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Method of Generating Very Intense Positive-Ion Beams

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The combination of multiply reflected electrons and positive ion flow in a reflex triode arrangement is analyzed. Under certain conditions it is possible to generate very intense beams of positive ions with this device. The analysis demonstrates that the energy loss and scattering of the electrons as they pass through the anode have a major effect on the ion and electron currents. Solid fractional-range anodes are shown to produce more intense ion beams than semitransparent mesh anodes.

Both laser and electron beams[1,2] are possible energy sources for the heating and compression required to release fusion energy from dense, inertially contained plasmas. The feasibility of using intense beams of positive ions for this purpose has also been investigated.[3,4] The main problem is that the current density of the available ion beams is far too low. However, there is no inherent technological limitation on ion-beam intensity, and we propose here a method of producing short pulses of MeV ions at current densities of ~10 kA/cm^2.

Recently, Humphries, Lee, and Sudan[5] have produced intense beams of positive ions with the "reflex-triode" or "double-diode" configuration shown in Fig. 1. Multiple transits of electrons are used to increase the ratio of ion current to electron current. The principal disadvantage of the ion beams measured so far is that the current densities are low (20–50 A/cm^2). In connection with experiments[6,7] employing the double-diode configuration of Fig. 1, Smith has pointed out that the use of a foil anode, instead of the

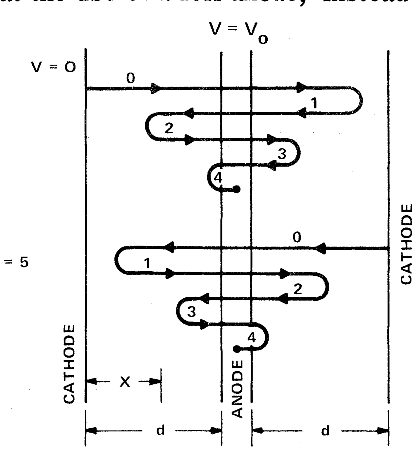

FIG. 1. Multiple reflection of electrons in a "reflex triode." A fractional-range anode is located between two cathodes so that the electrons make many transits before stopping. Positive ions flow from the anode plasma to both cathodes.

mesh anode used by Humphries, Lee, and Sudan,[5] can cause a concentration of electrons close to the anode so that there is a greatly increased positive-ion-current density. The details of this argument will be presented here; and it will be shown that very high ion-current densities are possible. The diode characteristics observed by Prono *et al.*[8] provide evidence for the validity of this model.

A steady-state nonrelativistic treatment is used for this problem, and the effects of two-stream instabilities and collisions have been neglected. The cathode-anode separation d is assumed to be much less than the dimensions of the cathode so that the problem becomes one-dimensional and the only relevant spatial coordinate is the distance X measured from the cathode (Fig. 1). The effects of the self-magnetic field have been neglected. These assumptions are similar to those used in the derivation of the well-known Child-Langmuir result for planar diodes. The anode plasma is assumed to be sufficiently hot and dense so that the ion current emitted is limited only by space-charge effects (i.e., the electric field is zero at the anode). The energy loss and scattering of the electrons, as they pass through the anode, have a major effect on the operating characteristics of the diode, and must be included in the analysis.

The potential V has a value of V_0 at the anode and zero at the cathode. The electrons are assumed to leave the cathode with zero velocity. For an electron with total energy W, we define the parallel energy W_X by

$$W=\tfrac{1}{2}m_e v_X^{\,2}+\tfrac{1}{2}m_e v_\perp^{\,2}-eV=W_X+\tfrac{1}{2}m_e v_\perp^{\,2}, \qquad (1)$$

where v_X and $v_\perp$ are the magnitudes of the velocities parallel and perpendicular to the X direction, and $-e$ and m_e are the charge and mass of the electron.

The electric field E is independent of time, and parallel to the X coordinate. For these conditions, W_X is a constant of the motion for that portion of an electron's trajectory which lies between successive transits through the anode. All changes in W_X occur while an electron is passing through the anode.

Let f be the total number of electrons per unit area per unit time which are traveling in the negative X direction at the anode surface. This flux density f consists of electrons which have made one or more transits through the anode, or have been backscattered from the anode. Because of the steady-state nature of the problem, the flux density f has a constant spectral distribution with respect to the parallel energy W_X. Define df/dW_X to be the spectral amplitude of this distribution at the anode surface. Because changes in W_X and W can only occur while the electrons are passing through the anode, the scattering and energy-loss processes produced by η transits through an anode of thickness τ are essentially the same as those occurring during passage through a single slab of thickness $\eta\tau$. Thus, in principle, df/dW_X can be supplied as an input to the problem and is completely independent of the potential distribution in the diode.

