Self-Focusing Relativistic Electron Streams in Plasmas

J. L. Cox, Jr.
Department of Physics and Geophysical Sciences, Old Dominion University, Norfolk, Virginia 23508
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A relativistic electron stream propagating through a dense plasma induces current and charge densities which determine how the stream can self-focus. Magnetic self-focusing is possible because stream-current neutralization, although extensive, is not complete. Electric self-focusing can occur because the stream charge becomes overneutralized when the net current is smaller than a critical value. Under some circumstances, the latter process can cause the stream to focus into a series of electron bunches.

An electron stream propagating through a plasma can self-focus either magnetically or electrically depending on the magnitude of the net stream current and on the extent to which the stream space charge is compensated by positive plasma ions. If its space charge is partially neutralized and if its net current is large enough, the stream will self-focus magnetically.[1] On the other hand, an electron stream with net current too low to experience magnetic self-focusing will self-focus electrically if the net charge density in the stream is positive and sufficiently large.[2,3] The former focusing process is typically manifest in high-current streams, while the latter has been studied primarily in relation to low-current streams propagating through tenuous plasmas generated by collisions between the stream electrons and background neutral atoms. A recent paper by McCorkle,[4] however, shows that the electric self-focusing process can also be important in high-current, relativistic electron streams propagating through neutral gases.

The net current, net charge density, and, consequently, the self-focusing properties of a relativistic electron stream propagating through a dense plasma depend on the current and charge density induced in the plasma by the stream. Previously published work by the author and coworkers[5-7] and by others[8-20] indicates that such a stream is nearly current neutralized by an induced reverse current which counterstreams along the direction of the stream axis and is nearly space-charge neutralized by the induced plasma charge density if the frequency of momentum-transfer collisions between plasma electrons and ions is not too great and if the dimensions of the stream are large compared to c/ω_p. (ω_p is the plasma frequency of the plasma electrons and c is the speed of light.) The role of the induced reverse current along the axial direction in limiting the extent to which the injected stream can magnetically self-focus, thereby allowing the propagation of streams with currents in excess of the Alfvén limit, has been recognized previously. However, the relationship between the induced axial current, the induced current in the radial direction, and the induced charge density has not been explored adequately. It is shown below that this relationship prohibits total current neutralization but allows charge neutralization and even charge overneutralization for the general conditions cited above. Thus, these streams can experience electric self-focusing and also, for large enough injected currents, magnetic self-focusing.

The stream considered in the present discussion is an axially symmetric pulse of relativistic electrons which has finite length and finite radius. Its current density can have arbitrary dependence on the axial coordinate, z, but is assumed to be uniform in the radial coordinate, r, for $0 \leq r \leq a$ and to be zero for $r > a$, where a is the stream radius. The pulse is assumed to travel at a constant velocity V in the positive z direction through a dense uniform plasma of infinite spatial extent.

The behavior of this system can be described by Maxwell's equations and the generalized Ohm's law:

$$\nabla\cdot\vec{E} = \frac{\rho_b+\rho_p}{\epsilon_0}; \quad \nabla\times\vec{B} = \mu_0(\vec{j}_b+\vec{j}_p) + c^{-2}\frac{\partial\vec{E}}{\partial t}; \tag{1}$$

$$\nabla\times\vec{E} = -\,\partial\vec{B}/\partial t; \quad \nabla\cdot\vec{B} = 0; \tag{2}$$

$$\frac{\partial\vec{j}_p}{\partial t} + \nu_m\vec{j}_p = \frac{ne^2}{m}\vec{E} + \frac{e}{m}\vec{j}_p\times\vec{B}. \tag{3}$$

Rationalized mks units have been used. The current density of the injected stream is $\vec{j}_b$, the current density induced in the plasma is $\vec{j}_p$, and ρ_b and ρ_p are the corresponding charge densities. The electronic mass is m, and the electronic charge is $-e$. Since the electron density, n, of plasma electrons is assumed to be much greater than the density of the injected electron stream, n and the frequency, ν_m, of momentum-transfer collisions between plasma electrons and ions are treated as constants.

