suggested in Ref. 3 is not observed in the present experiment. We conservatively set a limit on the intensity of such structure at 1% of the continuum for lines between 50 and 200 MeV. Since on the average there are three γ rays emitted per $\bar{p}$ annihilation,[3] our upper limit corresponds to an intensity of less than one γ ray of discrete energy per thirty annihilations. This result is consistent with theoretical predictions[2] of very small γ-ray branching ratios in the $N\bar{N}$ system.

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Phenomenology of Goldstone Neutrinos[*]

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The hypothesis that the physical neutrino is a Goldstone particle associated with spontaneously broken supersymmetry is studied with emphasis on the ensuing low-energy theorems. A picture of low-energy neutrino phenomenology is found which is attractive for neutral-current processes, but is in strong disagreement with experimental end-of-lepton-spectrum behavior in β decays.

Supersymmetric field theories are an elegant theoretical development,[1] but unfortunately no clear physical application is yet apparent. Since there are no degeneracies in mass between bosons and fermions in nature (expect between photon and neutrino, which is considered below), the supersymmetry, if applicable at all, must be broken. If the breakdown is spontaneous, the Goldstone theorem[2] requires the existence of a massless chiral spin-$\frac{1}{2}$ particle, and it is an attractive hypothesis[3] that the physical neutrino is this particle.

The present theoretical status of this hypothesis is that a general proof[4] of the Goldstone theorem in the case of supersymmetry has been given, and Lagrangian models of spontaneously broken supersymmetry[5-7] have been constructed. These models are generalizations of the original Wess-Zumino-Iliopoulos Lagrangian and of Lagrangian models in which supersymmetry is combined with either Abelian[8] or non-Abelian[9] gauge invariance. The most realistic model, due to Fayet,[6] is a unified description of the weak and electromagnetic interactions of an electron, its neutrino, and other heavy particles. Present models do not naturally describe two neutrinos (ν_e and ν_μ) but the general framework may be flexible enough to accomodate them.

It is to be expected that low-energy theorems[10] of the Nambu-Adler and Adler-Weisberger type apply to Goldstone neutrinos and that the resulting suppression of low-energy neutrino processes would be incompatible with experiment. Indeed Bardeen[11] has pointed out that the theoretical low-energy behavior of a β-decay amplitude is inconsistent with experimental information[12] on end-of-spectrum behavior in tritium β decay.

A careful theoretical study of the validity of low-energy theorems of spontaneously broken supersymmetry is therefore of considerable importance to the Goldstone neutrino hypothesis. We report here the results of such a study in a general class of field theories with combined supersymmetry and gauge invariance. We extract and concentrate upon the model-independent features of those theories. Technical matters associated with the combined theories, which support our discussion, will be presented separately.[13] Our results show that such models have a rich and characteristic low-energy phenomenology which is quite attractive for neutral-current processes but in serious disagreement with charged-current results.[11]

The expected suppression due to low-energy theorems might be inoperative because of special points of principle in the combined theories which do not occur in chiral pion theories, or because of Born terms which are singular at low energy as a result of mass degeneracies, a phenomenon which is well known to occur in current-algebra applications.[10] Our investigation has not given indication of major points of principle, but we have found singular Born terms, arising from the photon-neutrino degeneracy, and an unconventional singular Schwinger term, which contributes to low-energy theorems.

Our results can be described succinctly for physical processes involving the neutrino ν, the photon γ, and states A and B, which are single-particle or multiparticle states involving only massive particles. We consider three different types of neutrino processes: (I) processes of the type $A\to\nu+B$ (or $A\to\bar{\nu}+B$); (II) radiative processes of the type $A\to\nu+\gamma+B$ (or $A\to\bar{\nu}+\gamma+B$); and (III) neutral-current processes of the type $\nu+A\to\nu+B$ (or $\bar{\nu}+A\to\bar{\nu}+B$).

We let q denote the final neutrino momentum, and $\bar{u}_L(q)$ $[\bar{u}_R(q)]$ the final neutrino (antineutrino)[14] spinor. Similarly, k and $\epsilon^*(k)$ are the photon momentum and polarization, and l and $u_L(l)$ $[u_R(l)]$ are the corresponding quantities for an incident neutrino (antineutrino). We then introduce amplitudes[15] $\bar{u}(q)M_{\rm I}(q)$ for type-I processes, $\epsilon_\mu^*(k)\times\bar{u}(q)M_{\rm II}{}^\mu(q,k)$ for type-II processes, and $\bar{u}(q)\times M_{\rm III}(q,l)u(l)$ for type-III processes.

