Restrictions on the Structure of the $|\Delta S| = 1$ Nonleptonic Hamiltonian*

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Restrictions concerning the chiral structural of the $|\Delta S| = 1$ nonleptonic weak Hamiltonian are derived by studying kaon and hyperon decay amplitudes via current algebra and partial conservation of axial-vector current.

There appears to be increasing motivation for extending the structure of the basic currents which underlie weak interactions. The nonleptonic $\Delta I = \frac{1}{2}$ decays have thus far resisted theoretical efforts to explain both their apparent lack of suppression by $\sin\theta_c \cos\theta_c$ and their dominance over accompanying $\Delta I = \frac{3}{2}$ transitions.[1,2] Effects seen by Cline in neutrino experiments[3] and the large e^+e^- annihilation cross section measured by Augustin *et al.*[4] suggest the existence of quarks with hitherto unobserved "flavors." Such particles would naturally be expected to contribute in some manner to the weak current.

In view of this, investigation of the basic structures of weak currents is presently an active research area, and probably will remain so for some time to come. It is therefore important to be cognizant of constraints placed on any model of the weak interactions by existing experimental data. In this Letter, we discuss implications of the fact that a matrix element describing a particular decay mode is often a *coherent* superposition of isospin and/or of partial-wave amplitudes, Thus, the relative phases between contributing isospin and partial-wave channels are *experimentally observable* quantities. This knowledge can be used to test the merits of competing models of the $\Delta S = \pm 1$ nonleptonic Hamiltonian.

In order to present our ideas in the context of a specific example, we shall at points refer to a model recently suggested by De Rújula, Georgi, and Glashow (DGG).[5] However, general consequences of our study will be presented as well.

In an attempt to solve some of the problems alluded to above, DGG suggest modifying the conventional charged weak current,

$$J^\alpha = \bar{\mathcal{P}}\gamma^\alpha(1+\gamma_5)(\mathfrak{N}\cos\theta + \lambda\sin\theta) + \bar{\mathcal{P}}'\gamma^2(1+\gamma_5)(-\mathfrak{N}\sin\theta + \lambda\cos\theta) + \bar{\nu}_e\gamma^\alpha(1+\gamma_5)e + \bar{\nu}_\mu\gamma^\alpha(1+\gamma_5)\mu, \tag{1}$$

by appending a right-handed term

$$J^{\alpha\prime} = J^\alpha + \bar{\mathcal{P}}'\gamma^\alpha(1-\gamma_5)\mathfrak{N}, \tag{2}$$

which is charm changing and hence does not affect the structure of $\Delta C = 0$ β-decay phenomena. The effective Hamiltonian for nonleptonic $\Delta C = 0$, $\Delta S = \pm 1$ decay,

$$\mathcal{H}_\omega = (G\cos\theta/\sqrt{2})\{\bar{\mathcal{P}}'\gamma^\alpha(1+\gamma_5)\lambda\bar{\mathfrak{N}}\gamma_\alpha(1-\gamma_5)\mathcal{P}' + \sin\theta[\bar{\mathcal{P}}\gamma^\alpha(1+\gamma_5)\lambda\bar{\mathfrak{N}}\gamma_\alpha(1+\gamma_5)\mathcal{P} - \bar{\mathcal{P}}'\gamma^\alpha(1+\gamma_5)\lambda\bar{\mathfrak{N}}\gamma_\alpha(1+\gamma_5)\mathcal{P}']\}, \tag{3}$$

then contains a $\Delta I = \frac{1}{2}$ term which is dominant over its $\Delta I = \frac{3}{2}$ partner by $\cos\theta$ and is no longer suppressed by the factor $\sin\theta$. In addition, the new current appears consistent with other phenomena such as the copious production of $\mu^-\mu^+$ events in neutrino experiments[3] and the K-to-π ratio in the annihilation experiments,[4] both of which are anomalous within the framework of the conventional theory. However, as we shall see, there are difficulties with this kind of model having to do with its transformation properties under chiral SU(4)⊗SU(4).

