Reactive Content of the Single-Scattering Optical Potential*

P. C. Tandy, Edward F. Redish, and D. Bollé†‡
Department of Physics and Astronomy, University of Maryland, College Park, Maryland 20742
(Received 23 June 1975)

A unitarity relation is derived for the nucleon-nucleus scattering amplitude which displays the reaction mechanisms associated with the absorptive part of the single-scattering optical potential. The reactions implicit in the absorption are nucleon pickup and knockout. The transition amplitudes for these processes are extracted from the unitarity relation and their accuracy is discussed.

Most intermediate- and high-energy ($E > 100$ MeV) calculations of the nucleon-nucleus optical potential are carried out using the multiple-scattering theory[1] in the form given by Kerman, McManus, and Thaler[2] (KMT). Within this framework, however, the physical meaning of the absorptive part has remained obscure since the reactive content of the absorption cross section has not been investigated. The absorptive part implicitly contains information about the reaction mechanisms assumed; therefore, a knowledge of these processes gives an important self-consistency check on the validity of the approximations involved. In this Letter we explicate the reactive content of the single-scattering optical potential.

It is observed experimentally that single-nucleon knockout accounts for most of the reaction cross section above about 100 MeV.[3] For the intermediate states, most KMT calculations approximate the motion of the two nucleons by plane waves. Since these should have good overlap with the three-body knockout states, it is generally assumed that the calculation yields a reasonable imaginary part. However, calculations have now reached the point where small corrections to the optical potential are being investigated in order to search for nuclear correlations.[4] At this level, qualitative arguments concerning the accuracy of the first term are insufficient.

We here consider the single-scattering optical potential including the recoil of the struck nucleon and the influence of the nuclear medium. We derive a unitarity relation for the full nucleon-nucleus T matrix which explicitly displays the inelastic processes described by the absorptive part.[5] The reactive processes implicit in the first-order optical potential are single-nucleon pickup and knockout reactions. We also extract the reaction amplitudes for these inelastic processes from the unitarity equation. These amplitudes fail to include sufficient absorption in the final states (when compared with distorted-wave approximations). Consequently, we expect the imaginary part of the optical potential to be significantly overestimated. We consider the magnitude of this effect and suggest corrections to the KMT formalism.

The operator $T(E)$ describing elastic nucleon-nucleus scattering due to an optical potential $U(E)$ is given by the solution of the two-body Lippmann-Schwinger equation

$$T(E) = U(E) + U(E)P_e G'(E)T(E), \tag{1}$$

where $G'(E) = (E + i\epsilon - K_0 - H_T)^{-1}$ is the propagator of a free projectile with kinetic energy operator K_0 and a many-body target whose total Hamiltonian is H_T. The projection operator $P_e = \int d^3k \times |\vec{k}\varphi_T\rangle\langle\varphi_T\vec{k}|$ ensures that only the elastic channel,[6] in which the target remains in its ground state $|\varphi_T\rangle$, enters into (1). Thus the nonelastic channels, described by the complementary projector $Q_e = 1 - P_e$, are used to construct the optical potential. In particular, $U(E)$ is given by[1]

$$U(E) = V + VQ_e G'(E)U(E), \tag{2}$$

where $V = \sum_i v_i$ and v_i represents the interaction of the projectile with the ith target nucleon. By eliminating v_i in favor of the scattering operator

$$\tau_i'(E) = v_i + v_i Q_e G'(E)\tau_i'(E), \tag{3}$$

and iterating (2), we recover the Watson multiple-scattering expansion[1]

$$U(E) = \sum_i \tau_i'(E) + \sum_{i \neq j} \tau_i'(E)Q_e G'(E)\tau_j'(E) + \sum_{i \neq j \neq k} \cdots . \tag{4}$$

The Green function in Eq. (3) involves the full Hamiltonian of the target. Therefore, the solution of this equation is still a many-body problem. Following the many-body theories of the bound state, we write $H_T = K_i + K_{C_i} + H_{C_i} + V_i$, where the components are, respectively, the kinetic energy of the struck particle, the center-of-mass energy of the residual nucleus C_i, its in-

ternal Hamiltonian, and its interaction with the ith particle. It is the term V_i which makes Eq. (3) a many-body problem. If we write $V_i = V_i - u_i + u_i$, where u_i is a single-particle potential, and expand in powers of the residual interaction $V_i - u_i$, then the lowest-order term in the equation for τ_i' becomes

$$\tau_i(E) = v_i + v_i Q_e G_i(E) \tau_i(E), \tag{5}$$

with $G_i(E)$ given by

$$G_i(E) = (E + i\epsilon - K_0 - K_i - K_{C_i} - u_i - H_{C_i})^{-1}. \tag{6}$$

Equation (5) is now a three-body equation for τ_i since the coordinates of the nucleons of C_i only appear in H_{C_i}. This results in the introduction of a spectral density[7] instead of a one-body density matrix as a weighting function for the nucleon-nucleon effective interaction, τ_i. When the projection operator is handled by the KMT technique, Eq. (5) becomes the same as KMT's Eq. (2.24) if we ignore the single-particle potential, u_i. We retain it for this work in order to be able to suggest modifications of the KMT prescription.

