Relativistic Hydrodynamic Theory of Heavy-Ion Collisions*

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By use of finite-difference methods we solve the classical relativistic equations of motion for the head-on collision of two heavy nuclei. For ^{16}O projectiles incident onto various targets at laboratory bombarding energies per nucleon ≤ 2.1 GeV, curved shock waves develop. The target and projectile are deformed and compressed into crescents of revolution. This is followed by rarefaction waves and an overall expansion of the matter into a moderately wide distribution of angles.

The idea that a nuclear shock wave could be formed when a projectile moves through a nucleus at a velocity exceeding the nuclear sound speed was proposed back in 1959 by Glassgold, Heckrotte, and Watson.[1] They also pointed out that the angular distribution of the nucleons ejected after the shock wave passes through the nuclear surface could be used to determine the nuclear compressibility coefficient. However, they had in mind light projectiles such as protons and pions, as well as the propagation of moderately weak density variations at nuclear sound speeds, and their idea remained largely unnoticed until 1973.

It was then realized that whereas the passage of a light projectile through a nucleus might not be sufficient to create a nuclear shock wave, the use of a moderately heavy ion as a projectile would stand a much better chance of increasing the nuclear density to several times its equilibrium value. This realization, coupled with the increasing availability of heavy-ion projectiles, has led recently to a flurry of theoretical papers on nuclear shock waves and other aspects of a high-energy nuclear hydrodynamic model.[2-9] Some experimental data[7,10-12] already support such a model, and more refined experiments are in progress.[13]

The previous theoretical discussions are based on a variety of simplifying assumptions. To eliminate many of the resulting difficulties, we solve numerically the relativistic hydrodynamic equations of motion for the head-on collision of two heavy nuclei. Compared to the other recent work on nuclear shock waves,[2-9] our approach represents three important improvements: (1) The influence of the finite target and projectile sizes on the propagation of the shock waves is calculated accurately. (2) Relativistic effects are included. (3) Energy spectra and angular distributions of the outgoing matter, which form the bases for comparisons with experimental data, are calculated.

As all the other workers in this area have done,[2-9] we neglect nuclear viscosity, surface energy, Coulomb energy, and single-particle effects. These energies are small compared to the kinetic energies involved at high bombarding energies, but their neglect nevertheless precludes an accurate description of the coalescence of matter into clusters following the demolition of the system. We also neglect the production of additional particles and the associated radiative loss of energy from the system; this approximation becomes increasingly serious as the bombarding energy increases.

Arguments based on the nucleon mean free path and momentum transfer per collision suggest that a conventional nuclear hydrodynamic model should be valid only when the bombarding energy per nucleon is less than about 1 GeV, whereas at higher bombarding energies the colliding nuclei should become somewhat transparent to each other.[3,8] The expected partial transparency of nuclei at relativistic energies possibly can be taken into account by means of a two-fluid generalization of the present calculations, in which relativistic hydrodynamic equations are solved for separate target and projectile nuclear fluids.[14] One expects *a priori* that the terms in the equations that couple the two nuclear fluids are related to nucleon-nucleon cross sections and momentum transfers. However, because the collisions in heavy-ion reactions involve many nucleons rather than two, the coupling terms ultimately must be determined experimentally, as must the nuclear equations of state satisfied by the target and projectile fluids. In the limit of low bombarding energies, in which the nuclear equations of state would reduce to a single equation of state for the two fluids treated as a single entity,[14] the present one-fluid hydrodynamic model is recovered. Both because of its intrinsic interest and because several speculations[3-8] have been made concerning its consequences, it is important to have accurate computations for the conventional nuclear hydrodynamic model that we consider here.

The covariant relativistic hydrodynamic equations that we solve express the conservation of nucleon number, momentum, and energy,[15] for a specified nuclear equation of state. For our purposes these equations are written conveniently as

$$\partial N/\partial t + \nabla\cdot(\vec{v}N) = 0, \tag{1a}$$

$$\partial \vec{M}/\partial t + \nabla\cdot(\vec{v}\vec{M}) = -\nabla p, \tag{1b}$$

and

$$\partial E/\partial t + \nabla\cdot(\vec{v}E) = -\nabla\cdot(\vec{v}p), \tag{1c}$$

where N, $\vec{M}$, and E are, respectively, the nucleon number density, momentum density, and energy density (including rest energy) in the laboratory reference frame. The velocity of matter relative to the laboratory frame is denoted by $\vec{v}$, and p is the pressure in the rest frame. The three laboratory-frame quantities are related to rest-frame quantities by

$$N = \gamma n, \tag{2a}$$

$$\vec{M} = \gamma^2(\epsilon + p)\vec{v}, \tag{2b}$$

and

$$E = \gamma^2(\epsilon + p) - p, \tag{2c}$$

where n and ϵ are, respectively, the nucleon number density and energy density in the rest frame and $\gamma = (1 - v^2)^{-1/2}$, with the velocity measured in units of the light speed.

