cording to the uncertainty principle, as shown by curves I′ and II′, this agreement no longer obtains.[10]

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[1]J. J. Sakurai and D. Schildknecht, Phys. Lett. 40B, 121 (1972).

[2]M. Greco, Nucl. Phys. B63, 398 (1973); A. Bramón, E. Etim, and M. Greco, Phys. Lett. 41B, 609 (1972).

[3]R. P. Feynman, *Photon-Hadron Interactions* (Benjamin, New York, 1972).

[4]S. Drell, D. Levy, and T. M. Yan, Phys. Rev. Lett. 22, 744 (1969); P. V. Landshoff and J. C. Polkinghorne, Phys. Rep. 5C, 1 (1972).

[5]H. Fraas, B. J. Read, and D. Schildknecht, Nucl. Phys. B86, 346 (1975).

[6]It is possible to have vector-dominance models in which σ_L is small; see G. J. Gounaris, Nucl. Phys. B88, 451 (1975). Also, throughout our treatment the newly discovered mesons $\psi(3.1)$, etc., have been ignored. This is justified, because of the smallness of the ψ-nucleon cross section.

[7]T. L. Curtright, J. E. Mandula, and F. Ravndal, Phys. Lett. 55B, 393 (1975).

[8]J. Kuti and V. Weisskopf, Phys. Rev. D 4, 3418 (1971).

[9]D. C. Cundy, in *Proceedings of the Seventeenth International Conference on High-Energy Physics, London, England, 1974,* edited by J. R. Smith (Rutherford High Energy Laboratory, Didcot, Berkshire, England, 1975).

[10]Moreover, if the usual (one-component) GVDM were to be compared with the usual QPM, then Eq. (1) rather than Eq. (4) would have to be used. The requirement that $W_{\mu\nu}{}^{\rho\omega}=0$ would then lead to $u(x)+\bar{u}(x)=d(x)+\bar{d}(x)$ contrary to expectations.

Test of Scale Invariance in Ratios of Muon Scattering Cross Sections at 150 and 56 GeV

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Scale invariance is tested in ratios of muon scattering cross sections from an iron target at pairs of q^2 values ranging to 40 $(\mathrm{GeV}/c)^2$ and differing by a factor of $\frac{3}{8}$. The apparatus was changed with incident energy to preserve acceptance and resolution in scaled variables. The scale-noninvariant departure from unity of these ratios displays a statistically significant ω dependence. The effect exceeds the systematic uncertainty by a factor of 1.7.

The strikingly simple character of deep-inelastic lepton scattering has led to models of the nucleon as a composite of nearly structureless constituents. For scattering angles $\ll 1$ the scattering cross section is

$$\frac{d^2\sigma}{dx\,dy} \cong \frac{4\pi\alpha^2}{2ME}\frac{\nu W_2(x,q^2)}{x^2y^2}\left(1-y+\frac{y^2}{2(1+R)}\right), \qquad (1)$$

where $x=\omega^{-1}=q^2/2M\nu$, $y=\nu/E$, and $R=\sigma_s/\sigma_t$, with q^2, ν, σ_s, and σ_t the usual lepton scattering variables, M the nucleon mass, and E the laboratory energy. "Pointlike" structure implies that the structure function νW_2 depends only upon the scale-invariant variable x.[1] Since evidence for scale invariance first appeared,[2] field-theoretic descriptions of the behavior of products of hadron currents at short distances have predicted specific forms of scaling breakdown.[3]

In tests of scaling, the high precision of electron-nucleon scattering data collected at the Stanford Linear Accelerator Center[4] has been offset by ambiguity in parametrizing the approach to the scaling region at low ω, and by kinematic bounds on q^2 at high ω. Significant deviations from scaling in ω were seen nearly to vanish if instead the scaling variable $\omega' = \omega + M^2/q^2$ was used.[5] In the higher-energy lepton beams available at Fermilab, scattering data may be interpreted more directly in terms of asymptotic behavior.