We write $H_0 = K_0 + K_i + K_{C_i} + H_{C_i}$ as the noninteracting three-body Hamiltonian. The operator Q_e in this model then projects onto the scattering states of the Hamiltonian $H_0 + u_i$. The target is taken to contain A identical nucleons, so that $|\varphi_T\rangle$ is fully antisymmetrized in the A coordinates. In this situation we can delete any reference to a particular target nucleon, and represent τ_i by τ_e, where the subscript labels the elastic channel. The first-order optical potential becomes simply $A\tau_e(E)$, where

$$\tau_e(E) = v + v Q_e G(E) \tau_e(E), \tag{7}$$

and $G(E) = (E + i\epsilon - H_0 - u)^{-1}$. The elastic-scattering operator is the solution of

$$T_e(E) = A\tau_e(E) + A\tau_e(E)\Gamma_e(E)T_e(E), \tag{8}$$

where we have defined $\Gamma_e(E) = P_e G(E)$. The operator form of two-body unitarity associated with (8) is readily obtained by constructing the anti-Hermitian part $\Delta T_e(E) = [T_e(E) - T_e(E)^\dagger]/2$. The result is[8]

$$\Delta T_e(E) = T_e(E)^\dagger \Delta\Gamma_e(E) T_e(E) + [T_e(E)^\dagger \Gamma_e(E)^\dagger + 1] A \Delta\tau_e(E) [1 + \Gamma_e(E) T_e(E)], \tag{9}$$

where

$$\Delta\Gamma_e = -i\pi \int d^3k |\vec{k}\varphi_T\rangle \delta(E - k^2 - E_T) \langle \varphi_T \vec{k}|, \tag{10}$$

and $\Delta\tau_e(E)$ is the anti-Hermitian part of $\tau_e(E)$. When on-shell elastic matrix elements of (9) are taken we obtain the statement of conservation of flux in the form $\sigma_{tot} = \sigma_{el} + \sigma_{abs}$. We now decompose the second term of (9) representing the total absorption into its contributions from specific reaction processes. This requires knowledge of the anti-Hermitian part of $\tau_e(E)$ in the form of three-body unitarity relations. Apart from the projection operator Q_e, Eq. (7) represents a three-body scattering problem with two pairwise interactions.

The unitarity relations are easily expressed once the problem is recast in the form of Faddeev-type equations in which the boundary conditions for all physical processes are clear. This is achieved by eliminating the two interactions v and u in (7) in favor of associated two-body t matrices. Defining $G_0(E) = (E + i\epsilon - H_0)^{-1}$, we introduce

$$t_e(E) = u + u G_0(E) t_e(E), \tag{11}$$

$$t_p(E) = v + v G_0(E) t_p(E). \tag{12}$$

The subscript p is for later convenience in identifying the physical channel associated with the bound state of the Hamiltonian $H_0 + v$ (the pick-up state).

Three-body integral equations for $\tau_e(E)$ are derived by multiplying (7) on the left by $[1 + t_p(E) \times G_0(E)]$ and rearranging to obtain

$$\tau_e(E) = t_p(E)\{1 + [Q_e G(E) - G_0(E)]\tau_e(E)\}. \tag{13}$$

Representing Q_e by $1 - P_e$ and eliminating the interaction u in $G(E)$ by the identity $G(E) = G_0(E) + G_0(E) t_e(E) G_0(E)$, Eq. (13) can be written in the coupled-channel form

$$\tau_e(E) = t_p(E) G_0(E) \tau_p(E), \tag{14}$$

$$\tau_p(E) = (E - H_0) + [t_e(E) G_0(E) - (E - H_0)\Gamma_e(E)]\tau_e(E). \tag{15}$$

Equations (14) and (15) represent a practical realization of the Watson first-order optical-potential operator $\tau_e(E)$ in terms of three-body integral equations of Alt, Grassberger, and Sandhas.[9] The kernel of Eq. (15) has the desired property of prohibiting on-shell propagation of the elastic channel in in-

termediate states, and thus takes account of the projection operator, Q_e. The auxililiary operator $\tau_p(E)$ describes transitions from the elastic channel to the one-nucleon pick-up channel in which the projectile and struck nucleon are bound together and asymptotically free from the core. This interpretation for $\tau_p(E)$ will be clarified by the unitarity relation for $\tau_e(E)$ given below.