Let us begin our analysis with the following observation. A general $\Delta S = 1$ current-current model of the nonleptonic Hamiltonian can be constructed out of basic operators

$$O_{ij}{}^x = \bar{\mathfrak{N}}_i x_i \bar{x}_j \lambda_j,$$

where i and j denote the chirality (either right or left handed) and x designates some given quark flavor. It is useful to characterize such forms by means of their transformation properties under chiral SU(2)⊗SU(2). Except for the case where quark x caries isospin and $i \neq j$, the operator $O_{ij}{}^x$ transforms as $(0, m)$ or $(m, 0)$, where m can take on the values $\frac{1}{2}$ or $\frac{3}{2}$. One method for probing the representation content is to employ partial conservation of axial-vector current in nonleptonic kaon decay. By taking a single soft-pion limit, we can relate $\Delta I=\frac{1}{2}$ $(\frac{3}{2})$ parameters of the $K\to 3\pi$ system to experimentally determined $\Delta I=\frac{1}{2}$ $(\frac{3}{2})$ $K\to 2\pi$ amplitudes. When this calculation is performed with the conventional nonleptonic Hamiltonian ($i,j\to$ left handed and x is a $\mathcal{P}$-type quark), the results are quite satisfactory for both amplitude and slope.[6] Unless this is to be regarded as fortuitous, we can conclude the following:

(1) The $\Delta I=\frac{1}{2}$ Hamiltonian is predominantly composed of operators O_{ij}^{x} with the *same* representation content. Otherwise, the operator which results after commutation bears no relation to the original, so that comparison with experimental values is precluded (an entirely analogous agreement can be made for the $\Delta I=\frac{3}{2}$ Hamiltonian).

(2) The nonleptonic Hamiltonian cannot be predominantly composed of operators O_{ij}^{x} transforming as (m,n) with both $m,n\neq 0$. Otherwise, after commutation it would not maintain its original isospin structure. This would undermine any attempt to predict $\Delta I=\frac{1}{2}$ $(\frac{3}{2})$ effects in $K\to 3\pi$ from corresponding $\Delta I=\frac{1}{2}$ $(\frac{3}{2})$ terms in $K\to 2\pi$.

Hereafter, we shall consider only candidates for the nonleptonic Hamiltonian which do not violate conditions (1) and (2). Writing

$$H_w=H_w^{\ 1}+H_w^{\ 3},$$

where 1, 3 refer, respectively, to the $\Delta I=\frac{1}{2}$, $\frac{3}{2}$ components of the Hamiltonian, we define four classes of models via their commutation properties with $F_i^{\ 5}$ ($i=1,2,3$): class I,

$$[F_i^{\ 5},H_w]=[F_i,H_w^{\ 1}]+[F_i,H_w^{\ 3}]; \tag{4a}$$

class II,

$$[F_i^{\ 5},H_w]=-[F_i,H_w^{\ 1}]+[F_i,H_w^{\ 3}]; \tag{4b}$$

class III,

$$[F_i^{\ 5},H_w]=[F_i,H_w^{\ 1}]-[F_i,H_w^{\ 3}]; \tag{4c}$$

class IV,

$$[F_i^{\ 5},H_w]=-[F_i,H_w^{\ 1}]-[F_i,H_w^{\ 3}]. \tag{4d}$$

Note that the conventional model belongs to class I while the DGG version is (approximately) a member of class II.[7]

