,  $^3P_1$ , and  $^3P_2$ . Other curves are the same as in Fig. 1.

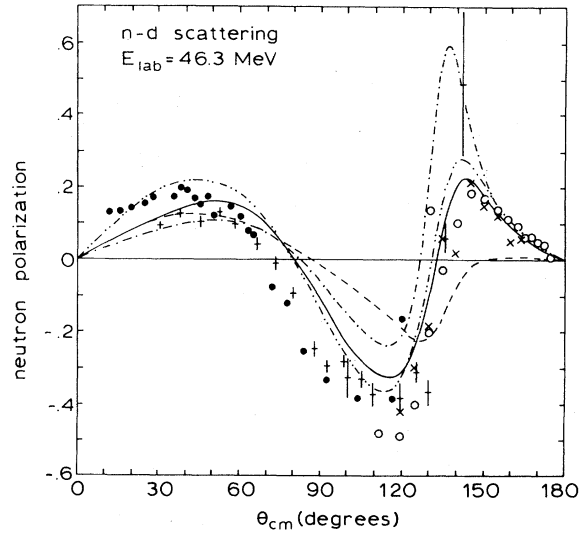


FIG. 4. Neutron polarization at 46.3 MeV. The curves are the same as in Fig. 3. The data are from Refs. 8 and 11.

tial cross section at 14.1 MeV is shown in Fig. 1, together with experimental points near this energy.<sup>6</sup> From this we see that if we include only *s*-wave components in the two-body *T* matrix and keep only the *s*-wave part of the deuteron with normalization 1 the results are similar to the case of the simple *s*-wave potentials.<sup>2</sup> By comparing Fig. 1 with Fig. 7 of P, it is clear that inclusion of the *p*-wave forces gives an increase in the forward direction which is less for the local potential than for the separable potential. This is intuitively expected because the repulsive core in the local *s*-wave potential reduces the effect of the higher partial-wave forces. In Fig. 2 is shown the cross section at 46.3 MeV.<sup>7,8</sup> The very deep minimum which is found by using only *s*-wave parts of the two-body *T* matrix is raised significantly by including the *d*-wave component of the deuteron. This strongly suggests that the *d*-state probability of the deuteron may be obtained by detailed study of this region. It should be noted that a similar effect has been suggested to explain the discrepancy found at the position of a minimum in the breakup cross section.<sup>9</sup>

Figures 3 and 4 show the nucleon polarization at 14.1 and 46.3 MeV.<sup>8, 10, 11</sup> Our results confirm the conclusions of the other authors, that the dominant contribution comes from the *p*-

wave forces but that the other partial-wave components also give important contributions. For this polarization we find a reasonable agreement with the data only at higher energies. At 14.1 MeV the separable potentials give a better fit, while at all angles, but especially in the

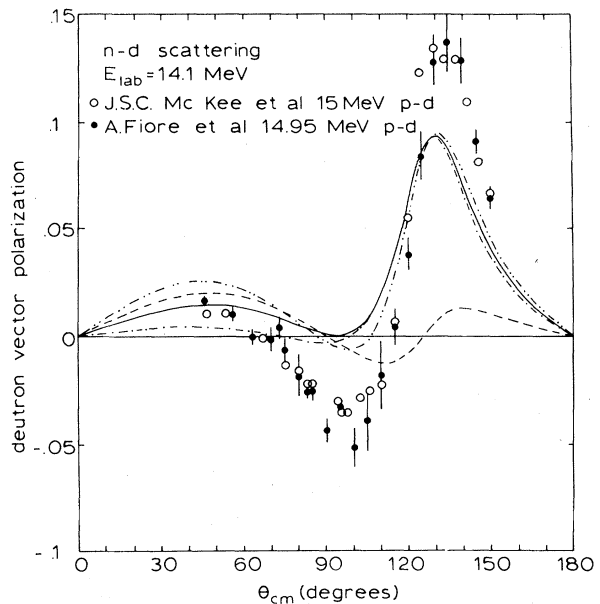


FIG. 5. Deuteron vector polarization at 14.1 MeV. The curves are the same as in Fig. 3.

backward maximum, our curve is too low. Moreover, the calculated height of the backward maximum is energy dependent, whereas experimentally it is found to be nearly constant between 15 and 50 MeV.<sup>10</sup> Finally, the deuteron vector polarization at 14.1 MeV is displayed in Fig. 5. There is only qualitative agreement with the data at 14.95 MeV. The results of Fayard, Lamot, and Elbaz<sup>5</sup> and Doleschall<sup>12</sup> indicate that here part of the differences can be attributed to the use of perturbation theory. The dip for intermediate angles is not deep enough. However, since the experimental minimum is found to be strongly energy dependent, the situation is probably slightly better than shown.<sup>13</sup>

To conclude, with the notable exception of the forward differential cross section where the presence of the repulsion in the local *s*-wave potentials reduces the effect of the higher partial-wave forces, the sensitivities of the *n-d* observables are qualitatively the same for the local potentials as for the separable potentials. Furthermore, the pronounced dip near the minimum in the cross section which is found at higher energies using only *s*-wave potentials is filled in predominantly by the contribution from the *d*-wave component of the deuteron. As a result, the description with local interactions is in reasonable agreement with the data for the differential cross section over the whole energy range considered.

We thank Dr. C. Fayard and Dr. S. C. Pieper for discussions and for providing us with numerical details of their calculations.

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## $\alpha + \alpha$ Reaction and the Origin of ${}^7\text{Li}$

C. H. King, H. H. Rossner,\* Sam M. Austin, and W. S. Chien

*Cyclotron Laboratory and Physics Department, Michigan State University, East Lansing, Michigan 48824*†

and

G. J. Mathews, V. E. Viola, Jr., and R. G. Clark

*Cyclotron Laboratory and Chemistry Department, University of Maryland, College Park, Maryland 20742*‡

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Cross sections are presented for the production of  ${}^7\text{Li}$  and  ${}^7\text{Be}$  in the  $\alpha + \alpha$  reaction between threshold and 140 MeV. Implications of these measurements for the problem of the origin of  ${}^7\text{Li}$  in the universe are discussed.

The observed abundances of most stable nuclides can be understood in terms of two main processes: (1) nucleosynthesis during stellar evolution,<sup>1</sup> which applies principally to carbon

and heavier elements, and (2) spallation of interstellar matter by galactic cosmic rays,<sup>2</sup> which is most important for the elements with  $A < 12$ . There are, however, a few nuclides whose abun-