One advantage of the integral equations (14) and (15) is that $\Delta\tau_e(E)$ can be readily obtained from the known expressions of three-body unitarity.[10] The only nonstandard feature is the presence of the projection off the elastic channel which serves to eliminate the elastic-channel term from the standard unitarity relation. The result is[11]

$$\Delta\tau_e(E)=\tau_p(E)^\dagger\Delta\Gamma_p(E)\tau_p(E)+\tau_p(E)^\dagger\Omega_p(E)^\dagger\Delta G_0(E)\Omega_p(E)\tau_p(E), \tag{16}$$

where $\Gamma_p(E)$ is the propagator of the pick-up channel defined by $P_p(E+i\epsilon-H_0-v)^{-1}$, and P_p is a projector onto this channel defined analogously to P_e. The wave operator, $\Omega_p(E)=1+t_p(E)G_0(E)$, maps a three-body plane-wave state into a scattering state of two-nucleon relative motion and a plane-wave state of their center-of-mass motion. Inserting (16) into (9) yields

$$\Delta T_e(E)=T_e(E)^\dagger\Delta\Gamma_e(E)T_e(E)+A[T_p(E)^\dagger\Delta\Gamma_p(E)T_p(E)+T_0(E)^\dagger\Delta G_0(E)T_0(E)], \tag{17}$$

where we have introduced the reaction operators

$$T_p(E)=\tau_p(E)[1+\Gamma_e(E)T_e(E)], \tag{18}$$

$$T_0(E)=\Omega_p(E)T_p(E). \tag{19}$$

From the definition of the propagators $\Gamma_p(E)$ and $G_0(E)$ made earlier, it is easily established that $\Delta\Gamma_p(E)$ and $\Delta G_0(E)$ are operators projecting onto all on-shell intermediate states of one-nucleon pick-up and knockout, respectively. Thus the unitarity relation (17) identifies $T_p(E)$ and $T_0(E)$ as the associated transition operators for these reactions which provide the total absorption cross section. The reaction amplitudes can be calculated from (18) and (19) in terms of the effective interaction for pick-up $\tau_p(E)$, and the elastic-scattering wave function, which is produced when $1+\Gamma_e(E)T_e(E)$ operates on an incident-channel state. This factorization into distorted-wave form in the incident channel allows contact to be made between the theory of the optical potential and the distorted-wave method for the associated direct reactions. It is clear from (18) and (19) that the distortion in the final state is included in the operators $\tau_p(E)$ and $\Omega_p(E)$ for pick-up and knockout, respectively. This distortion is primarily due to the purely real interaction, u, between the struck nucleon and the residual nucleus. As a result, there is too little absorption in the final state, and the reaction amplitudes will be too large.

The KMT single-scattering model is obtained from the present formalism by ignoring u in the continuum states of the struck nucleon. Therefore, KMT's implicit reaction amplitudes contain even less final-state absorption and will also be too large. Distorted-wave $(p, 2p)$ calculations indicate that eliminating the final-state absorption results in an overestimate of the break-up cross section. Specifically,[12] for the reaction $C^{12}(p, 2p)B^{11}$ at 100 MeV with typical parameters, this overestimate is about 10% at the quasi-free peak. Furthermore, the effect can not be expected to decrease with increasing energy.[13]

We therefore conclude that the absorption contained in the standard single-scattering optical potential arises from reaction mechanisms which contain too little absorption. There is no component in the standard theory which allows the intermediate-state nucleons to interact absorptively with the nuclear medium.

This can be rectified by starting from three-body equations of the type given in Eqs. (14) and (15). Some additional absorption can be included by letting the single-particle potential u become energy dependent and complex. A completely realistic treatment of the absorption for the reaction processes could be described by introducing a projectile–residual-nucleus interaction into the three-body model. Both of these changes will result in modifications of the unitarity relations presented above. Further details will be presented in a future publication.

The authors would like to thank Dr. J. C. Redish for editorial assistance. One of us (D.B.) acknowledges the hospitality of the nuclear theory group of the University of Maryland.

*Work supported in part by the U. S. Energy Research and Development Administration.

†Onderzoeker IIKW, Belgium. Work supported in part by NATO.

‡On leave from the Institute for Theoretical Physics,

University of Leuven, B-3030, Heverlee, Belgium.

[1]K. M. Watson, Phys. Rev. 89, 575 (1953); M. Goldberger and K. M. Watson, *Collision Theory* (Wiley, New York, 1964).