In order to understand the distinction between these classes, we return to an examination of the nonelptonic decays of the kaon. Here, as long as only $\Delta I=\frac{1}{2}$ effects are considered, one cannot distinguish between any of the four classes since the relative phase between $K\to 2\pi$ and $K\to 3\pi$ amplitudes is unobservable. However, in both the $K\to 2\pi$ and $K\to 3\pi$ systems there exist small but detectable $\Delta I=\frac{3}{2}$ effects, which can be related by means of the current-algebra methods discussed above. Assuming a linear expansion in energy for the $K\to 3\pi$ amplitudes, we find[6]

$$\frac{1}{2}\frac{\Gamma_{++-}}{\Gamma_{+-0}}-1=\begin{cases}\left(\dfrac{1-y}{1+2y}\right)^2-1 \text{ for class I, IV}\\[2ex] \left(\dfrac{1+y}{1+2y}\right)^2-1 \text{ for class II, III},\end{cases}$$
$$-\frac{1}{2}\frac{\lambda_{+00}}{\lambda_{++-}}-1=\begin{cases}-\frac{27}{2}y \text{ for class I, IV}\\ +\frac{27}{2}y \text{ for class II, III},\end{cases} \tag{5}$$

where Γ_{abc} and λ_{abc} are, respectively, the rate (divided by phase space) and slope for the decay $K\to\pi^a\pi^b\pi^c$, and

$$y=\frac{-\sqrt{2}A(K^+\to\pi^+\pi^0)}{2A(K^0\to\pi^+\pi^-)-A(K^0\to\pi^0\pi^0)}$$
$$=-0.032\pm 0.001. \tag{6}$$

The experimental and theoretical values are compared in Table I. For the decay rates there is good quantitative agreement for class-I and class-IV models but strong disagreement for models belonging to class II or III. In the case of the slope, which being a ratio of coefficients is much more sensitive to omitted terms of $O((m_\pi/m_k)^2)$, none of the predictions agree with the experimental value, but again models in class I or IV have the correct sign whereas class-II or class-III models do not.

A similar analysis can be performed for nonleptonic hyperon decay.[8] Here the S-wave ampli-

TABLE I. $K\to 3\pi$ observables. Γ_{abc} and λ_{abc} are, respectively, the decay rate with phase space divided out, and slope parameter in the Dalitz plot for the reaction $K\to\pi^a\pi^b\pi^c$. The numerical values appearing in the theoretical columns are based on the model of Ref. 7, and the division into classes pertains to Eqs. (4).

	Theoretical		
	Class I, IV	Class II, III	Experimental
$\frac{\Gamma_{++-}}{2\Gamma_{+-0}}-1$	0.215	-0.172	0.251 ± 0.038
$\frac{-\lambda_{+00}}{2\lambda_{++-}}-1$	0.43	-0.43	0.219 ± 0.051

tude is given by the commutator

$$\langle\beta\pi^2|H_w|\alpha\rangle_{S\text{-wave}}=\frac{-i}{F_\pi}\langle\beta|[F_a{}^5,H_w{}^{\text{p.v.}}]|\alpha\rangle\,, \tag{7}$$

while the P-wave amplitude is assumed to be given by the hyperon pole terms,[9]

$$\langle\beta\pi^a|H_w|\alpha\rangle_{P\text{-wave}}=\bar{u}(p')\gamma_5u(p)\times i\left[\frac{m_\alpha+m_\beta}{m_\beta+m_\gamma}\frac{g_{\pi a\beta\gamma}S_{\gamma\alpha}}{m_\gamma-m_\alpha}+\frac{m_\alpha+m_\beta}{m_\alpha+m_\gamma}\frac{g_{\pi a\delta\alpha}S_{\beta\delta}}{m_\beta-m_\delta}\right], \tag{8}$$

where $\langle\beta|H_w{}^{\text{p.c.}}|\alpha\rangle\equiv S_{\beta\alpha}\bar{u}(p')u(p)$ and the phase of $F_\pi g_{\pi\beta\alpha}$ is fixed by the Goldberger-Treiman relation. In this approximation, the same matrix elements appear in both S-wave and P-wave amplitudes so that (1) because of the sign differences in the commutators in Eqs. (4a)–(4d), the relative phase of the dominant $\Delta I=\frac{1}{2}$ S-wave and P-wave amplitudes is opposite for models in classes I, III to that for classes II, IV. The experimental situation is indicated in Table II, which favors classes I or III. Also (2) the relative phase of S-wave $\Delta I=\frac{3}{2}$ and $\Delta I=\frac{1}{2}$ terms is opposite in classes I, IV to that in classes II, III models. That is, defining for Σ decay (subscripts +, −, 0 label the charge of the decay pion)

