produced bremsstrahlung causes the inner shells to expand before velocity multiplication occurs, reducing their effectiveness somewhat.[11] This does not seem to be a problem for ion-beam-driven targets.

As has been shown, ion-beam currents of 10 MA or less are required to achieve breakeven. These are considerably lower than the electron-beam currents required for similar targets. Electron currents in excess of 1 MA at 14 MeV have been produced[12] (but with too long a pulse length). However the ion-beam current densities achieved to date in this relatively new field are orders of magnitude below the electron-beam current densities that have been achieved. Further work, presently underway in several laboratories, is needed in order to understand the production and focusing of intense ion beams before ion-beam fusion can be compared quantitatively with other approaches to fusion.

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Interaction of Relativistic Electron Beams with Fusion-Target Blowoff Plasmas

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The relativistic Boltzmann equation with a Fokker-Planck collision term is solved assuming that elastic scattering characterizes the shortest time scale of the beam-plasma system. One-dimensional solutions containing the self-consistent return-current electric field show that, although range shortening occurs, the electric field prevents high-current-density beams from penetrating into the dense-plasma region.

The goal of relativistic electron-beam-initiated fusion reactions in targets containing a deuterium-tritium mixture[1,2] has prompted theoretical interest in the interaction between the beam and a plasma blown off of a high-atomic-number shell surrounding the fusionable material. Investigations have included collisional scattering and energy loss of the beam due to plasma in the absence of electromagnetic fields,[3,4] while others have discussed effects due to these fields neglecting the self-consistent collisional interaction.[1,5] Here, a theory for beam deposition including both collisional and field effects is presented. The relativistic Boltzmann equation with a Fokker-Planck collision term is solved assuming that the elastic-scattering collision time scale is the shortest of the system. A one-dimensional solution is obtained which includes the electric field generated by the plasma return current. The energy transfer from beam to plasma is estimated for this solution and the constraints placed on beams for fusion applications are discussed.

The equation describing the momentum distribution function of relativistic electrons interacting with a cold, high-Z plasma may be written[6]

$$\frac{\partial f}{\partial t}+\frac{\vec{p}}{m\gamma}\cdot\nabla f-e\left(\vec{E}+\frac{\vec{p}\times\vec{B}}{m\gamma}\right)\cdot\nabla_p f$$
$$=\nabla_p\cdot[\nu_S(p)(p^2\overleftrightarrow{I}-\overleftrightarrow{pp})\cdot\nabla_p f]$$
$$+\nabla_p\cdot[\nu_E(p)\vec{p}f], \tag{1}$$

where $\gamma^2=1+p^2/(mc)^2$. The quantities ν_S and ν_E are scattering and energy-loss frequencies:

$$\nu_S=\Omega_S\gamma/(\gamma^2-1)^{3/2},\quad \nu_E=\epsilon\gamma\nu_S, \tag{2}$$

where $\Omega_S=2\pi n_i r_0^{\,2}c(Z^2+Z)\ln\Lambda$ and $\epsilon=2/(Z+1)\ll 1$. Here, n_i is the plasma ion density, r_0 is the classical electron radius, c is the velocity of light, Z is the plasma atomic number, and $\ln\Lambda$ is in the range 10–20.

When the beam dynamics are dominated by elastic scattering, characteristic time and length scales (τ and χ) and the field variables can be ordered according to

$$\nu_S\sim\epsilon^{-1/2}c\chi^{-1}\sim\epsilon^{-1/2}eE/mc\sim\epsilon^{-1/2}eB/m\sim\epsilon^{-1}\tau^{-1},$$

so that the terms of Eq. (1) have relative magnitudes given by

$$O(\epsilon):O(\epsilon^{1/2}):O(\epsilon^{1/2})=O(1):O(\epsilon).$$

When f is expanded in powers of $\epsilon^{1/2}$, so that $f=f_0+f_1+\ldots$, Eq. (1) can be iteratively solved. The solution to second order is given by

$$f_0=f_0(p,\vec{x},t), \tag{3}$$

that is, f_0 is isotropic in momentum space; and

$$f_1=\vec{A}\cdot\vec{p}, \tag{4}$$

$$f_2=\frac{1}{6\nu_S}\left(\frac{e\vec{E}}{p}\frac{\partial\vec{A}}{\partial p}-\frac{1}{m\gamma}\nabla\vec{A}\right):\overleftrightarrow{pp}+\frac{e}{2\nu_S m\gamma}(\vec{B}\times\vec{A})\cdot\vec{p}, \tag{5}$$

where

$$\vec{A}=\frac{1}{2\nu_S}\left(\frac{e\vec{E}}{p}\frac{\partial f_0}{\partial p}-\frac{1}{m\gamma}\nabla f_0\right). \tag{6}$$

