is tested, b shifts upward by 0.047 [Fig. 1(b)]. An increase of 0.057 results from suspending Monte Carlo corrections, which affect only the end bins in the ω distribution. Dropping these end bins raises b by 0.025. The effect of using a scale-noninvariant form of R which fits data from Stanford Linear Accelerator Center is negligible.[10] The ± 1% systematic error in relative-energy calibration creates an uncertainty of ± 0.056 in b. Therefore, the scale noninvariance observed by using only this method of analysis is not fully conclusive.

We are indebted to L. Litt and T. Markiewicz for their contributions to the data analysis, and are grateful to the Fermilab staff and our professional and technical personnel for their sustained efforts in support of the experiment. Two of us (S.C.L. and M.S.) wish to acknowledge our support by the Cornell Laboratory of Nuclear Studies during earlier phases of the experiment.

*Present address: Texaco Exploration Canada, Ltd., Calgary, Alta., Canada.

‡Work supported in part by the National Science Foundation.

†Deceased.

§Now at Stanford Linear Accelerator Center, Stanford, Calif. 94305.

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[1]J. D. Bjorken, Phys. Rev. 179, 1547 (1969).

[2]E. D. Bloom *et al.*, Phys. Rev. Lett. 23, 930 (1969); M. Breidenbach *et al.*, Phys. Rev. Lett. 23, 935 (1969).

[3]For a review, see F. J. Gilman, in *Proceedings of the Seventeenth International Conference on High-Energy Physics, London, England, 1974,* edited by J. R. Smith (Rutherford High Energy Laboratory, Didcot, Berkshire, England, 1975), p. IV-149.

[4]For example, see J. S. Poucher *et al.*, Phys. Rev. Lett. 32, 118 (1974).

[5]E. D. Bloom and F. J. Gilman, Phys. Rev. D 4, 2901 (1971); E. M. Riordan *et al.*, Phys. Lett. 52B, 249 (1974).

[6]D. J. Fox *et al.*, Phys. Rev. Lett. 33, 1504 (1974).

[7]C. Chang *et al.*, following Letter [Phys. Rev. Lett. 35, 901 (1975)].

[8]For a description, see Y. Watanabe, thesis, Cornell University, 1975 (unpublished).

[9]An internally consistent subset of these data was used for normalization. The same fiducial cuts were applied to the 56-GeV small-angle sample in order to maintain scale invariance of the analysis.

[10]We used $R(q^2) = 1.28q^2/(q^2+1.16)^2$ (E. M. Riordan, private communication).

Observed Deviations from Scale Invariance in High-Energy Muon Scattering

C. Chang, K. W. Chen, D. J. Fox,† A. Kotlewski, and P. F. Kunz*
Physics Department, Michigan State University, East Lansing, Michigan 48824‡

and

L. N. Hand, S. Herb, A. Russell,§ and Y. Watanabe
Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14853‡

and

S. C. Loken and M. Strovink
Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720 ‖

and

W. Vernon
Physics Department, University of California, San Diego, La Jolla, California 92037‡

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Scale noninvariance is observed in 150- and 56-GeV muon scattering from an iron target. In the range $1 < q^2 < 40$ (GeV/c)^2, νW_2 rises with q^2 at fixed $\omega' \gtrsim 6$ and falls at $\omega' \lesssim 6$. The scale breaking is statistically and systematically significant, and persists with alternative choices of scaling variable. It is parametrized roughly by a constant $b = \partial^2 \ln(\nu \times W_2)/\partial \ln(\omega')\partial \ln(q^2)$ with a value near 0.09.

Indications of scale noninvariance in inelastic muon scattering from an iron target have been reported,[1] most recently in ratios[2] of cross sections in beams of 150 and 56 GeV. This Letter compares these cross sections with lower-energy electron-scattering values.[3] Within fixed bands of ω, variations of νW_2[4] are evaluated over the full q^2 range of the data. Thereby, with accurate Monte Carlo simulation of the experiment, a scaling test more precise than in Ref. 2 is possible.

The muon spectrometer was described earlier.[1] Events were identified, fitted, and cut as recently reported,[2] except that event selection to equate the 150- and 56-GeV beam-radius distributions was not required. Data collected with two spectrometer configurations at each beam energy first were tested for internal consistency by comparing ratios of data to Monte Carlo rates in common bins smaller than the experimental resolution. Similar comparisons then tested the consistency of data at the two energies. In all cases the confidence levels established good consistency, permitting the merger of the four samples.