Equations (1) imply total charge conservation; i.e., that $\partial(\rho_b+\rho_p)/\partial t+\nabla\cdot(\vec{j}_b+\vec{j}_p)=0$. If it is assumed that the stream charge is conserved as it propagates through the plasma, it then follows that $\partial\rho_b/\partial t+\nabla\cdot\vec{j}_b=0$. These two equations then imply that $\partial\rho_p/\partial t+\nabla\cdot\vec{j}_p=0$; i.e., that the plasma charge is conserved.

Consider the cylindrical volume illustrated in Fig. 1 which is coaxial with the injected stream pulse and has a radius equal to the stream radius, a. Let it extend to infinity in the positive z direction, and let it be bounded on the left by a plane which is normal to the z axis and is located at arbitrary axial position z. As the stream pulse enters this volume from the left, currents induced in the plasma flow across the bounding surface causing the removal of plasma electrons from the volume. If the continuity equation for the plasma charge is integrated over this cylindrical volume, it is found that $I_{pz}(\eta)=Vq_p(\eta)+I_{pr}(\eta)$, where $\eta=Vt-z$; $I_{pz}(\eta)=\int_0^a j_{pz}(r,\eta)2\pi r\,dr$ is the induced plasma current flowing across the plane located at z, $I_{pr}(\eta)=2\pi a\int_{-\infty}^{\eta} j_{pr}(a,\eta')\,d\eta'$ is the induced plasma current flowing across the cylindrical surface, and $q_p(\eta)=\int_0^a\rho_p(r,\eta)2\pi r\,dr$ is the charge per unit length induced in the plasma within the stream channel. This result can be added to $I_b(\eta)=Vq_b(\eta)$, where $I_b(\eta)$ is the injected stream current and $q_b(\eta)$ is the injected stream charge per unit length. One thus obtains

$$I(\eta)=Vq(\eta)+I_{pr}(\eta), \tag{4}$$

where $I(\eta)=I_b(\eta)+I_{pz}(\eta)$ is the net current within the stream channel and $q(\eta)=q_b(\eta)+q_p(\eta)$ is the net charge per unit length within the stream channel.[21]

The Bennett critical-current condition[1] and Eq. (4) can be used to determine whether or not the injected stream can self-focus magnetically. According to the Bennett condition, magnetic self-focusing is possible if the net current within the stream I is greater than $c(8\pi\epsilon_0 N\psi+q^2)^{1/2}$, where N is the total number of particles (including plasma particles) per unit length of stream, ψ is the mean kinetic energy per particle due to motion transverse to the stream axis, and q is the net charge per unit length of stream. If the net current given by Eq. (4) is substituted into the Bennett condition, one finds that magnetic self-focusing can occur if

$$I_{pr}^2+2VqI_{pr}>8\pi\epsilon_0 c^2N\psi+c^2\gamma^{-2}q^2, \tag{5}$$

where $\gamma^{-2}=1-V^2/c^2$. Thus, it is clear that the net stream current is large enough to produce magnetic self-focusing only if the induced plasma current in the radial direction is sufficiently large to satisfy Eq. (5). The physical reason for this is that the divergence of plasma electrons in the radial direction provides the charge which partially neutralizes the space charge of the net current I, thereby making magnetic self-focusing possible. If I_{pr} were zero, then I would equal Vq; i.e., the net current and net charge per unit length would be related as are the current and charge per unit length in a stream which has no charge neutralization.

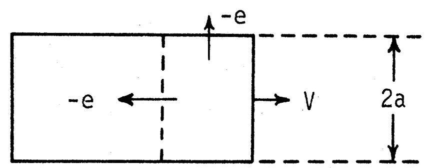

FIG. 1. Dashed lines denote the volume of integration; solid lines denote the stream pulse.

By the use of previously published results,[6,7] it can be shown[22] that for $a\gg V/\omega_p$, $\nu_m\ll\omega_p$, and $\beta^2>0$,

$$j_{pr}(a,\eta)=(V/2)\int_{-\infty}^{\eta}\exp[-\alpha(\eta-\xi)/2]\sin[\beta(\eta-\xi)][\partial\rho_b(\xi)/\partial\xi]\,d\xi \tag{6}$$

and

$$\rho_p(\eta)=-(\omega_p^2/\beta V^2)\int_{-\infty}^{\eta}\exp[-\alpha(\eta-\xi)/2]\sin[\beta(\eta-\xi)]\rho_b(\xi)\,d\xi, \tag{7}$$