Our nuclear equation of state, which relates p to n and ϵ, is obtained from a Thomas-Fermi treatment of the effective two-nucleon interaction that consists of an attractive Yukawa function multiplied by a quadratic momentum-dependent term.[16] This leads to a rest-frame energy per nucleon ϵ/n of the form

$$\frac{\epsilon}{n} = m_0 + a\left(\frac{n}{n_0}\right)^{2/3} - b\frac{n}{n_0} + c\left(\frac{n}{n_0}\right)^{5/3} + \frac{I}{n}, \tag{3}$$

where m_0 is the nucleon rest mass, $n_0 = 3/4\pi r_0^{\,3}$ is the equilibrium value of n, and I/n is the rest-frame internal (heat) energy per nucleon. For the specific choices[16] of 1.2049 fm for r_0 and -15.677 MeV for the nonrelativistic energy per nucleon at equilibrium (excluding rest energy), the values of the three constants that appear are $a = 19.88$ MeV, $b = 69.02$ MeV, and $c = 33.46$ MeV. The resulting value of the nuclear compressibility coefficient is $K = 9n_0^{\,2}\partial^2(\epsilon/n)/\partial n^2|_0 = 294.8$ MeV.

The pressure p is obtained from the relationship $p = n^2\partial(\epsilon/n)/\partial n|_S$, with differentiation at constant entropy S. The relationship between I/n and the nuclear temperature is taken from a nonrelativistic Fermi-gas model for the thermal motion of the nucleons relative to the hydrodynamic flow, for which $n^2\partial(I/n)/\partial n|_S = \frac{2}{3}I$. This is the exact result for a nonrelativistic Fermi-gas model, instead of being true only to second order in the nuclear temperature, as is often implied. The pressure is given finally by

$$p = [\tfrac{2}{3}a(n/n_0)^{5/3} - b(n/n_0)^2 + \tfrac{5}{3}c(n/n_0)^{8/3}]n_0 + \tfrac{2}{3}I = [-\tfrac{2}{3}m_0(n/n_0) - \tfrac{1}{3}b(n/n_0)^2 + c(n/n_0)^{8/3}]n_0 + \tfrac{2}{3}\epsilon. \tag{4}$$

However, to prevent the formation of small clusters that at this stage of our work are not physically meaningful, we set the pressure to zero when it would otherwise be negative.

To solve Eqs. (1), (2), and (4) we have developed a relativistic generalization[17] of a standard particle-in-cell finite-difference computing method. This technique is applicable to supersonic flow and combines some of the advantages of both Eulerian and Lagrangian methods.

As an example we present in Fig. 1 a calculated sequence of shapes for the head-on collision of an ^{16}O projectile with ^{107}Ag at a laboratory bombarding energy per nucleon of 2.1 GeV. For this energy and our equation of state, the rest-frame density is increased initially to 7.4 times its equilibrium value in an infinitesimal volume near the contact point. However, the maximum compression is reduced substantially below this value as a result of the rarefaction from the top surface and the divergence of the curved shock waves. For example, 4.1×10^{-23} sec after impact, when the system has been deformed into the crescent of revolution shown in the fourth frame of Fig. 1, the maximum compression is only 2.3.

Energy and angular distributions for the expanding matter are constructed from the velocity vectors just prior to the arrival of matter at the bottom boundary of the computational mesh, corresponding to the last frame in Fig. 1. The small amount of matter that already has passed through the top and side boundaries is also included.

The resulting angular distribution $dN/d\theta$ for the

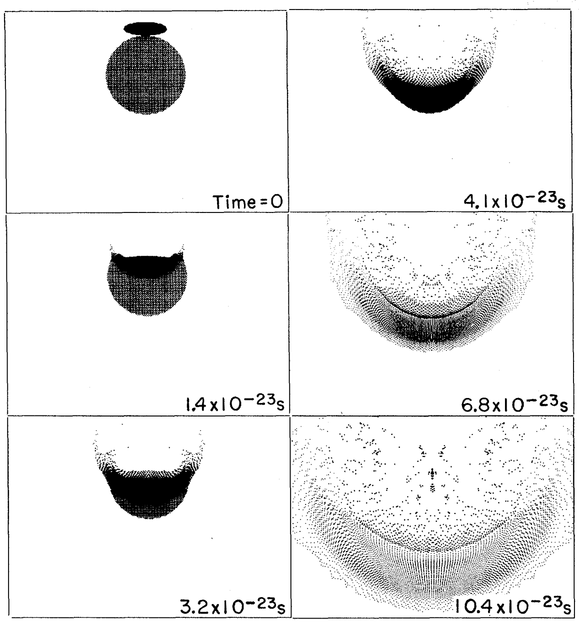

FIG. 1. Characteristic stages in the head-on collision of a relativistic heavy ion with a medium-weight nucleus, as viewed in the laboratory reference frame. The projectile (^{16}O, energy per nucleon of 2.1 GeV), which is initially Lorentz contracted in the incident direction, is represented by heavy points, and the target (^{107}Al) is represented by light points.