$$S_3=-\sqrt{2}S_0+S_+-S_-\,,\qquad P_3=-\sqrt{2}P_0+P_+-P_-\,, \tag{9}$$

we predict for classes I and IV,

$$\frac{P_+S_3}{S_-P_3}=+1.48, \tag{10a}$$

and for classes II and III,

$$\frac{P_+S_3}{S_-P_3}=-1.48, \tag{10b}$$

TABLE II. Hyperon S-wave and P-wave nonleptonic decay amplitudes. Each entry gives the *ratio* of P-wave to S-wave amplitudes. See Ref. 9 for values of SU(3) parameters employed here.

	Theoretical Conventional	Theoretical DGG	Experimental
$\Lambda\to n\pi^0$	5.2	−5.2	6.74±0.55
$\Sigma^+\to p\pi^0$	−5.4	5.4	−8.14±0.50
$\Sigma^-\to n\pi^-$ [a]	−0.88	0.88	−0.34±0.04
$\Xi^0\to\Lambda\pi^0$	−2.6	2.6	−3.86±0.73

[a]The result for this mode depends far more sensitively than the others on our choice of pole model and accompanying parameters. We include it for completeness.

while the experimental value is[10]

$$\frac{P_+S_3}{S_-P_3}=+0.81\pm0.48, \tag{10c}$$

which, although hardly conclusive, again favors class-I and class-IV models.

Of course, our conclusions are not definitive since (1) a considerable piece of the $\Delta I=\frac{3}{2}$ amplitudes might conceivably be electromagnetic in origin. This would invalidate our conclusions in both the kaon and hyperon sectors concerning $\Delta I=\frac{3}{2}$ terms. However, such effects should be of $O(\alpha/\pi)$ and thus negligible.[11] Also one would have to assume that the reasonably good quantitative results for $\Delta I=\frac{3}{2}$ terms obtained in Ref. 6 for both amplitudes and slopes is fortuitous. (2) Pole terms constitute only a portion of the P-wave hyperon amplitudes. However, simple $\frac{1}{2}^+$ pole fits indicate that this is the dominant component of the P-wave amplitudes[8] so that the overall sign should be correct.

What then can be concluded in general from our analysis? We have obtained several conditions regarding the structure of the $|\Delta S|=1$ nonleptonic Hamiltonian. Let us discuss them in order of increasing "tightness" of the constraints which they impose.

First, from the rather successful prediction of the structure of the $K\to3\pi$ amplitude in terms of experimental $K\to2\pi$ parameters, it follows that the commutation properties indicated in Eqs. (4) are approximately valid.

Secondly, from the comparison of $\Delta I=\frac{3}{2}$ and $\Delta I=\frac{1}{2}$ amplitudes, we require that the Hamiltonian belongs either to class I or to class IV in order to avoid the consequences of opposite signs for these terms after commutation, e.g., as occurs for the DGG Hamiltonian.

Finally, although the theoretical justification for this conclusion is weaker than the others we have given, we require from the comparison of hyperon S- and P-wave decay amplitudes that the $\Delta I=\frac{1}{2}$ Hamiltonian should be either from class I or from class III.