The experimental method and preliminary results based on a subset of the data have been described previously.[6] Results reported here are based upon bombarding an iron target of 622, (233) g/cm^2 with 1.5×10^9 (4×10^9) μ^+ of energy 150 GeV (56 GeV). At 150 GeV, the spectrometer was operated in two settings to accept scattered-muon angles θ over the full azimuth in the ranges $0.011 \lesssim \theta \lesssim 0.048$ ("small angle") and $0.017 \lesssim \theta \lesssim 0.065$ ("large angle"). At 56 GeV, the apparatus was scaled[6] to preserve the acceptance and resolution in scaled variables [e.g., x and y in Eq. (1)]. All data from each configuration are included here.

Scaling the apparatus implies concurrent alteration of the longitudinal dimensions of the spectrometer, the magnetic field integral, and the amount of material encountered by the scattered muon when the beam energy is changed from 150 to 56 GeV. Scattered-muon trajectories equivalent under a scaling transformation and initially related at the two energies by $E'' = \lambda E'$ and $\theta'' = \theta'/\sqrt{\lambda}$, where $\lambda = \frac{150}{56}$ (single and double primes refer to final-state muon energies and angles at 56 and 150 GeV, respectively) are then transported and multiply scattered in an identical fashion throughout the spectrometer with respect to the points actually measured in the experiment. The variable $v = q^2/2ME$ is invariant under the scaling transformation and presents q^2 scaled by the incident energy.

Two tests of scaling are possible: (a) The differential ratio of counting rates at 150 and 56 GeV is compared with unity [Eq. (1)]; (b) with Monte Carlo simulation of the spectrometer acceptance and resolution, the q^2 dependence of νW_2 at fixed ω is evaluated. Method (a) is the basis of this Letter; results of method (b) appear in the following Letter.[7]

Muon tracks with scattered energy $E' > E/3$ were recognized[8] and momentum-fitted with full