The Monte Carlo simulation generated events using a parametrization of $\nu W_2(\omega')$ from electron-scattering data,[5] with $R=0.18$. The iron target nucleus was modeled as a collection of nucleons in Fermi motion.[6] Radiative corrections used a peaking approximation differing from the more exact correction[7] by less than 3%. The correction for coherent wide-angle bremsstrahlung rose from less than 3% for $\omega<20$ to 15% at ω near 50.[8] The simulated muons suffered Coulomb scattering, energy loss, straggling from μ-e scattering and bremsstrahlung in the iron, and mismeasurement by the spark chambers. Further analysis treated real and simulated data identically.

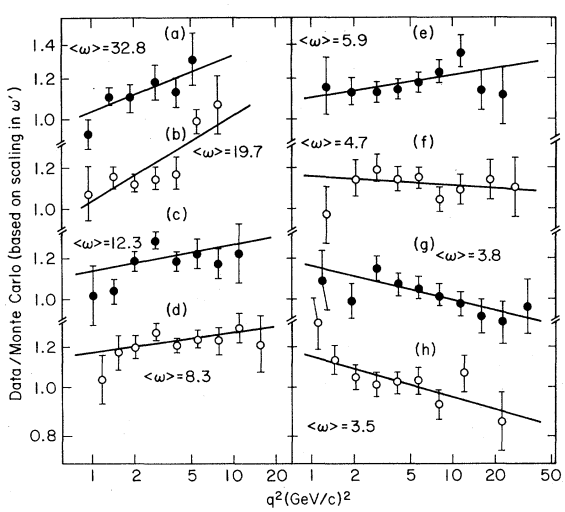

FIG. 1. Ratio of observed to simulated event rate versus q^2 for eight ranges of ω. Widths of these ranges and parameters of the straight-line fits are detailed in Table I. Errors are statistical.

A number of tests establish the accuracy of the simulation. Shapes of real and simulated distributions in azimuth and radius of tracks near fiducial boundaries agree well. The simulation accounts for tails in distributions of χ^2 and reconstructed energy through 2.5 decades in population. Also, it accurately models differences between momentum reconstruction algorithms which use different points on the muon trajectories. We believe that inaccuracies in simulating the acceptance and resolution contribute negligibly to the overall error.

Ratios of observed to simulated event rates are shown in Figs. 1(a)–1(h) versus q^2 for eight overlapping bands of ω.[9] Results of straight-line fits to these data points along with the widths of the ω bands are given in Table I. The points show a rising trend with q^2 at high ω, and a falling trend at low ω. Fitting a power-law q^2 dependence in each ω band yields an overall χ^2 of 45.6 for 51 degrees of freedom. Scale invariance in ω' requires each set of points to exhibit no q^2 dependence. This hypothesis raises the χ^2 by 36.6 with the addition of eight degrees of freedom. Statistically, such a fluctuation has a probability smaller than 2×10^{-5}. These eight fitted slopes $\partial\ln[\nu W_2(\omega',q^2)]/\partial\ln(q^2)$ are plotted versus ω in Fig. 2(a). In the range of these data, the scale breaking may be parametrized by a quantity $b=\partial^2\ln(\nu W_2)/\partial\ln(\omega')\,\partial\ln(q^2)$. If independent of both q^2 and ω, b has the magnitude 0.099 ± 0.018.

The scale-noninvariant result is stable. Subsamples of data from individual spectrometer configurations or beam energies give values of b consistent with the overall result but not with zero. Agreement is excellent with the value of b recently obtained[12] using a different analysis

TABLE I. Logarithm of ratio of observed to simulated event rates fitted by $c+b\ln(q^2/3)$.

$\langle\omega\rangle^a$	$\delta(\omega)^a/\omega$	c	b	% confidence
3.5	0.60	0.04 ± 0.07	-0.076 ± 0.031	54
3.8	0.53	0.07 ± 0.07	-0.064 ± 0.028	80
4.7	0.52	0.12 ± 0.04	-0.017 ± 0.033	73
5.9	0.56	0.13 ± 0.03	0.043 ± 0.032	69
8.3	0.60	0.19 ± 0.02	0.038 ± 0.031	77
12.3	0.58	0.17 ± 0.03	0.052 ± 0.036	10
19.7	0.58	0.20 ± 0.03	0.166 ± 0.043	19
32.8	0.66	0.14 ± 0.04	0.107 ± 0.049	38
Fits to all eight ω bands.				52

[a]Defined in Ref. 9.

