COMMENTS

Leptonic Processes in a Scalar Model of Weak Interactions

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$\nu_\mu e$ scattering, $\nu_e e$ scattering, $e^+e^- \to \mu^+\mu^-$ asymmetry, and $e^+e^- \to \mu^+\mu^-$ muon polarization are calculated for a renormalizable model of weak interactions mediated by scalar bosons $B^\pm$, B^0, and $\overline{B}^0$. The predictions differ considerably from those of other weak-interaction theories.

In a recent Letter, Segrè[1] discussed a renormalizable model in which the weak interactions are mediated by scalar bosons and heavy leptons.[2] The purpose of the present note is to extend that work by investigating some of the experimental consequences of his scalar model of weak interactions (SMWI). Calculational details and evaluation of higher-order corrections will be published separately.

The interaction Lagrangian of Ref. 1 is

$$\mathcal{L}_{\rm int} = if\{\sum_{l=e,\mu} \overline{L}_l(1-\gamma_5)lB^0 + \overline{L}_l(1-\gamma_5)\nu_l B^- + [\overline{\mathfrak{N}}_c(1-\gamma_5)\mathcal{P} + \overline{\lambda}_c(1-\gamma_5)\mathcal{P}']B^- + [\overline{\mathfrak{N}}_c(1-\gamma_5)\mathfrak{N}_c + \overline{\lambda}_c(1-\gamma_5)\lambda_c]B^0\} + \text{H.c.}, \tag{1}$$

where L_e and L_μ are heavy leptons and $\mathcal{P}$, $\mathfrak{N}$, λ, and $\mathcal{P}'$ are the conventional SU(4) quarks.[3] The diagram for μ decay is shown in Fig. 1. The effective $V-A$ interaction is

$$H_{\rm eff} = (f^2/4\pi)^2 m^{-2}\,\overline{\nu}_\mu\gamma^\alpha(1-\gamma_5)\mu\,\overline{e}\,\gamma_\alpha(1-\gamma_5)\,\nu_e, \tag{2}$$

so that the weak Fermi coupling constant is identified as

$$G/\sqrt{2} \simeq (f^2/4\pi)^2 m^{-2}, \tag{3}$$

where m is the mass of the scalar meson.

(I) Universality bounds $f^2/4\pi$ from below. If we assume that the quark masses (M_Q) can be neglected in comparison with the heavy-lepton masses M (and take the charged- and neutral-boson masses equal, which gives the least restrictive bound), the full contribution to Fig. 1 (and to its analog for β decay) replaces Eq. (3) by

$$\frac{G_\mu}{\sqrt{2}} = \left(\frac{f^2}{4\pi}\right)^2 \frac{1}{m^2}\left[1 + \frac{M^2}{m^2}\left(3 - 2\ln\frac{m^2}{M^2}\right)\right], \tag{4a}$$

$$\frac{G_\beta}{\sqrt{2}} = \left(\frac{f^2}{4\pi}\right)^2 \frac{1}{m^2}\left[1 + \frac{M^2}{m^2}\left(1 - \ln\frac{m^2}{M^2}\right)\right]. \tag{4b}$$

Requiring $G_\mu = G_\beta$ to within, say, 5% and $M \geq 5$ GeV gives $f^2/4\pi \geq 0.08$. We will ignore the two other ways of attaining universality, namely M_Q

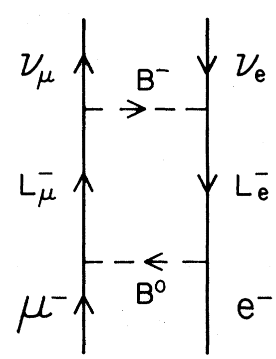

FIG. 1. Diagram for μ decay.

$\simeq M$ or $\ln(m^2/M^2) \simeq 2$, because we regard them as physically unlikely.[4] We further restrict our discussion to the region $f^2/4\pi < 1.0$ in order that perturbation calculations be sensible.