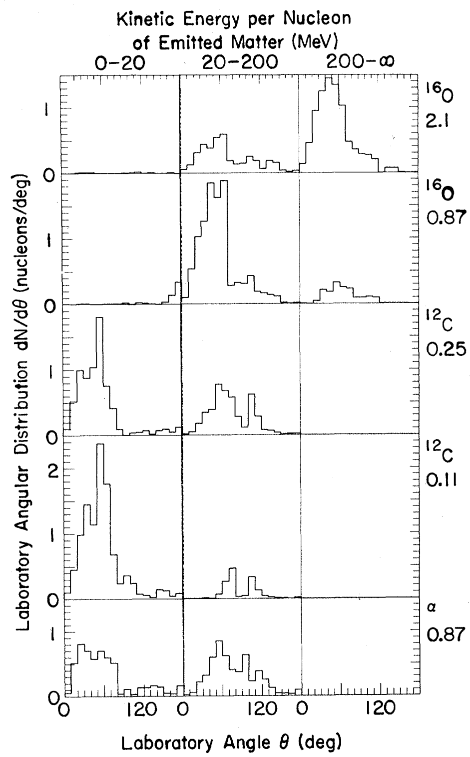

FIG. 2. Calculated angular distributions per unit angle for the head-on collision of various projectiles with ^{107}Ag at various bombarding energies per nucleon, in units of GeV.

system that we have been considering is shown in the top part of Fig. 2, separated into three groups according to the kinetic energy of the emitted matter. The major fraction of matter has a kinetic energy per nucleon $\geqslant 200$ MeV; the angular distribution for this group peaks at about 45° and is 50° wide at its half-maximum value.

The lower parts of Fig. 2 show that for a fixed ^{107}Ag target a decrease in either bombarding energy per nucleon or projectile size decreases the kinetic energy of the emitted matter. Also, as the bombarding energy per nucleon decreases, the peak in the angular distribution shifts slightly to a larger angle of about 55°. This is opposite to the qualitative conjectures of Refs. 5–7 and to the recent experimental results of Refs. 7 and 10 for the bombardment of AgCl with the projectiles and energies of Fig. 2.

Because the detectors used in Refs. 7 and 10 are insensitive to particles whose kinetic energy per nucleon exceeds about 200 MeV, the major hydrodynamic contributions to their results from head-on collisions with ^{107}Ag nuclei are obtained by adding the two lower-energy groups in Fig. 2.

We also performed the same calculations with a ^{35}Cl target, which is much lighter than ^{107}Ag. The angular distributions analogous to Fig. 2 peak at somewhat smaller angles and have larger components in the higher-energy groups than those for ^{107}Ag. The relative contribution from the more-forward-peaking Cl target therefore increases as the bombarding energy per nucleon decreases. It is nevertheless impossible to explain the data of Refs. 7 and 10 in terms of a superposition of head-on collisions with Ag and Cl nuclei.

In conclusion, the possibility may be at hand for determining the nuclear equation of state by comparing experimental data with accurate numerical solutions of the classical relativistic hydrodynamic equations of motion. For the achievement of this goal, we are incorporating a variety of extensions into the calculations, including finite impact parameters in a fully three-dimensional configuration and the possibility that nuclei become partially transparent at relativistic energies.

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Search for Two-Photon Decay in Thermal *np* Capture

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An upper limit of 10^{-4} has been obtained for the branching ratio for the emission of two photons ($E_\gamma > 0.57$ MeV) following thermal neutron capture in hydrogen. This limit was obtained with two Ge(Li) detectors shielded so as to reduce γ-ray scattering from one detector into the other and is a factor of 10 lower than has been recently reported.

A recent publication[1] reported a branching ratio of 10^{-3} for two-photon decay following *np* radiative capture. This ratio is at least a factor of 200 higher than current theoretical estimates.[2] We have searched for this decay mode with Ge(Li) detectors in a configuration that reduced γ-ray scattering from one detector into the other and have obtained an upper limit for this branching ratio of 10^{-4}.

The present experiment was performed with 0.025-eV neutrons obtained by Bragg reflecting a beam of neutrons from the NRU reactor thermal column with a Ge monochromator. The beam traveled down a flight tube lined with 5 mm of ^{6}LiF to a distilled H_2O sample contained in a thin-walled Lucite cylinder (4.1 cm× 4.2 cm long). The ^{6}LiF shielded the detectors from neutrons in the beam and from neutrons scattered by the sample. Two Ge(Li) detectors having photopeak efficiencies of 11.3 and 6.7% (relative to a 3-in. × 3-in. NaI detector at 25 cm) were placed as close to the target as possible. These detectors were shielded such that the counting rate of 2.223-MeV γ rays with the H_2O sample removed was less than 10^{-4} of the counting rate with it in place.

The linear signals from the Ge(Li) detectors were summed and gated by pulses from a standard fast-slow coincidence circuit before analysis. The fast-coincidence time resolution was 5.7 nsec full width at half-maximum and thresholds associated with the slow-coincidence circuit required the γ-ray energy deposited in each detector to be >0.57 MeV. This energy discrimination was selected to reject singly Compton-scattered γ rays and positron-annihilation photons that crossed from one detector to the other.