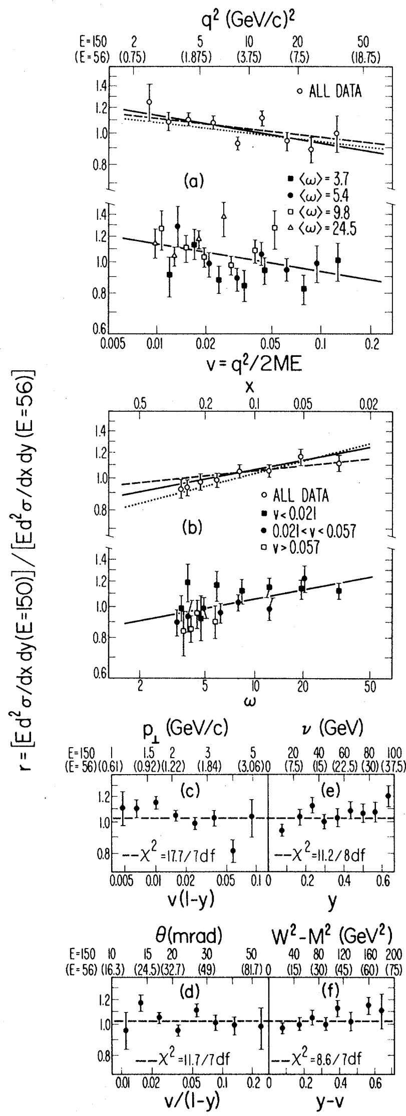

FIG. 1. r versus (a) v, (b) ω, (c) $v(1-y)$, (d) $v/(1-y)$, (e) y, and (f) $y-v$. v is $q^2/2ME$. Other scaled variables are proportional, respectively, to (c) $p_\perp^2$, (d) θ^2, (e) ν, and (f) W^2-M^2, which (with r, ω, y, M, and E) are defined in the text. Errors are statistical. There is an additional normalization error of ± 7%. Typical rms measurement errors are q^2, 17%; $\ln\omega$, 0.5; $p_\perp$, 15%; θ, 5%; $1-y$, 16%; $y-v$, 0.12. Bands of reconstructed ω are assigned rms values determined by the simulations. Solid lines in (a) and (b) are power-law fits to combined data. These fits are also drawn through data broken into bands of (a) ω and (b) v, in order to test the hypothesis that any scale noninvariance depends (a) only on q^2, and (b) only on ω. Fits by a constant are indicated by dashed lines in (c)–(f); df refers to degrees of freedom. In (a) and (b), the effects of increasing E' at 150 GeV by 1% are indicated by dashed lines, and the effects of assuming scaling in ω rather than ω' in the Monte Carlo simulation by dotted lines.

allowance for Coulomb scattering, energy loss, and bending in the iron magnets. Only spark chambers shielded from the target by at least 1240 g/cm^2 of iron (at 150 GeV) were used. Intensities were such that fewer than 30% of *random* triggers yielded one or more tracks.

The track-finding-inefficiency correction, based on studies using auxiliary detectors and omitting various chambers, was less than 7% and varied by less than 1% between the two energies. An exception occurred in the 150-GeV small-angle sample where additional inefficiencies averaging ~ 10% were observed in restricted data sets and fiducial regions. These were cut out with little loss of statistical precision.[9] The relative uncertainty in the magnetic field integral of the spectrometer at the two beam energies is ± 0.7%. Absolute-momentum calibration was based on these magnetic field maps, dE/dx measurements, steering beam muons into the spectrometer, and studying the end point of the E' spectrum. We ascribe uncertainties of 1% to the relative-momentum calibration and less than 7% to the relative normalization of data taken at the two energies.

Internal consistency of the small-angle and large-angle samples was verified by comparing ratios $r=[E\,d^2\sigma/dx\,dy\ (E=150)]/[E\,d^2\sigma/dx\,dy\ (E=56)]$ for each sample in common bins smaller than the experimental resolution (16% in $1/E'$). The χ^2 was 50 and 58 degrees of freedom, permitting the samples to be combined.

The ratios r were corrected for experimental nonscaling effects by means of a Monte Carlo simulation assuming that νW_2 depends only on ω'. Details of the simulation are described elsewhere.[7] In decreasing order of importance, the corrections were made necessary by (i) different radial distributions of the beam, (ii) inexact scaling of spectrometer resolution and acceptance, and (iii) radiative corrections. Effect (i) was greatly reduced *before* Monte Carlo correction by selecting the 56-GeV events to produce agreement between the beam distributions. Combined corrections for (i), (ii), and (iii) typically are less than 10%.

A test of scaling was made by fitting r by a constant in bins of E' and θ with widths smaller than the experimental resolution. The result is consistent with unity (1.02 ± 0.02) with a χ^2 of 117 for 108 degrees of freedom.

Further interpretation of the data is made with the help of Fig. 1, which depicts r as a function of q^2, ω, $p_\perp = E' \sin\theta$, θ, ν, and $W^2 - M^2$. W is the invariant mass of final-state hadrons. Data in Figs. 1(a) and 1(b) are presented both in combined form and (respectively) in bins of ω and (scaled) q^2. The hypothesis that any scale noninvariance is a function only of q^2 is tested in Fig. 1(a) by drawing the fit to combined data through data in the four ω bands. This hypothesis has 16% confidence for either a power-law or a propagator fit in q^2 (Table I). The latter is a poor representation of the data since its value at $q^2=0$ exceeds unity.

A statistically more favorable hypothesis (71% confidence) is that r depends only on ω. In this case the scale-breaking parameter $b = \partial^2 \ln(\nu W_2)/\partial \ln(\omega')\theta \ln(q^2)$ is nearly equal to the exponent of the power-law fit to r versus ω (Table I). If assumed to be ω independent, the parameter b is 0.098 ± 0.028. Its dependence (and also that of the q^2 fit) upon systematic effects is indicated in Fig. 1. In particular, if scaling in ω rather than ω'

TABLE I. Fits to r (defined in the text).