Setting secular terms in the second-order equation equal to zero yields

$$\frac{\partial f_0}{\partial t}+\frac{p^2}{3m\gamma}\nabla\cdot\vec{A}=\frac{1}{p^2}\frac{\partial}{\partial p}\left[p^3(\nu_E f_0+\tfrac{1}{3}e\vec{E}\cdot\vec{A})\right]. \tag{7}$$

Moments of f which yield the beam-electron and energy fluxes are of interest. To lowest order, these quantities are

$$\vec{\Phi}=\int\frac{\vec{p}}{m\gamma}f_1d^3p=\int\frac{\overleftrightarrow{pp}}{m\gamma}\cdot\vec{A}\,d^3p, \tag{8}$$

$$\vec{q}=\int mc^2(\gamma-1)\frac{\vec{p}}{m\gamma}f_1d^3p$$
$$=mc^2\int(\gamma-1)\frac{\overleftrightarrow{pp}}{m\gamma}\cdot\vec{A}\,d^3p. \tag{9}$$

The rate at which energy is transferred from the beam to a unit volume of plasma is obtained by taking the divergence of Eq. (9) and substituting from Eq. (7). In the steady state,

$$Q=-\nabla\cdot\vec{q}=e\vec{E}\cdot\vec{\Phi}+4\pi\int_0^\infty\frac{p^4\nu E}{m\gamma}f_0dp. \tag{10}$$

In the same way, Eq. (8) yields

$$\nabla\cdot\vec{\Phi}=-4\pi(p^3\nu_E f_0)_{p=0}$$
$$=-4\pi\epsilon\Omega_S(mc)^3f_0(0,\vec{x}). \tag{11}$$

The right-hand side of Eq. (11) represents the merging of beam electrons, slowed by dynamic friction to very low energies, with the cold-plasma–electron background.

Steady-state solutions of f in one dimension are now considered. The plane $x=0$ divides a semi-infinite, uniform plasma in the region $x>0$ from a vacuum. A monoenergetic, well-collimated beam of relativistic electrons (particle flux Φ_i) is normally incident on the plasma. The plasma is assumed to be sufficiently conductive to exclude the vacuum magnetic field associated with the incident beam.[1] Thus, a return current $j=e\Phi$ of thermal electrons flows in the plasma. The electric field associated with this current is $E=\eta j$, where η is the plasma resistivity.[7] Plasma electrons which reach the interface flow as $\nabla P\times\vec{B}$ surface currents in the $x=0$ plane.

The region $0\leq x\lesssim c\gamma_0^{\,2}/5\Omega_S$ constitutes a scattering boundary layer in which the ordering assumed in this work fails. In this region, the angular distribution of the incident beam decays to the nearly isotropic form derived above. However, the beam transmission at $x=0$ can be estimated by matching the calculated solutions, valid for larger values of x, to the incident beam distribution.[8] This approach is used in what follows.

To first order, the beam-electron distribution function in the plasma can now be written

$$f=f_0(p,x)+\mu pA, \tag{12}$$

where μ is p_x/p and A is determined from

$$\frac{\partial A}{\partial x}=\frac{m\gamma}{p^4}\frac{\partial}{\partial p}\,[p^3(3\nu_E f_0+eEA)]. \qquad (13)$$