Thus, models of class I, such as the conventional nonleptonic Hamiltonian, are the only ones favored by all criteria, whereas class-II versions, such as the DGG modification, are ruled out by each test. Moreover, given from the study of β decay that there exists an interaction of the form

$$(G/\sqrt{2})\cos\theta\,\sin\theta\bar{\lambda}\gamma^{\alpha}(1+\gamma_5)\mathcal{P}\bar{\mathcal{P}}\gamma_{\alpha}(1+\gamma_5)\mathfrak{N} + \text{H.c.}, \tag{11}$$

a $\Delta I=\frac{1}{2}$ modification which seeks to avoid the difficulties described previously should have the form

$$\bar{\lambda}\gamma^{\alpha}(1\pm\gamma_5)x\bar{x}\gamma_{\alpha}(1+\gamma_5)\mathfrak{N}, \tag{12}$$

where x is an SU(2) singlet quark which may or may not coincide with the $\mathcal{P}'$. If x is the $\mathcal{P}'$, then this term is suppressed by $\sin\theta$, although if the $\lambda\mathcal{P}'$ current is right-handed this may be somewhat compensated by enhancement effects calculated within the renormalization-group framework.[12] If, however, x carries some new flavor, this left-handed $\mathfrak{N}$ quark cannot be accommodated within the usual gauge-theory framework, and the existence of an additional weak interaction mediated by some new gauge boson must be assumed. It is not our purpose here to construct such a model, however, but merely to emphasize some strictures on its form.

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[1]M. K. Gaillard and B. W. Lee, Phys. Rev. Lett. 33, 108 (1975), use renormalization-group methods to generate an enhancement for the $\Delta I=\frac{1}{2}$ and suppression for the $\Delta I=\frac{3}{2}$ nonleptonic $\Delta S=1$ interactions. However, the effect is too small to fit the data.

[2]J. Donoghue, E. Golowich, and B. R. Holstein, "Phenomenological Analysis of a Fixed-Sphere MIT Bag Model—I" (to be published), calculate $\Delta T=\frac{1}{2}$ hyperon S-wave decay amplitudes employing a reasonable model for hadron wave functions and the operators of Ref. 1. Again the amplitudes are too small.

[3]D. Cline, Bull. Am. Phys. Soc. 20, 635 (1975).

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[6]B. Holstein, Phys. Rev. 183, 1228 (1969); Y. Nambu and Y. Hara, Phys. Rev. Lett. 16, 875 (1966); C. Bouchiat and Ph. Meyer, Phys. Lett. 25B, 282 (1967). Also see A. Neveu and J. Scherk, Ann. Phys. (N.Y.) 57, 39 (1970) who consider final-state interactions in the $K\to 3\pi$ system.

[7]Here we assume that the $\Delta I=\frac{1}{2}$ Hamiltonian is composed strictly of the new $\bar{\lambda}\mathcal{P}'\bar{\mathcal{P}}'n$ piece. If the conventional $\Delta I=\frac{1}{2}$ component is comparable, then there is no reason to have the new model. If this component enters at the $\sin\theta$ level, it could make ~20% corrections to the size of our predictions but will *not* alter our basic conclusions. Thus, we omit this term for the sake of simplicity.

[8]B. W. Lee, Phys. Rev. 170, 1359 (1969); B. Holstein, Nuovo Cimento A 2, 561 (1971); M. Gronau, Phys. Rev. Lett. 28, 188 (1972), and Phys. Rev. D 5, 118 (1972).

[9]The F/D ratio for the hadron coupling constants and weak Hamiltonian matrix elements are, respectively, 0.45 and −1.25, the latter value being taken from Ref. 8. We have employed pseudovector pion-hadron coupling constants in order to avoid contact terms which would otherwise appear, and have used the generalized Goldberger-Treiman relation in deriving Eq. (8).

[10]V. Chaloupka *et al.*, Phys. Lett. 50B, 1 (1974).

[11]See, however, C. Bouchiat and M. Veltman, in *Topical Conference on Weak Interactions, CERN, Geneva, Switzerland, 1969*, edited by J. S. Bell (CERN Scientific Information Service, Geneva, Switzerland, 1969), p. 225.

[12]G. Altarelli and L. Maiani, Phys. Lett. 52B, 351 (1974).