(II) $\nu_\mu e \to \nu_\mu e$ is forbidden in fourth order because of the form of the interaction Lagrangian, Eq. (1). It proceeds in order f^6 and in order $f^2 e^2$ through the diagrams of Fig. 2. The cross section in the lab frame can be written

$$d\sigma/dT = (G^2/2\pi)m_e\left[(C_V{}' - C_A{}')^2 + (C_V{}' + C_A{}')^2(1 - T/\omega)^2 - (C_V{}'^2 - C_A{}'^2)m_e T/\omega^2\right], \tag{5}$$

where m_e is the electron mass, T the final electron's kinetic energy, and ω the energy of the initial neutrino. The diagrams of Fig. 2 give

$$C_A{}' = \frac{f^2}{4\pi}\frac{1}{\pi}\frac{R-1}{\ln R} I(R), \tag{6a}$$

$$C_V{}' = C_A{}' + \alpha\left(\frac{f^2}{4\pi}\right)^{-1}\frac{R-1}{R\ln R}\left(1 - \frac{2}{3}\ln\frac{m_+{}^2}{M^2}\right), \tag{6b}$$

where m_+ and m_0 are the masses of the charged and neutral scalars and $R = m_+{}^2/m_0{}^2$.[5] $I(R)$ is a known integral with $I(R=1) = 1$ which comes from evaluating the diagram of order f^6 in Fig. 2. We plot in Fig. 3 $C_V{}'$ and $C_A{}'$ for several values of R as $f^2/4\pi$ varies from ~ 0.1 to 1.0. The Weinberg-Salam[6] model gives $C_A{}' = 0.5$ and $-1.5 < C_V{}' < 0.5$. Another spontaneously broken gauge theory for lepton-lepton scattering, based on SU(2) $\otimes$ SU(2)$\otimes$ U(1) and motivated by strong-interaction duality,[7] gives $C_A{}' > \frac{1}{8}$ and $-\frac{1}{12} - C_A{}' < C_V{}' < \frac{1}{4} - C_A{}'$. That model and the SMWI allow for a cross section for $\nu_\mu e$ which is not zero but is distinctly less than the cross sections for other processes mediated by neutral currents and distinctly less than the $\nu_\mu e$ cross section in the Weinberg-Salam theory. This smaller, but nonzero, cross section is more in line with the present experimental situation.

(III) The process $\nu_e e \to \nu_e e$ is allowed in fourth order and reduces to

$$\frac{G}{\sqrt{2}}\bar{\nu}\gamma^\alpha(1-\gamma_5)\nu\bar{e}\gamma_\alpha(C_V - C_A\gamma_5)e, \tag{7}$$

with $d\sigma/dT$ given by (5) above; to this order in the SMWI, $C_V = C_A = 1$; in the Weinberg-Salam spontaneously broken gauge theory, $C_A = \frac{1}{2} \leq C_V \leq \frac{5}{2}$. The prediction $C_V = C_A = 1$ is independent of the ratio $R = m_+{}^2/m_0{}^2$.

(IV) A weak contribution to the process $e^+e^- \to \mu^+\mu^-$ is also allowed in order f^4. The diagram is a box, as in Fig. 1, but with the exchange of two B^0's. Its contribution is

$$M_W = -\left(\frac{f^2}{4\pi}\right)^2\frac{1}{m_0{}^2}\bar{\mu}\gamma^\alpha(1-\gamma_5)\mu\bar{e}\gamma_\alpha(1-\gamma_5)e. \tag{8}$$

The dominant s-channel pole term with which it interferes is

$$M_\gamma = \frac{e^2}{4E^2}\bar{\mu}\gamma^\alpha\mu\bar{e}\gamma_\alpha e, \tag{9}$$

where $2E$ is the total c.m. energy. The M_γ contribution to the differential cross section at scattering angles φ and θ ($z = \cos\theta$) for e^+ and e^- with opposite polarizations (s) perpendicular to the scattering plane is

$$\frac{d\sigma^{(0)}}{d\Omega} = \frac{\alpha^2}{16E^2}\left[1 + z^2 - s^2(1-z^2)\cos(2\varphi)\right] \equiv \frac{\alpha^2}{16E^2}W_0. \tag{10}$$

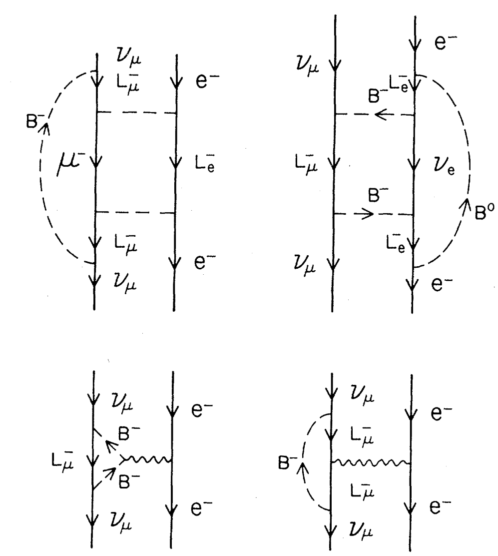

FIG. 2. Diagram for $\nu_\mu e$ elastic scattering.