The results are as follows. For type-I processes, the low-energy suppression is present and therefore

$$\text{(I)}\quad \lim_{q\to 0} M_{\rm I}(q)=0. \tag{1}$$

This result is a general consequence of the Goldstone neutrino idea, and would obtain even if this idea were realized outside the framework of supersymmetric field theories. The right-hand side of (1) would not vanish in conventional $V-A$ theories, and this leads to the experimental difficulty mentioned above. In type-II and type-III processes there are singular Born terms, and there is no low-energy suppression. The results are

$$\text{(II)}\quad \epsilon_\mu^*(k)M_{\rm II}{}^\mu(q,k)\xrightarrow[q\to 0]{} -cf_\nu^{-1}\not{\epsilon}^*\not{k}\gamma_5 M_{\rm I}(k)+O(q), \tag{2}$$

in which the amplitude $M_{\rm I}(k)$ for the related nonradiative process appears. The constants c and f_ν are defined below. For neutral-current processes

$$\text{(III)}\quad \bar{u}(q)M_{\rm III}(q,l)u(l)\xrightarrow[q\to 0]{} -[\bar{u}(q)\gamma_\mu\gamma_5 u(l)]ecf_\nu^{-1}\langle B|J_{\rm em}{}^\mu(0)|A\rangle+O(q), \tag{3}$$

which reflects the interesting fact that the effective neutral current in the Goldstone neutrino framework is proportional to the electromagnetic current. Further information is obtained in the limit where both neutrino momenta become small, viz.,

$$\begin{aligned}\text{(III)}\quad \bar{u}(q)M_{\rm III}(q,l)u(l)\xrightarrow[\substack{q\to 0\\ l\to 0}]{} &-[\bar{u}(q)\gamma_\mu\gamma_5 u(l)]ecf_\nu^{-1}\langle B|J_{\rm em}{}^\mu(0)|A\rangle\\ &+[\bar{u}(q)\gamma_\mu u(l)]f_\nu^{-2}(q+l)_\rho\langle B|\theta^{\rho\mu}(0)|A\rangle+O(q_\alpha q_\beta,q_\alpha l_\beta,l_\alpha l_\beta),\end{aligned} \tag{4}$$

where the energy-momentum tensor appears as a current commutator contribution.

The supersymmetry current $S_\mu(x)$, which is a gauge-invariant operator transforming under Lorentz transformations as a combined four-vector and Majorana spinor, is the central operator in our study of the low-energy theorems. This operator is not strictly conserved because of the gauge-field quantization procedure, but the matrix elements between physical states are gauge invariant and conserved,[13] viz.,

$$\partial_\mu \langle \text{phys} | S^\mu(x) | \text{phys} \rangle = 0. \tag{5}$$

The results (1)–(3) are derived from this simple one-current Ward identity.

We assume that we are dealing with a field theory in which spontaneous breakdown of supersymmetry occurs and gauge invariance is broken except for a residual electromagnetic gauge group. In this situation a massless photon and massless neutrino, described by a Majorana field, appear. The Fayet model,[6] for example, has exactly these properties. The general proof[4] that a Goldstone fermion appears needs minor modification in gauge theories, and this has been given.[13]

The relevant terms in the supersymmetry current are

$$S^\mu(x) = -if_\nu \gamma^\mu \nu(x) + \tfrac{1}{2} c F_{\alpha\beta}(x) \gamma^\alpha \gamma^\beta \gamma^\mu \gamma_5 \nu(x) + \dots , \tag{6}$$

where $\nu(x)$ is the neutrino field and $F_{\alpha\beta}(x)$ is the electromagnetic field tensor. In the Fayet model $c = 1$, but we may have $c \neq 1$ in more complicated models.

The derivation of low-energy theorems is very similar to the case of chiral pions,[10] namely, we consider various contributions to Eq. (5) in the limit of vanishing momentum q associated with the current. In that limit the neutrino-pole contribution, which originates from the first term in (6), survives. Unless there are additional singular contributions due to boson-fermion mass degeneracies, the low-energy neutrino emission amplitude vanishes. This is the case for processes of type I, since there are no such mass degeneracies known[15] among the massive particles in nature, and we obtain (1). For processes of types II and III, the mass degeneracy of photon and neutrino is relevant and leads to a singular Born term, determined by the second term in the current (6). This leads to (2) and (3). These conclusions have been verified in several explicit tree approximation calculations.

The two-current Ward identity which incorporates information on the commutator of supersymmetry currents will give additional information on neutral-current amplitudes (III). This identity[13] may be written as

$$(\partial/\partial x_\mu) \langle \text{phys} | T S^\mu(x) \overline{S}^\rho(y) | \text{phys} \rangle = -\delta^4(x-y) \gamma_\mu \langle \text{phys} | \{ 2\theta^{\rho\mu}(x) + c f_\nu \epsilon^{\rho\mu\alpha\beta} F_{\alpha\beta}(x) \} | \text{phys} \rangle . \tag{7}$$

The first term gives the connection between the commutator of supersymmetry currents and the energy-momentum tensor $\theta^{\rho\mu}$. The second term, which has no analog for chiral pions, may be described as a covariant Schwinger term which contributes to the low-energy theorem because it brings in a photon propagator which is singular. It is closely related to the second term in the current (6). Other Schwinger and seagull terms[13] which do not contribute to the low-energy theorem have been omitted in (7).