where j_{pr} is the radial component of $\vec{j}_p$, $\beta^2=\omega_p^2/V^2-\nu_m^2/4V^2$, and $\alpha=\nu_m/V$. Equations (6) and (7) can be used to estimate the magnitudes of I_{pr} and q once the functional form of $\rho_b(\eta)$ has been specified. As a particular example, let $\rho_b(\eta)$ be chosen as follows: $\rho_b(\eta)=(\rho_{b0}/2\pi)[2\pi\eta/l-\sin(2\pi\eta/l)]$ for $0\le\eta\le l$; $\rho_b(\eta)=\rho_{b0}$ for $l\le\eta\le L-l$; $\rho_b(\eta)=(\rho_{b0}/2\pi)\{-2\pi(\eta-L)/l+\sin[2\pi(\eta-L)/l]\}$ for $L-l\le\eta\le L$, where ρ_{b0}, the length of the stream head l, and the total stream length L are constants. This stream pulse is illustrated in

Fig. 2. If it is also assumed that $l \gg V/\omega_p$, it can be shown that for $0 \leqslant \eta \leqslant l$, $I_{pr} \simeq VI_{b0}\eta/al\omega_p$ and

$$q(\eta) \simeq (\alpha I_{b0}V/l\omega_p^2)[1-\cos(2\pi\eta/l)] + (2\pi VI_{b0}/l^2\omega_p^2)\sin(2\pi\eta/l);$$

for $l \leqslant \eta \leqslant L-l$, $I_{pr} \simeq VI_{b0}/a\omega_p$ and

$$q(\eta) \simeq (4\pi^2V^2I_{b0}/l^3\omega_p^3)\{\exp[-(\alpha/2)(\eta-l)]\sin[\beta(\eta-l)] - \exp[-(\alpha/2)\eta]\sin(\beta\eta)\},$$

where $I_{b0} = \rho_{b0}V\pi a^2$. Since I_{pr} is always greater than Vq for the conditions cited, Eq. (4) shows that $I(\eta) \neq 0$; i.e., that current neutralization cannot be complete although it may be nearly so.

By substituting these results into Eq. (5), one can show that for either $0 \leqslant \eta \leqslant l$ or $l \leqslant \eta \leqslant L-l$, magnetic self-focusing is possible if $I_{b0} > (2\epsilon_0 N\psi/\pi)^{1/2}a\omega_p$. This result indicates that magnetic self-focusing is enhanced by injecting the relativistic electron stream so that its energy associated with transverse motion and its radius are as small as possible.

That this condition for magnetic self-focusing is not easily satisfied can be illustrated by reference to the experiments performed by Roberts and Bennett.[8] According to the criterion presented above, their injected current would have had to be approximately 10 times larger than it was in order for magnetic self-focusing to have occurred. Thus it does not appear that their streams were focused by this mechanism after they passed through the anode of the stream-forming diode.

The possibility of electric focusing can be assessed by examination of the radial potential variation within the stream. Previously published work[6] can be used to show that for $a\omega_p/V \gg 1$ the magnitude of the electric scalar potential is nearly constant along the radial direction except within a thin shell at the stream radius where it decreases rapidly. It can be shown[23] that $\Delta\varphi \simeq (c^2/2\pi\epsilon_0\omega_p^2a^2)q$, where $\Delta\varphi = \varphi(0,\eta) - \varphi(a,\eta)$. Electric self-focusing results if $e\Delta\varphi \geqslant \psi$. Substitution of these expressions into Eq. (4) yields $I - I_{pr} \geqslant (\pi a^2 \times \epsilon_0\omega_p^2V/ec^2)\psi$ as the condition for electric self-focusing. Since $I = -|I|$ and $I_{pr} = -|I_{pr}|$, this condition can be written as follows:

$$|I_{pr}| - |I| \geqslant (\pi a^2\epsilon_0 V\omega_p^2/ec^2)\psi. \qquad (8)$$

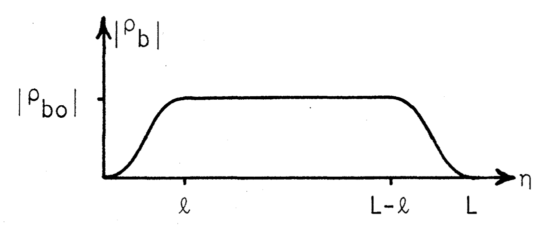

FIG. 2. The stream pulse.