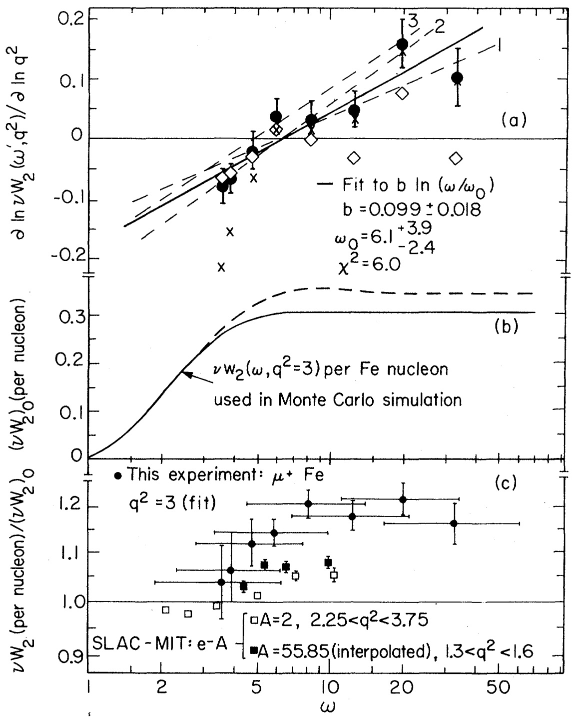

FIG. 2. (a) Fitted slopes in $\ln(q^2)$ for the eight ω ranges. Errors are statistical. Dashed lines depict effects of (1) raising E' by 1% at 150 GeV, (2) the same at 56 GeV, and (3) raising 150-GeV cross sections by 7%. Assuming scaling in ω rather than ω' in the Monte Carlo simulation yields the points indicated by ×. Using the form in Ref. 10 yields the diamond-shaped points. (b) $\nu W_2(\omega, q^2=3)$ per nucleon used in the Monte Carlo simulation, with scaling in ω' and $R=0.18$. The dashed line is the alternative form described in the text. (c) Ratio of νW_2 at $q^2=3$ to the form in (b). Errors on muon data, from fits detailed in Table I, include energy-calibration errors but not the normalization error of ±10%. Errors on SLAC-MIT e^-D scattering data do not include systematic uncertainties of 4–6% (Ref. 3). The e^-Fe points use the same deuterium data with A dependences from Ref. 11. Their systematic errors relative to deuterium are 7%.

philosophy.

Figure 2(b) displays the structure function used in the Monte Carlo simulation, and Fig. 2(c) shows ratios of the observed νW_2 to that function at $q^2 = 3$ $(\text{GeV}/c)^2$. The electron-nucleus[13] scattering results[3,11] are consistent with these values within the normalization uncertainties (Fig. 2 caption). It is apparent that the parametrization of $\nu W_2(\omega')$ used in the Monte Carlo simulation predicts smaller cross sections at large ω than observed here and in recent electron-scattering data.[11] A better parametrization, indicated by the dashed line in Fig. 2(b), makes the ratios in each of Figs. 1(a)–1(h) near unity when averaged over q^2. Refitting the slopes yields a result similar to that above, $b = 0.090 \pm 0.018$.

Normalization and energy calibration of the data followed the procedures recently discussed,[2] and dominate the systematic error in b [Fig. 2(a)]. Uncertainties of 10% (7%) are ascribed to absolute (relative) normalization of data at the two beam energies. The 1% uncertainties in scattered-muon-energy calibration at each beam energy are correlated to yield a 1% relative error. The systematic error in b is ± 0.032 with only this correlation, rising to ± 0.041 with worst-case correlation of 150-GeV energy calibration and normalization. That is, b would lie near zero if, at 150 GeV, scattered energies were raised by 2.6% and cross sections simultaneously were reduced by 18%. However, these shifts would only transform the form of the scale breaking into a relative depletion of event rate at 150 GeV. Moreover, the χ^2 which measures the overall smoothness and consistency of data at the two energies would rise by more than 40 if these shifts were imposed. With reasonable correlation of all errors, b is 0.09 ± 0.04, exceeding zero with 98% confidence.