If X is a single-valued function of V, then the negative-going electrons which can reach some particular value of X are those for which the corresponding V is energetically accessible, i.e., for which $W_X+eV\geq 0$. However, if there is a local potential minimum, so that two values of X correspond to the same V, then a potential barrier exists and the negative-going electrons with $W_X+eV=0$ will be reflected at the larger value of X. In what follows, X is assumed to be a single-valued function of V. The conditions under which this assumption fails will be discussed later.

Because of the steady and one-dimensional nature of the problem, the negative-going flux density of electrons with parallel energy in the interval $(W_X,\ W_X+dW_X)$ has the same value at all potentials V which are energetically accessible to those electrons. Because of reflections in the field of the diode (Fig. 1), the flux density of negative-going electrons at any X is accompanied by an equal positive-going flux density with exactly the same spectral distribution. At V, the density of negative-going electrons with energy in the interval $(W_X,\ W_X+dW_X)$ is given by dividing the flux density by the velocity v_X. If we define f_e as the flux density of electrons leaving the cathode and traveling in the positive X direction, then at any potential V these electrons have velocity $(2eV/m_e)^{1/2}$ and density $f_e/(2eV/m_e)^{1/2}$.

In order to calculate the total electron density n_e at V, it is necessary to include all of the electrons which have sufficient energy to reach this potential:

$$n_e=f_e\left[\frac{2eV}{m_e}\right]^{-1/2}+2\int_{-eV}^{0}\frac{df}{dW_X}\left[\frac{2}{m_e}(W_X+eV)\right]^{-1/2}dW_X. \qquad (2)$$

The integral in Eq. (2) is multiplied by a factor of 2 in order to include both negative- and positive-going electrons.

For the positive ions of charge Ze and mass m_i, the velocity v_p in the X direction is assumed to be zero at the anode so that the kinetic energy at V is given by $m_i v_p^2/2 = Ze(V_0 - V)$. If f_i is the flux density of positive ions leaving the anode, then the ion density is given by $n_i = f_i/v_p$.

In order to determine the potential V it is necessary to solve Poisson's equation in one dimension with the boundary conditions $V = dV/dX = 0$ for $X=0$ appropriate for space-charge-limited flow at the cathode. For convenience, we introduce the dimensionless variables λ and φ defined by $\lambda = X/d$ and $\varphi = V/V_0$, and define $j_e = ef_e$ and $j_i = ef_i$ as the current densities of electrons leaving the cathode and ions leaving the anode, respectively. Substituting the expressions for n_e and n_i into Poisson's equation and integrating once gives

$$d\varphi/d\lambda = \tfrac{4}{3}(j_e/j_0)^{1/2}\{\varphi^{1/2} - \alpha[1-(1-\varphi)^{1/2}] + F(\varphi)\}^{1/2} = \tfrac{4}{3}(j_e/j_0)^{1/2}[G(\varphi)]^{1/2}, \tag{3}$$

where $\alpha = (j_i/j_e)(m_i/Zm_e)^{1/2}$ and j_0 is the Child-Langmuir electron current density which would flow in the absence of positive ions and multiple reflections. For space-charge-limited flow at the anode, $\alpha = 1 + F(1)$. The function $F(\varphi)$ is given by

$$F(\varphi) = \frac{e}{j_e}\left(\frac{e}{V_0}\right)^{1/2}\int_0^{\varphi V_0} dV' \int_{-eV'}^{0} \frac{df}{dW_X}[W_X + eV']^{-1/2}\, dW_X. \tag{4}$$

The form of the function $F(\varphi)$ is determined by the spectral distribution of the flux density f. The magnitude of f can be specified by a parameter η defined by $f = (\eta - 1)f_e$. η is essentially the average number of transits through the anode and is given approximately by the ratio of the range of an electron with kinetic energy eV_0 to the thickness of the anode.

Equation (3) is only valid for those functions G which satisfy the condition that $G(\varphi) \geq 0$ for $0 \leq \varphi \leq 1$. This condition is related to the requirement that λ must be a single-value function of φ. The more restrictive condition that $G(\varphi) > 0$ for $0 < \varphi < 1$ is sufficient to insure that φ increases monotonically, and that λ is single valued. If φ has a local maximum or minimum for $0 < \lambda < 1$, then the relation between charge and potential becomes more complicated and the problem must be reformulated to find a new form for the function G.

