Correction to Muonic X-Ray Spectra for Nuclear Polarization and Extended Space

D. McLoughlin and S. Raboy
Department of Physics, State University of New York at Binghamton, Binghamton, New York 13901

and

E. Deci
New School of Liberal Arts and Institute for Nuclear Theory, Brooklyn College of The City University of New York, Brooklyn, New York 11210
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The nuclear-polarization and extended-space corrections are calculated for x-ray transitions of muonic atoms of ^{169}Tm, ^{181}Ta, and ^{165}Ho for acceptable values of the three parameters: the half-radius of the charge distribution, the surface thickness, and the intrinsic quadrupole moment. It is shown that there is enough variation in the values of the corrections for the transitions in a given nuclide to preclude the assignment of a constant amount to all the transitions.

The results of calculations of nuclear-polarization corrections to the x-ray spectra of muonic atoms of odd-A nuclei are presented to show that the mere addition of a constant energy to the traditional calculations involving the dynamical quadrupole interactions is not a sound procedure. These corrections are inspired by the earlier work of Cooper and Henley[1] and more recently by Chen.[2]

We have calculated the corrections for the nuclei ^{165}Ho, ^{169}Tm, and ^{181}Ta which are necessary to obtain satisfactory fits to experimental data obtained by the group of collaborators representing the State University of New York at Binghamton, Argonne National Laboratory, and Carnegie-Mellon University.[3-5] We have concentrated our work at present on the corrections for specific odd-A nuclei in the strongly deformed region. We will at some future date make similar sets of calculations for even-even nuclei in the strongly deformed region.

The details of the calculations will appear in another publication.[6] But briefly the nuclear-polarization correction[2] is made to reduce the systematic errors incurred when the matrix of the quadrupole interaction between the muon in the p or d orbit and the nucleus in the ground or excited state is diagonalized for a subset of the complete set of the basis states for this problem. In this subset muonic states of the same principal quantum number and only four or five low-lying nuclear states in the ground-state rotational band are chosen. For identification purposes the corrections to compensate for missing muonic states will be called the "extended-space corrections" and we will reserve the term "nuclear-polarization corrections" for those corrections to compensate for the missing nuclear states.

The nuclear-polarization corrections were calculated by expanding the electrostatic interaction between the muon and the nucleus to include the third-order spherical harmonics. The correc-

TABLE I. Corrections to x rays of muonic atoms of ^{181}Ta. $c_0 = 1.1055$ fm, $t = 2.389$ fm, and $Q_0 = 7.38$ b.

F	F'	Transitions Calculated Energies (KeV)	Nuclear Polarization Correction (KeV)	Extended Space Correction (KeV)	Total Correlations (KeV)
		K x-ray Transitions ($2F \to 1F'$)			
4 4	4 5	5097.42	6.54	1.87	8.41
5 5	4 5	5135.02	6.52	1.56	8.08
3 3	3 4	5138.45	6.46	1.07	7.53
4 4	3 4	5144.20	6.46	1.20	7.66
4 4	4 5	5205.40	6.55	1.93	8.48
4 4	3 4	5233.42	6.51	1.91	8.42
5	4	5271.01	6.54	1.87	8.41
5 5	4 5	5278.18	6.52	1.45	7.97
3 3	3 4	5309.21	6.59	1.45	8.04
5	4	5326.37	6.55	1.92	8.47
4 4	3 4	5341.39	6.51	1.90	8.41
2	3	5356.64	6.64	2.98	9.62
		L x-ray Transitions ($3F \to 2F'$)			
6	5	1925.19	3.74	1.74	5.48
5	5	2060.61	3.85	2.52	6.37
6	5	2068.35	3.82	2.52	6.34
5	4	2077.35	3.75	2.08	5.83

tions corresponding to $l=3$ were sufficiently diminished from the corrections due to the $l=1$ and $l=2$ terms of the interactions that we were able to neglect the corrections contributed by the $l=4$ term.

In the nuclear-polarization corrections the two-body nuclear correlation terms appeared. We formed this function from a set of single-particle states which are solutions to the problem of the isotropic harmonic-oscillator potential. The strength of the harmonic-oscillator potential was adjusted to give the same intrinsic quadrupole moment as the modified Fermi distribution of charge which we used.

The radial equations which had to be solved for the extended-space corrections and the nuclear-polarization corrections are inhomogeneous Dirac equations originating from the relativistic treatment of the muonic atom. A modified Fermi distribution of the type

$$\rho(r,\theta)=\rho_0\{1+\exp([r+\beta P_2(\cos\theta)-c]4.4/t)\}^{-1} \tag{1}$$

was assumed for the charge. The values of the parameters c_0, Q_0, and t were varied to mini-

TABLE II. Corrections to x rays of muonic atoms of ^{165}Ho. $c_0=1.1054$ fm, $t=2.23$ fm, and $Q_0=4.90$ b.