When $\nu_E f_0$ and eEA are comparable in magnitude, the equations must be solved numerically. Here, consideration is limited to sufficiently high beam currents to cause dynamic friction to be negligible in comparison to electric field slowing down of the beam. Other solutions are considered elsewhere.[9] For the case at hand, Eq. (11) predicts that Φ, and therefore E, are constant. The solution of Eq. (13) with $\nu_E=0$ for an incident monoenergetic beam yields

$$f=f_0+\mu p A$$
$$=\left\{\alpha-\beta\left[K(\gamma^*)-K(\gamma_0)-\frac{\mathcal{E}\mu}{\gamma^{*2}-1}\right]\right\}\times\delta(\gamma-\gamma^*) \qquad (14)$$

to first order, where α and β are constant, $\mathcal{E}=eE/mc\Omega_S$,

$$K(\gamma)=\frac{\gamma(\gamma^2+1)}{4(\gamma^2-1)^2}+\frac{1}{8}\ln\left(\frac{\gamma-1}{\gamma+1}\right), \qquad (15)$$

and $\gamma^*(x)=\gamma_0-eEx/mc^2$. The quantity β is determined from Eq. (8),

$$\Phi_0=(4\pi/3)m^3c^4\mathcal{E}\beta. \qquad (16)$$

The heating rate is $Q_E=eE\Phi_0$ for $\gamma^*\geqslant 1$.

For small values of x, Eq. (14) reduces to

$$f\approx\{\alpha+[\beta\mathcal{E}/(\gamma_0^{\,2}-1)](\mu-2x/\lambda)\}\delta(\gamma-\gamma_0), \qquad (17)$$

where $\lambda=c(\gamma_0^{\,2}-1)^2/\Omega_S\gamma_0^{\,2}$ is the scattering mean free path for incident electrons. This is the form obtained from a scattering boundary layer analysis[8] for $x\gtrsim\lambda/5$.

The transmission coefficient of current at the interface can be estimated by equating Φ_i with the positive-going current there.[8] To first order,

$$\Phi_i=2\pi\int_0^1 d\mu\int_0^\infty dp\,p^2 f\,\frac{p\mu}{m\gamma}$$
$$=\pi\int_0^\infty\frac{p^3 f_0(p,0)\,dp}{m\gamma}+\tfrac{1}{2}\Phi_0. \qquad (18)$$

With f_0 given by Eq. (14), the transmission coefficient is

$$T_E=\frac{4\mathcal{E}\beta/3}{(\gamma_0^{\,2}-1)\alpha+2\mathcal{E}\beta/3}. \qquad (19)$$

A boundary condition for large x is needed to determine α. Since ν_E increases with x according to the definition of γ^*, dynamic friction must dominate over the electric field at large x. The values of γ^* and x at which dynamic friction becomes important can be estimated by equating the two terms on the right-hand side of Eq. (13). Thus, $3\nu_e f_0=eEA$ at $\gamma^*=\gamma_c=\gamma_0-eEx_c/mc^2$, or

$$\mathcal{E}^2\beta=3\epsilon\gamma_c^{\,2}\{\alpha-\beta[K(\gamma_c)-K(\gamma_0)]\}. \qquad (20$$

The region $x>x_c$ represents an absorber of slowed-down electrons. The negative-going current at $x=x_c$ is then set equal to zero:

$$\Phi_R(x_c)=2\pi\int_{-1}^0\mu\,d\mu\int_0^\infty\frac{p^3 f_0(p,x_c)}{m\gamma}\,dp+\tfrac{1}{2}\Phi_0=0,$$

which results in

$$\alpha=\beta\left[\frac{2}{3}\frac{\mathcal{E}}{\gamma_c^{\,2}-1}+K(\gamma_c)-K(\gamma_0)\right]. \qquad (21)$$

Equations (20) and (21) must be considered to be approximate since the ordering employed again fails in the vicinity of $x=x_c$.

Simultaneous solution of Eqs. (20) and (21) yields γ_c and an approximate value of α. The resulting variation of transmission with electric field is shown in Fig. 1 for $\gamma_0=3$. The electric