Using the boundary conditions that $\varphi = 0$ for $\lambda = 0$, Eq. (3) can be integrated to give λ as a function of φ:

$$\lambda = \tfrac{3}{4}(j_e/j_0)^{-1/2}\int_0^{\varphi}[G(\theta)]^{-1/2}\, d\theta. \tag{5}$$

For any particular spectral distribution of the flux density f, the value of j_e/j_0 can be calculated from Eq. (5) by applying the condition that $\varphi = 1$ for $\lambda = 1$. The passage of electrons through matter has been investigated extensively, and both numerical and empirical results are available. It should be possible to estimate the spectral distribution of the flux density f for a given anode, and to design anodes with nonuniform thickness so that the resulting spectrum will optimize ion production.

To illustrate the properties of the solutions, Eq. (5) has been solved numerically using several simple functions for the spectral distribution of f. The results are shown in Fig. 2 for protons. For solid anodes, scattering and energy loss occurs during each transit through the anode so

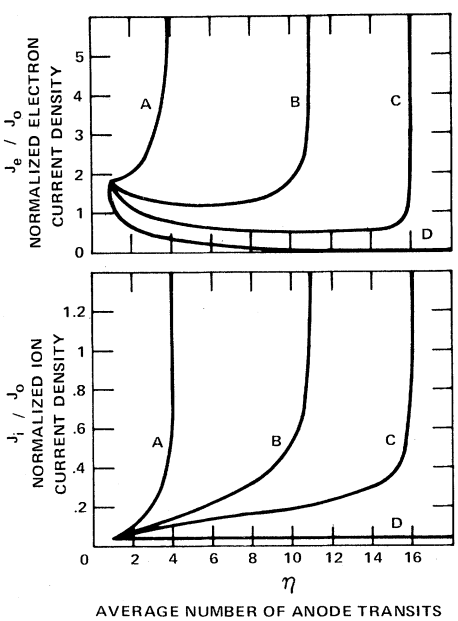

FIG. 2. Ratios of the electron (ion) current density j_e (j_i) to the Child-Langmuir current density j_0. Curves A, B, and C correspond to flux densities f which have a continuous spectral distribution with respect to W_X. For curve D, df/dW_X is given in terms of a delta function of W_X.

that f is continuously distributed with respect to W_x. The curves A, B, and C provide examples of the solution for this type of distribution. df/dW_x was taken to be a linearly decreasing function of W_x for curve A, a constant for curve B, and a linearly increasing function for curve C. For the mesh anodes used by Humphries, Lee, and Sudan,[5] the electrons either pass through with no energy loss or are stopped completely. For this case df/dW_x is given by a delta function of W_x and curves D in Fig. 2 represent the solution of Eq. (5).

For the curves A, B, and C shown in Fig. 2, the parameter η has an upper limit η_c imposed by the criterion that $G(\varphi) \geqslant 0$; and, as η approaches η_c, the ratio of the electron (ion) current density emitted from the cathode (anode) to the Child-Langmuir current density, j_0, exhibits a sharp resonancelike increase. This increase is associated with the fact that the potential is relatively constant over a large region where approximate charge neutrality occurs, and a large part of the potential difference occurs across a small spacing. None of the three flux spectra used to generate the curves A, B, and C is likely to be obtainable exactly in practice. However the calculations serve clearly to show that for a range of widely different flux spectra the result is always the same; i.e., a large increase of current density above the Child's-law values.

From Fig. 2, it can be seen that the use of a solid anode can result in a very intense ion-current density which may be as large as or larger than the Child-Langmuir electron current density j_0. Also, from Fig. 2, the ion-current density j_i can be an appreciable fraction of the total diode current $j_e + j_i$. As a numerical example, consider a diode where $V_0 = 1$ MV and $d = 0.5$ cm, for which j_0 has a value of ~ 8 kA/cm^2. If the diode operates in a regime where $j_i \sim j_0$, then ion-current densities of this intensity represent a two-order-of-magnitude increase over that reported by Humphries, Lee, and Sudan,[5] and add considerable viability to ion-beam-heating schemes.[3,4]

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Ion Heating by Expansion of Beam-Heated Plasma

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A column of plasma whose electrons are rapidly heated by an electron beam or laser will expand against the external magnetic field, setting up large-amplitude magnetosonic oscillations. We study the dissipation of these oscillations by anomalous resistance, ion Landau damping, and ion counterstreaming, and show that much of the initial T_e can be converted into T_i.

Relativistic electron beams and lasers show great promise for rapid plasma heating (e.g., 50 nsec for e beams), but unfortunately for applications to controlled thermonuclear fusion, energy is usually deposited preferentially in plasma electrons. We recently pointed out,[1] however,