F'	F	Calculated Energies (KeV)	Nuclear Polarization Correction (KeV)	Extended Space Correction (KeV)	Total Correlations (KeV)
		K x-ray Transitions (2F → 1F')			
4 4	5 4	4679.25	6.23	2.70	8.93
5 5 5 4 4 3 3	6 5 4 4 3 4 3	4706.09	4.77	1.53	6.30
5	4	4755.61	6.23	2.61	8.84
4 4	4 3	4772.17	4.48	2.67	7.15
4 4	4 5	4779.24	6.23	2.94	9.17
5	4	4798.88	4.15	2.39	6.54
5 5	5 4	4826.73	6.21	3.31	9.52
3 3	4 3	4852.19	5.29	3.39	8.68
4 4	5 3	4872.13	4.45	2.91	7.36
2	3	4894.41	5.92	3.53	9.45
5	4	4919.46	4.27	3.78	8.05
		L x-ray Transitions (3F → 2F')			
6 3	5 2	1659.84	2.05	1.66	3.71
2 1	2 2	1687.67	3.81	1.45	5.26
5 4	4 4	1701.01	2.20	2.08	4.28
5 2 3	5 3 3	1724.23	1.31	1.57	2.88
6	5	1731.01	2.30	2.40	4.70
5 4	4 4	1800.90	2.19	2.36	4.55
4 3	4 4	1844.50	2.10	3.50	5.60
5 4 3	4 3 3	1850.45	2.06	3.70	5.76

TABLE III. Total corrections to K and L x rays of muonic atoms of ^{169}Tm. $c_0=1.1085$ fm, $t=2.23$ fm, and $Q_0=8.179$ b.

F	F'	Calculated Energies (KeV)	Total Correction (KeV)
		K x-ray Transitions (2F → 1F')	
2 2	3 2	4809.14	8.32
1 1	1 2	4841.37	7.80
0	1	4845.07	7.28
1 1	1 0	4849.77	7.79
1 1	2 1	4854.42	8.04
2 1 1	1 0 1	4862.82	8.04
2 2	1 2	4918.91	8.79
2	1	4927.31	8.79
2	3	4938.68	9.64
1 1	1 2	4997.37	8.44
1 1	0 1	5005.77	9.43
		L x-ray Transitions (3F → 2F')	
3	2	1775.82	5.36
3 2	2 1	1782.70	4.36
2	1	1826.49	4.77
3	2	1904.81	5.42
2	1	1947.65	6.16
1	1	1958.33	6.39
1 1 2	0 1 2	1962.26	6.69

mize χ^2:

$$c = c_0 A^{1/3};$$

$$Q_0 = Q_0(\beta).$$

Complete details are given in a paper submitted elsewhere.[6]

The results of our calculation for ^{181}Ta, ^{165}Ho, and ^{169}Tm are displayed in Tables I, II, and III, respectively. We indicate the values of the parameters for which these calculations were made. The values of the parameters are generally within the acceptable region of fit for the respective nuclei. If more than one transition is indicated, the energy is the weighted average (weighted according to calculated intensities of the $E1$ radiation). The corrections listed with the weighted averages represent the difference we would obtain for the weighted result calculated with and without making the corrections.

One observes from the tables that there is considerable variation in the total corrections from nuclide to nuclide. The total corrections for the K x-ray transitions vary by as much as 3.09 keV and for the L x-ray transitions the corrections vary as much as 3.73 keV. We also note that within a single nuclide, there is a large variation. In ^{165}Ho we encounter a variation as large as 3.25 keV in K x-ray transitions and 2.88 keV in the L x-ray transition.

In looking at the contributions of the individual parts to the total correction we find that the extended-space part varies by 2.25 keV for both K and L x-ray transitions. The variation in the nuclear-polarization correction varies by as much as 2.50 keV in ^{165}Ho.

It is, therefore, not possible to account for the effects of either correction by the addition of constant amounts of energy to the values of the calculated energies. Nor can the total correction be obtained by adding a constant number to the energy of the transitions calculated without the corrections.

[1] L. N. Cooper and B. M. Henley, Phys. Rev. 92, 801 (1955).

[2] M. Y. Chen, Ph.D. thesis, Princeton University, 1968 (unpublished), and Phys. Rev. C 1, 1167 (1970).

[3] A. Gaigalas, Ph.D. thesis, Carnegie-Mellon University, 1967 (unpublished).

[4] E. Deci, Ph.D. thesis, State University of New York at Binghamton, 1973 (unpublished).

[5] D. McLoughlin, Ph.D. thesis, State University of New York at Binghamton, 1974 (unpublished).

[6] D. McLoughlin *et al.*, to be published.

Elastic *n-d* Scattering with the Reid Interaction

C. Stolk and J. A. Tjon

Institute for Theoretical Physics, University of Utrecht, Utrecht, The Netherlands

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For the case of the Reid soft-core potential the elastic neutron-deuteron scattering is studied. The three-nucleon equations are solved exactly for the pure s-wave parts of the two-nucleon T matrix while the higher partial-wave components of the force are treated perturbatively. Results are presented at 14.1 and 46.3 MeV. In particular, it is found that the pronounced minimum in the elastic cross section at higher energies, calculated with pure s-wave interactions, is filled in predominantly by the d-wave component of the deuteron.

Although there exist by now various three-body calculations on polarization phenomena, all of these studies have been carried out with separable forces.[1] On the other hand, solutions have been presented only for simple local s-wave potentials.[2] One of the remarkable results of the latter calculations is the considerable improvement of the elastic cross section in the forward direction compared to the results of s-wave separable interactions, whereas there is a significant effect in this region of the presence of p-wave forces in the separable case. Therefore it is interesting to study the effects of inclusion of higher partial-wave components in the nuclear force in the case of local interactions.

In this Letter we report on a calculation of n-d elastic scattering using the full Reid soft-core interaction.[3] For this interaction exact numerical solution of the Faddeev equations is precluded by computational limitations in view of the