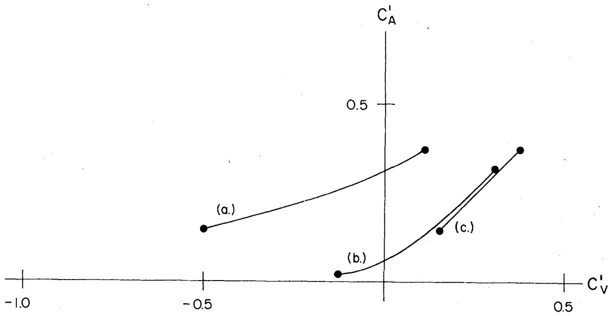

FIG. 3. The vector and axial-vector coupling constants for $\nu_\mu e \rightarrow \nu_\mu e$ for (curve a) $R=0.01$, (curve b) $R=1.0$, and (curve c) $R=100$. $f^2/4\pi$ is equal at the lower left of each curve to the minimum value for which universality holds to within 5% and is equal to 1.0 at the upper right. [$\ln(m^2/M^2)$ is taken to be 4.0.]

The effect of M_W is to multiply (10) by $1+\delta$ given by

$$\delta_z = -\frac{8\sqrt{2}G}{e^2}\frac{R-1}{\ln R}\frac{E^2}{W_0}z\,. \tag{11}$$

At SPEAR energies ($E=3.5$ GeV) and polarizations ($s^2=0.924$), the asymmetry in the cross section $(\delta_z/z)_{\max}$ for the present model is 2% when R is 1. This is twice the Weinberg-Salam value.[8] (The experimental prediction includes the two-photon contribution.[9]) This factor of 2 and the similar one in C_V and C_A for $\nu_e e$ scattering arise from the "isotopic spin" content of the two models; in the present model the heavy lepton has $I=0$ while ordinary leptons and the scalars (in coupling to leptons) have $I=\frac{1}{2}$. Hence the $e^+e^- \rightarrow \mu^+\mu^-$ process receives contributions from $B^0\overline{B}^0$ with both $I=0$ and $I=1$ while the Z exchange diagram in the Weinberg-Salam model has only $I=1$. If R in (11) is allowed to vary, $(\delta_z/z)_{\max}$ varies from 0.8% at $R=\frac{1}{10}$ to 8% at $R=10$.

(V) We may also calculate the polarization of the final μ^- in $e^+e^- \rightarrow \mu^+\mu^-$. Setting

$$P = \frac{d\sigma|_{h=+1} - d\sigma|_{h=-1}}{d\sigma|_{h=+1} + d\sigma|_{h=-1}}\,, \tag{12}$$

where h is the helicity of the μ^-, we have

$$P = \frac{4\sqrt{2}GE^2}{e^2}\frac{R-1}{\ln R}\left(1+\frac{2z}{W_0}\right)\,. \tag{13}$$

At $E=3.5$ GeV, $\cos^2\varphi=1$, $z=[(1-s^2)/(1+s^2)]^{1/2}$, and $R=1$, P is 3.1%. This is twice the value in the Weinberg-Salam model,[8]

$$P = -1.55(3\sin^2\theta_W - \cos^2\theta_W)\%\,, \tag{14}$$

at $\theta_W=0$. At the popular value[10] of $\theta_W=37°$ the Weinberg-Salam model gives $P=-0.7\%$ while for $R\neq 1$ (13) is multiplied by the factor $(R-1)/\ln R$ and therefore could be larger than 3.1%.