This condition is similar to the one derived by McCorkle[4] in that electric self-focusing occurs when $|I|$ is sufficiently small.

An important characteristic of the electric self-focusing experienced by the streams considered here is that the amount of focusing is not uniform along the stream axis. The special example considered above shows that $q(\eta)$ varies sinusoidally along the flat portion of the stream pulse ($l \leqslant \eta \leqslant L-l$), a result which does not depend on the shape of the rising portion of the pulse. Moreover, the distribution of net charge travels with the same speed as the stream. Thus, the net charge in a given segment of the stream is stationary, a circumstance which can cause the stream to focus into a series of coherent electron bunches separated successively by a distance $\sim\pi V/\omega_p$.

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[21]The result presented in Eq. (4) is valid for a stream pulse that travels with uniform velocity through a uniform plasma; i.e., a pulse which is a function of z and t only in the combination $Vt-z$. It does not depend on the assumption of an infinite plasma, although the detailed forms of the various terms are influenced by the presence or absence of boundaries.

[22]These expressions were derived by neglecting the $\vec{j}_p \times \vec{B}$ term in Eq. (3). This is consistent with the results obtained: The stream is quickly current-neutralized except within a thin shell at its radius. When the injected current is large enough to make the net current in this shell significant, as is now being considered, the magnetic force causes the plasma electrons in this shell to antipinch. Consequently, I_{pr} increases and the critical current for magnetic self-focusing is lowered below the value given here. However, the concomitant increase in positive space charge in the shell limits this process and keeps the critical current from being lowered greatly.

[23]This result was derived, under the assumption that $a\omega_p \gg 1$, by comparing the expression for $\tilde{\varphi}$ found in Ref. 6 with the expression for $\tilde{\rho}$ obtained by use of Eq. (10) in Ref. 7. The tilde denotes the Fourier transform with respect to η as defined in Ref. 6.

Strong-Turbulence Theory and the Transition from Landau to Collisional Damping

C. T. Dum

Max-Planck-Institut für Plasmaphysik, 8046 Garching bei München, Germany

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New methods are introduced for a quantitative evaluation of the dielectric constant describing the interaction of a long-wavelength test wave with electrons in the presence of electron-ion collisions or small-scale turbulence. It is shown that the usual resonance-broadening arguments of strong-turbulence theory do not apply.

Collisional effects on wave propagation have been investigated by many authors. In the strongly collisional regime, $\omega/\nu \ll 1$, $k\lambda \ll 1$, the dispersion relation may be obtained from the two-fluid transport equations.[1] In the weakly collisional regime corrections to Landau damping have been found either from the Landau collision term by iteration or by use of model collision terms such as the Bernstein-Greene-Kruskal or Fokker-Planck term with constant diffusion and friction coefficients. In the case of ion-sound and related modes where electrons of all speeds $v > v_{\rm ph} \ll v_e$ can resonate with the wave, such procedures become dubious on two grounds.[2] The collision frequency for the dominant process, pitch-angle scattering by electron-ion collisions, is strongly velocity dependent, $\nu(v) = \nu_e (v_e/v)^3$, and iterative procedures cannot be applied to resonant particles.

The breakdown of iterative procedures for resonant particles is the starting point of Dupree's perturbation theory for strong turbulence[3] and related theories. A principal result of Dupree's theory is the *broadening* of wave-particle resonances $\omega - \vec{k}\cdot\vec{v} = 0$. The broadening is estimated as $\Delta\omega = (\frac{1}{3}k^2 D)^{1/3}$, where $D(v)$ is the velocity diffusion coefficient. Not even the solution of a simplified diffusion equation for the ensemble-averaged orbits has been obtained however. By various methods or simply by ignoring the velocity dependence of $\overleftrightarrow{D}(v)$ one arrives at

$$\exp[i\vec{k}\cdot\vec{x}(-t)] = \exp[i\vec{k}\cdot(\vec{x}-\vec{v}t) - \tfrac{1}{3}\vec{k}\cdot\overleftrightarrow{D}\cdot\vec{k}t] \qquad (1)$$

replacing the usual unpertubed orbits. Accordingly, the usual resonant denominators are replaced by the Laplace transform of (1). Customarily even further approximations are made to replace $\delta(\omega - \vec{k}\cdot\vec{v})$ by a Lorentzian or square