The scale noninvariance is insensitive to the assumed form of $\nu W_2(\omega')$, because the data are analyzed within bands of ω.[14] Choices of $\nu W_2(\omega')$[11] which are different from those above produce perturbations in b smaller than 10%. The q^2 dependence of νW_2 for $\omega \lesssim 6$ is coupled to the assumed form of its approach to asymptotic behavior. Use of ω as a scaling variable increases the scale noninvariance in this range [Fig. 2(a)], while use of $\omega + 1.4/q^2$ largely cancels it. Conversely, the q^2 dependence of νW_2 for $\omega \gtrsim 6$ is inconsistent with scaling in any variable differing from ω by terms of order M^2/q^2 since νW_2 is nearly independent of ω in this range. If νW_2 depends upon ω and also upon a second variable ω_W defined by two adjustable parameters,[10] the data can be fitted within systematic errors [Fig. 2(a)]. Ignoring all data with $q^2 < 4$ raises the confidence level for scaling in ω' to 14%. Variation of R between ∞ at $q^2 = 1$ and 0 at $q^2 = 5$ $(\text{GeV}/c)^2$ can account for only $\frac{1}{4}$ of the scale noninvariance at $\omega \approx 25$.

The rising q^2 dependence of νW_2 at $\omega \gtrsim 6$ might be attributed to excitation of new hadronic degrees of freedom,[15] with the opposite behavior for $\omega \lesssim 6$ offset by a clairvoyant choice of scaling variable. Field theories[16] with anomalous dimensions or asymptotic freedom predict scale noninvariance at both low and high ω of a character[17] similar to that observed here.

While adequately describing the scale noninvariance, the logarithmic fits serve only to provide a numerical basis for discussion of the effect. They neither predict the behavior of data in extended ranges of q^2 and ω, nor anticipate the results of more exact fits to the form of νW_2 which will be published elsewhere. We express again our gratitude to all who have contributed to this experiment.

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†Deceased.

*Now at Stanford Linear Accelerator Center, Stanford, Calif. 94305.

‡Work supported in part by the National Science Foundation.

§Present address: Texaco Exploration Canada, Ltd., Calgary, Alta., Canada.

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[1]D. J. Fox *et al.*, Phys. Rev. Lett. 33, 1504 (1974).

[2]Y. Watanabe *et al.*, preceding Letter [Phys. Rev. Lett. 35, 898 (1975)].

[3]J. S. Poucher *et al.*, Phys. Rev. Lett. 32, 118 (1974).

[4]Our notation follows that of Ref. 2.

[5]This function is graphed in Fig. 2(b). Beyond $\omega' = 7$ we fixed $\nu W_2 = 0.305$.

[6]Results are insensitive to the parameters of this model, since the smearing in ω is scale invariant and small (12%) compared to the resolution.

[7]L. W. Mo and Y. S. Tsai, Rev. Mod. Phys. 41, 205 (1969).

[8]An approximate cross section is found in R. H. Siemann *et al.*, Phys. Rev. Lett. 22, 421 (1969).

[9]$\langle\omega\rangle$ and $\delta(\omega)/\omega$ are the antilog of the mean and the root variance of a distribution in $\ln(\omega)$ of simulated events which lie in the pertinent bin of reconstructed ω. The error in $\langle\omega\rangle$ is negligible.

[10]F. W. Brasse *et al.*, Nucl. Phys. B39, 421 (1972). We used $x_W = \omega_W^{-1} = (q^2 + 0.351)/(2M\nu + 1.512)$ and $\nu W_2 = (x/x_W)(1-x_W)^3 f(1-x_W)$, with f a polynomial of order four. The constants fit Stanford Linear Accelerator Center–Massachusetts Institute of Technology (SLAC-MIT) D_2 data (E. M. Riordan, private communication).

[11]S. Stein *et al.*, SLAC Report No. SLAC-PUB-1528, 1975 (to be published). Electron structure functions here and in Ref. 3 change by 5–10% with reasonable variations of R.

[12]The value reported was $b = 0.098 \pm 0.028$ (Ref. 2).

[13]Nuclear shadowing is discussed by N. Nikoleav and V. Zakharov, Phys. Lett. 55B, 397 (1975).

[14]The bands are defined by bins in reconstructed ω. Definition by ω' makes little difference since the bands are broad.

[15]D. Schildknecht and F. Steiner, Phys. Lett. 56B, 36 (1975).

[16]For a review see F. J. Gilman, in *Proceedings of the Seventeenth International Conference on High-Energy Physics, London, England, 1974,* edited by J. R. Smith (Rutherford High Energy Laboratory, Didcot, Berkshire, England, 1975), p. IV-149.

[17]W. Tung, University of Chicago Report No. EFI75/14 (to be published).