(VI) Finally we note that the SMWI does not give coherent Freedman scattering of neutrinos[11] off massive nuclei since, as pointed out by Segre,[1] the coupling to the quarks gives (for $\Delta S=0$) an effective $I=1$ vector current. Coherence requires $I=0$ while, at least for the application of this scattering to supernovas,[12] the targets are spin zero and therefore require vector currents.

In summary we have a small but nonzero neutral-current effect for $\nu_\mu e$ scattering, no neutral-current effect for $\nu_e e$ scattering, and a neutral-current effect in $e^+e^- \rightarrow \mu^+\mu^-$ which is approximately twice as large as that in the Weinberg-Salam theory.

Complete details of these calculations, a complete discussion of the limits imposed on the coupling $f^2/4\pi$ by universality (lower bound) and by the desire that the perturbation expansion give accurate answers (upper bound), a calculation of the weak correction to the muon magnetic moment, and specific predictions of neutrino-hadron scattering will be published elsewhere.

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[4]Bounds on $f^2/4\pi$ can also be obtained by considering the weak correction to the muon magnetic moment. This gives a lower bound since it is proportional to $(f^2/4\pi)(1/m^2) = G(f^2/4\pi)^{-1}$. Present magnetic-moment accuracy (1-standard-deviation error) together with the condition $\ln(m^2/M^2) \simeq 2$ actually gives a more restrictive lower bound, $f^2/4\pi \geq 0.15$, than the bound we

use.

[5]As discussed in Ref. 1, the SMWI gives an elastic cross section for ν_μ on hadrons ($2B^-$ exchange) which is too large by a factor of 3 when $R=1$. Agreement is achieved for $R\cong 3.5$ but as discussed in Ref. 1, agreement can also be achieved for $R=1$ by multiplying the B^- couplings in Eq. (1) by $(3)^{-1/8}$. In the latter case, the cross-section prediction of paragraph (II) is decreased, that of (III) is unchanged, and those of (IV) and (V) are multiplied by 3. These changes are in the direction of enhancing the difference between the SMWI and the Weinberg-Salam spontaneously broken gauge theory.

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Inelastic Neutrino Scatterings: Nonscaling Effects at Large ω*

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Stimulated by the SPEAR e^+e^--annihilation data and recent results of the neutrino experiment of Aubert *et al.* and Benvenuti *et al.*, we speculate on the various aspects of inelastic neutrino scatterings at large ω, or small x. We conjecture that there will be a breakdown of Bjorken scaling at small x, and, relatedly, an increase of the "excess" events at higher incident neutrino energies. The y dependence in the region of small x is also discussed.

It has been suggested for some time that a two-component point of view[1] should apply to inelastic lepton-hadron scattering: While the parton mechanism is dominant in the small-ω, or large-x, region, it is the generalized vector-dominance mechanism that is relevant in the large-ω, or small-x, region. It has also been emphasized[2] that because of the qualitative physical differences between the two mechanisms, a breakdown of Bjorken scaling is likely to occur when ω is sufficiently large, even though scaling is valid for small ω. The discovery of nonscaling behavior,[3] though maybe transitory, of the e^+e^--annihilation cross section has provided fresh impetus toward settling the question of scaling breakdown in electroproduction. Since a correlation between timelike (annihilation) and spacelike (inelastic scattering) regions is an intrinsic characteristic of the generalized vector-dominance viewpoint, a sizable breakdown[4] of Bjorken scaling in electroproduction, at sufficiently large ω, is naturally predicted.[5] It has also been suggested in a recent note[6] that another related phenomenon is a rising total photoproduction cross section beginning at a photon energy of 40 or 50 GeV.

Because of the underlying symmetry between electromagnetic and weak currents, the following picture emerges. In the electromagnetic case we have in the e^+e^--annihilation channel a nonscaling hadronic cross section that starts at a center-of-mass energy of roughly 3 GeV. Superimposed on the continuum of final states are also the newly discovered narrow 1^- resonances[7] at 3105 and 3695 MeV. In the weak-interaction case we should also have in the hypothetical $\nu_\mu\mu^+$-annihilation channel a nonscaling hadronic cross section starting at 1.5 to 2 GeV.[8] Superimposed on the continuum are narrow $1^\pm$ and 0^- (possibly 0^+ also) resonant states. These continuum states and resonant states will participate in inelastic neutrino scattering, in a manner similar in many respects to hadron-hadron scattering, at suffi-