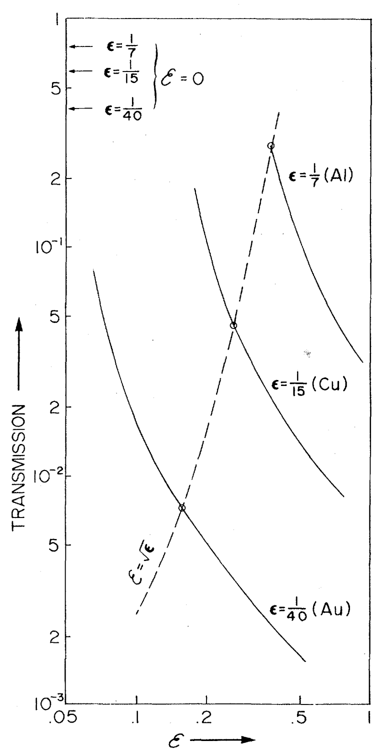

FIG. 1. Transmission coefficient versus normalized electric field strength in plasma for $\gamma_0=3$ and three atomic numbers. Return-current heating dominates to the right of the $\mathcal{E}=\epsilon^{1/2}$ curve. Values of transmission for $\mathcal{E}=0$ are from Ref. 9.

field is related to the penetrating beam current by $E=\eta e\Phi_0$, or

$$e\Phi_0 = 8.4\times10^{-17}\frac{Z^2+Z}{Z_i}n_i\theta^{3/2}\mathcal{E}\,\frac{\mathrm{A}}{\mathrm{cm}^2}. \qquad (22)$$

Here, θ is the plasma electron temperature in eV, and Z_i is the plasma ionization level. The small values of T_E predicted are due to dynamic friction being insufficient to stop scattered incident electrons before they drift back across the interface under the influence of the electric field.

The importance of the electric field to heating can be estimated by comparing Q_E with $\overline{Q}_C$ (the heating rate due to dynamic friction[9] averaged over x). For $\gamma_0=3$, $Q_E/\overline{Q}_C\approx\mathcal{E}/\epsilon^{1/2}$, so that electric field heating dominates when $\mathcal{E}\gtrsim\epsilon^{1/2}$. The region indicated by this inequality is shown in the figure. Electric field (or return-current) heating is important only when $\mathcal{E}$ is large enough to reflect all but a small portion of the incident beam. Thus, in order to maximize energy transfer from beam to plasma, the plasma should be prepared in a manner which keeps $\mathcal{E}$ small everywhere. In terms of the incident current $J_i=e\Phi_0/T_E$, this condition is

$$J_i\lesssim10^{-15}n_i\theta^{3/2}\ \mathrm{A/cm^2}. \qquad (23)$$

A similar condition can be derived for hydrogenic plasmas by neglecting the enhanced reflection due to the electric field and requiring that return-current heating be small compared to beam-plasma collisional heating.[1] For incident currents of 10^9 A/cm^2 and 1-keV temperatures, Eq. (23) suggests that poor penetration into plasmas occurs when $n_i<10^{19}$ cm^{-3}. If, during irradiation, the blowoff has time to expand to a low-density region of large extent, poor coupling of the beam to the dense-plasma or solid portion of the shell is predicted. However, if the beam is not strongly scattered in the low-density blowoff, high transmission to the dense-plasma region can occur. A condition that the beam not be strongly scattered in the region $n_i\leqslant10^{15}J_i\theta^{-3/2}$ is obtained by requiring that λ be larger than the thickness of the blowoff plasma (approximately the beam duration times the plasma sound speed):

$$J_i\tau<10^2\gamma_0^2\theta/Z^2\ \mathrm{C/cm^2}.$$

Here τ is the beam duration in seconds. Relativistic electron beams for fusion as proposed[1] do satisfy this requirement.

The assumption of magnetic-neutralization-created electric fields in the plasma is not usually valid in present-day experiments because of the low plasma temperatures encountered. Comparing the electromagnetic skin depth[5] with the blowoff thickness suggests that magnetic neutralization occurs when $\tau>5\times10^{-5}Z_i\theta^{-5/2}$ sec, so that temperatures in excess of about 10 eV are required for substantial return currents.

Finally, two phenomena which can effect beam-plasma collisions and plasma resistivity must be shown not to alter our results: plasma-electron runaway, and excitation of microturbulence. Using the above normalization for the electric field strength, the critical field for runaway[5] is $\mathcal{E}_c\approx10^5Z_i^2/Z^2\theta$, which is much larger than fields considered here. Also, it has been shown[10] that an enhanced collision rate due to two-stream turbulence is not likely once the relativistic electron beam is scattered. Thus, our results indicate that the existence of large return-current-generated electric fields in pellet blowoff plasmas will not increase beam-energy deposition because they inhibit beam penetration to high plasma densities.

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