Figure reference	r	Confidence level (%)	Fitted parameters	
1(a)	$(\nu/\nu_0)^n$	16 (16^a)	$n=-0.083\pm0.032$	$\nu_0=0.041\pm0.011$
	$N(1+q_2^2\Lambda^{-2})^2/(1+q_1^2\Lambda^{-2})^{2\,b}$	9 (16^a)	$\Lambda^{-2}=(50^{+29}_{-26})\times10^{-4}$	$N=1.079\pm0.035$
1(b)	$(\omega/\omega_0)^n$	94 (71^c)	$n=0.096\pm0.028$	$\omega_0=6.08^{+8.86}_{-3.61}$

[a] Fit is made to data at all ω. This confidence level applies to the same best fit compared to data broken into four bands of ω [Fig. 1(a)].

[b] q_1^2 (q_2^2) refers to q^2 at 150 (56) GeV. This propagator fit is constrained to $N=1.0$ with a Gaussian error of ± 0.07. The best-fit Λ^{-2} corresponds to $\Lambda > 10.7$ GeV (90% confidence).

[c] Fit is made to data at all ν. This confidence level applies to the same best fit compared to data broken into three bands of ν [Fig. 1(b)].

is tested, b shifts upward by 0.047 [Fig. 1(b)]. An increase of 0.057 results from suspending Monte Carlo corrections, which affect only the end bins in the ω distribution. Dropping these end bins raises b by 0.025. The effect of using a scale-noninvariant form of R which fits data from Stanford Linear Accelerator Center is negligible.[10] The ± 1% systematic error in relative-energy calibration creates an uncertainty of ± 0.056 in b. Therefore, the scale noninvariance observed by using only this method of analysis is not fully conclusive.

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[1] J. D. Bjorken, Phys. Rev. 179, 1547 (1969).

[2] E. D. Bloom *et al.*, Phys. Rev. Lett. 23, 930 (1969); M. Breidenbach *et al.*, Phys. Rev. Lett. 23, 935 (1969).

[3] For a review, see F. J. Gilman, in *Proceedings of the Seventeenth International Conference on High-Energy Physics, London, England, 1974*, edited by J. R. Smith (Rutherford High Energy Laboratory, Didcot, Berkshire, England, 1975), p. IV-149.

[4] For example, see J. S. Poucher *et al.*, Phys. Rev. Lett. 32, 118 (1974).

[5] E. D. Bloom and F. J. Gilman, Phys. Rev. D 4, 2901 (1971); E. M. Riordan *et al.*, Phys. Lett. 52B, 249 (1974).

[6] D. J. Fox *et al.*, Phys. Rev. Lett. 33, 1504 (1974).

[7] C. Chang *et al.*, following Letter [Phys. Rev. Lett. 35, 901 (1975)].

[8] For a description, see Y. Watanabe, thesis, Cornell University, 1975 (unpublished).

[9] An internally consistent subset of these data was used for normalization. The same fiducial cuts were applied to the 56-GeV small-angle sample in order to maintain scale invariance of the analysis.

[10] We used $R(q^2) = 1.28q^2/(q^2+1.16)^2$ (E. M. Riordan, private communication).

Observed Deviations from Scale Invariance in High-Energy Muon Scattering

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Scale noninvariance is observed in 150- and 56-GeV muon scattering from an iron target. In the range $1 < q^2 < 40$ (GeV/c)^2, νW_2 rises with q^2 at fixed $\omega' \gtrsim 6$ and falls at $\omega' \lesssim 6$. The scale breaking is statistically and systematically significant, and persists with alternative choices of scaling variable. It is parametrized roughly by a constant $b = \partial^2 \ln(\nu \times W_2)/\partial \ln(\omega')\partial \ln(q^2)$ with a value near 0.09.

Indications of scale noninvariance in inelastic muon scattering from an iron target have been reported,[1] most recently in ratios[2] of cross sections in beams of 150 and 56 GeV. This Letter compares these cross sections with lower-energy electron-scattering values.[3] Within fixed bands