[2]A. K. Kerman, H. McManus, and R. M. Thaler, Ann. Phys. (N.Y.) 8, 551 (1959).

[3]F. R. Kroll and N. S. Wall, Phys. Rev. C 1, 138 (1970); J. R. Wu *et al.*, in Proceedings of the Second International Conference on Clustering Phenomena in Nuclei, April 1975, University of Maryland (to be published).

[4]E. Lambert and H. Feshbach, Ann. Phys. (N.Y.) 76, 80 (1973).

[5]Unitarity relations for nucleon-nucleus elastic scattering due to the KMT potential have been discussed previously [D. J. Ernst, C. M. Shakin, and R. M. Thaler, Phys. Rev. C 9, 1374 (1974)]. In that work the total-absorption cross section is described only in terms of the anti-Hermitian part of the optical potential. The reaction amplitudes are not explicated.

[6]We ignore excited bound states of the target. These can easily be accommodated into the formalism. We concentrate on the more difficult rearrangement and continuum excitations.

[7]D. H. E. Gross and R. Lipperheide, Nucl. Phys. A150, 449 (1970).

[8]The energy variable, E, is understood to have an infinitesimally small positive imaginary part.

[9]E. O. Alt, P. Grassberger, and W. Sandhas, Nucl. Phys. B2, 167 (1967).

[10]K. L. Kowalski, Phys. Rev. 188, 2235 (1969).

[11]We have chosen to express the break-up contribution in terms of the rearrangement operator.

[12]N. Chant, private communication.

[13]The mean free path of a nucleon in an optical well with imaginary part W is given by $\lambda = -k/(2mW)$. Since W increases somewhat with E [e.g., A. Bohr and B. Mottleson, *Nuclear Structure* (Benjamin, New York, 1969), Vol. I, p. 237] while k increases as $E^{1/2}$, λ does not increase rapidly with E.

Rotational Excitation of Reaction Products: $C^+ + O_2 \rightarrow CO^+ (A\ ^2\Pi) + O$

Ch. Ottinger and J. Simonis
Max-Planck-Institut für Strömungsforschung, Göttingen, Germany
(Received 17 March 1975)

Chemiluminescent reactions between C^+ and O_2 were studied in a beam experiment. At $E_{c.m.} = 3.6$ eV, $CO^+(A\ ^2\Pi)$ is formed with abnormally high rotational excitation.

Chemiluminescence is a convenient way of studying excited products of bimolecular gas-phase reactions with high precision. Infrared chemiluminescence has even revealed the rotational excitation of reaction products.[1] Here we are reporting on a uv-chemiluminescent ion-molecule reaction, a type of process which has only recently come under study.[2,3] In this work rotational excitation of the products of an ion-molecule reaction was directly observed for the first time.

The formation of CO^+ ions in $C^+ + O_2$ collisions is known from mass-spectrometric work.[4-6] Measurements of the translational endoergicity showed that energies up to several eV could be stored in the product ion.[6] This suggested a search for emission from the known electronically excited states $CO^+(A\ ^2\Pi)$ or $CO^+(B\ ^2\Sigma^+)$.

Our apparatus consists of a plasma ion source, magnetic mass separator, collision chamber, optical spectrometer, photomultiplier, and photon-counting system. The C^+ ions were produced from CO; their energy in the collision chamber was varied between 1 and 1000 eV with a spread of about 0.9 eV full width at half-maximum and an absolute uncertainty, due to plasma and surface charges, of about 1 eV. The target-gas pressure was 10^{-2} Torr; the observed emission intensity varied linearly with the pressure between 2×10^{-4} and 1.6×10^{-2} Torr. The beam current into the collision chamber was 2×10^{-9} A at $E_{lab} = 5$ eV. The emission spectra were scanned repetitively from 1800 to 5000 Å with an optical resolution of 20 Å and signal averaged for up to 20 h.

At high collision energy the emission consisted mainly of O_2^+ band systems. They disappeared below 25 eV_{lab}.[7] A typical low-energy result is shown in Fig. 1(a) for a collision energy of $E_{lab} = 5$ eV, corresponding to $E_{c.m.} = 3.6$ eV. This spectrum is due to the $CO^+(A\ ^2\Pi \rightarrow X\ ^2\Sigma^+)$ transition,[8,9] where the CO^+ ions are formed in the reaction

$$C^+ + O_2 \rightarrow CO^+(A\ ^2\Pi) + O. \qquad (1)$$

There is no clear evidence of $CO^+(B\ ^2\Sigma^+)$ emission.

For comparison, Fig. 1(b) shows a $CO^+(A \rightarrow X)$ spectrum which was excited by charge transfer