To relate the amplitude $M_{\text{III}}(q,l)$ to the right-hand side of (7), we use the same combinatoric technique applied by Weinberg[16] to multiple-pion emission. We define amplitudes $T^{\mu\rho}(q,l)$, $T^\mu(q,l)u(l)$, and $\overline{u}(q)\overline{T}^\rho(q,l)$ as the Fourier transforms of the matrix elements $\langle B | T S^\mu(x) \overline{S}^\rho(y) | A \rangle$, $\langle B | S^\mu(x) | A \nu(l) \rangle$, and $\langle B \nu(q) | \overline{S}^\rho(y) | A \rangle$, respectively. The amplitude $M_{\text{III}}(q,l)$ occurs as a neutrino pole residue in $T^{\mu\rho}$, T^μ, and $\overline{T}^\rho$. We can then write

$$M_{\text{III}}(q,l) = -if_\nu^{-2} q_\mu l_\rho T^{\mu\rho}(q,l) - if_\nu^{-1} l_\rho \overline{T}^\rho(q,l) - if_\nu^{-1} q_\mu T^\mu(q,l) + if_\nu^{-2} q_\mu l_\rho N^{\mu\rho}(q,l), \tag{8}$$

where $N^{\mu\rho}(q,l)$ is that part of $T^{\mu\rho}$ consisting of all graphs without neutrino poles. On the mass shell $\not{l} = \not{q} = 0$ (but $l_\rho \neq 0$, $q_\mu \neq 0$), the second and third terms vanish by (5). We now take the low-energy limit $q_\mu \to 0$, $l_\rho \to 0$, and the last term vanishes because there are no boson-fermion mass degeneracies in states A and B. Only the first term survives in the low-energy limit and it can be evaluated using the Ward identity (7). The result (4) is obtained after sandwiching between spinors and using a little Dirac algebra.

It is worth pointing out the exact nature of the experimental disagreement with the result (1). In three-body β-decay processes the limit of low neutrino energy corresponds to high charged-lepton energy, and (1) suppresses the lepton spectrum at high energy in roughly the same way as would a nonzero neutrino mass. Experiments to determine neutrino mass upper limits therefore provide a direct check on (1), and we learn that (1) is inconsistent with experiment unless the scale of validity of the low-energy theorems is of the order of present neutrino mass limits, i.e., 60 eV for ν_e and 1.2 MeV for ν_μ.

Theoretically the expected scale of the low-energy theorems is not fully clear but should be set by particle states which are excited[11] when the current $S^\mu(x)$ acts on an external line of the states A and B. If the line concerned is a lepton, then the particle state could be one of the vector and scalar bosons which mediate the weak interaction in the models. If the line concerned is a hadron, then the particle state would have to be a new particle which carries the lepton quantum number. In view of the present experimental situation it is unlikely that either type of particle could have a mass less than several GeV. The most optimistic attitude one could take would be that the scale of the low-energy theorem would be determined by the lightest mass particle of the states A and B. Even in this case, (1) would remain inconsistent with experiment.

In the case of neutral-current processes (III), theoretical considerations provide two terms in the low-energy expansion of amplitudes, as shown in (4), and we can therefore estimate with more confidence the energy scale in which the first term dominates. Since we estimate $e f_v^{-1} \approx G_F$, the second term is suppressed with respect to the first for lepton momenta satisfying $G_F e^{-2} q p \ll 1$, where p is a typical particle energy in the state A or B. This suggests that the neutral weak interaction is dominated by $J_{\text{em}}{}^\rho(0)$ for energies $qp < 10^3$ GeV. In view of the problems posed by (1), any exultation over this nice result should be suppressed.

We believe that we are making a strong criticism of the Goldstone neutrino hypothesis, and, because we ourselves are reluctant to see an elegant idea put to rest, it is worth discussing the theoretical basis of our work and possible ways to avoid the conclusions.

Once the Goldstone theorem and the Ward identity (5) are established, our results follow ineluctably. In our work[13] these results were established using formal manipulations in the path-integral formalism. These techniques are correct in the tree approximation but could conceivably fail for loop graphs if there were anomalies in the supersymmetry current. The existence of such anomalies is an interesting theoretical question, but rather than a modification of (1) in the direction of agreement with experiment, the most likely result of anomalies would be that massless fermions associated with supersymmetry breakdown would not be required. It is then an open question how the physical neutrinos would fit in the resulting framework. Another method of eliminating Goldstone fermions could be a generalized Higgs mechanism involving spin-$\frac{3}{2}$ particles,[17] but the introduction of high-spin fields in quantum field theory poses new and severe difficulties.

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